

STROOCK-VARADHAN SUPPORT THEOREM FOR RANDOM EVOLUTION EQUATION IN BESOV-ORLICZ SPACES

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ABSTRACT. We consider the family of stochastic processes $X = \{X_t, t \in [0; 1]\}$, where X is the solution of the Itô stochastic differential equation

$$dX_t = \sigma(X_t, Z_t)dW_t + b(X_t, Y_t)dt$$

whose coefficients Lipschitzian depend on $Z = \{Z_t, t \in [0; 1]\}$ and $Y = \{Y_t, t \in [0; 1]\}$. We prove that the trajectories of X a.s. belong to the Besov-Orlicz space defined by the function $M(x) = e^{x^2} - 1$ and the modulus of continuity $\omega(t) = \sqrt{t \log(1/t)}$. The aim of this work is to characterize the support of the law X in this space.

1. INTRODUCTION

Let $X := \{X_t, t \in [0, 1]\}$ be the solution of the following random evolution equation, see [6]:

$$(1.1) \quad \begin{cases} dX_t = \sigma(X_t, Z_t)dW_t + b(X_t, Y_t)dt \\ X_0 = x \end{cases}$$

where $x \in \mathbb{R}^d$ is the starting point and (W_t) is the standard Brownian motion taking values in $\mathbb{R}^d$ defined on some well filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$.

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We suppose that Y and Z are progressively measurable random process belong to $\mathbf{L}^q(q \geq 1)$ and their respectively topological support are a compact subset in $B_M^{1/2,0}$ which is an separable subset of $B_M^{1/2}$. Furthermore, W is independent of (Y, Z) and we always assume that the coefficients $\sigma : \mathbb{R}^d \times \mathbb{R}^l \longrightarrow \mathbb{R}^d \otimes \mathbb{R}^k$ and $b : \mathbb{R}^d \times \mathbb{R}^m \longrightarrow \mathbb{R}^d$ satisfy the following hypotheses:

(H₀) : The function b is jointly measurable in (x, y) and there exists a constant $K > 0$ such that:

$$\begin{aligned} |b(x, y)| &\leq K(1 + |x|), \quad \forall (x, y) \in \mathbb{R}^d \times \mathbb{R}^m \\ |b(x, y) - b(x', y')| &\leq K(|x - x'| + |y - y'|), \quad \forall x, x' \in \mathbb{R}^d; y, y' \in \mathbb{R}^m. \end{aligned}$$

(H₁) : The function σ is jointly measurable in (x, z) and there exists $K > 0$ such that:

$$\begin{aligned} |\sigma(x, z)| &\leq K, \quad \forall (x, z) \in \mathbb{R}^d \times \mathbb{R}^l \\ |\sigma(x, z) - \sigma(x', z')| &\leq K(|x - x'| + |z - z'|), \quad \forall x, x' \in \mathbb{R}^d; z, z' \in \mathbb{R}^l. \end{aligned}$$

(H₂) : b is $\mathcal{C}^1$, σ is $\mathcal{C}^2$ and there exists some positive constant K such that:

$$\max \left\{ \text{tr}(\sigma \sigma^*(x, z)), \langle x, b(x, y) \rangle \right\} \leq K(1 + |x|^2), \quad \forall x \in \mathbb{R}^d; y \in \mathbb{R}^m; z \in \mathbb{R}^l.$$

Here $\langle x, y \rangle$ is the Euclidean inner product in $\mathbb{R}^d$.

Let $\Omega = \mathcal{C}([0, 1], \mathbb{R}^d)$ be the set of continuous functions from $[0, 1]$ to $\mathbb{R}^d$ equipped with the usual topology of uniform convergence defined by the norm $\|f\|_\infty = \sup_{0 \leq t \leq 1} |f(t)|$ and let $\mathcal{H} = \{h(t) = \int_0^t \dot{h}(s) ds, \dot{h} \in \mathbf{L}^2([0, 1])\}$ be the Cameron-Martin space, that is the subset of Ω and for all $h \in \mathcal{H}$, $\psi \in \text{supp}Y$, and $\chi \in \text{supp}Z$, $S(h, \psi, \chi)$ is the solution of ordinary differential equation(ODE):

$$(1.2) \quad \begin{cases} dS(h, \psi, \chi)_t = (b(S(h, \psi, \chi)_t, \psi_t) - \frac{1}{2}(\nabla_x \sigma) \sigma(S(h, \psi, \chi)_t, \chi_t)) dt \\ \quad \quad \quad + \sigma(S(h, \psi, \chi)_t, \chi_t) \dot{h}_t dt \\ S(h, \psi, \chi)_0 = x \end{cases}$$

where $\text{supp}Y$ is the support of the distribution of Y , $\text{supp}Z$ is the support of the distribution of Z .

In this paper, we characterize the support theorem of X in Besov-Orlicz spaces $B_M^{1/2}$. The aim is to prove the characterization of the support $P \circ X^{-1}$ as the closure $\mathbb{S}$ of the set $\{S(h, \psi, \chi), h \in \mathcal{H}, \psi \in \text{supp}Y \text{ and } \chi \in \text{supp}Z\}$ in $B_M^{1/2,0}$. We use the approximation theorem of the stochastic system adapted linear interpolation of ω^n of ω and Millet result, see [8], to prove our result. So, we check the convergence in probability of $\|S(\omega^n, \psi, \chi) - X\|_{\omega_{1/2}, M, \infty}$ and

of $\|X(\omega^n - \omega + h) - S(h, \psi, \chi)\|_{\omega_{1/2}, M, \infty}$ to 0 where the law of transformation T_n of ω defined by $T_n(\omega) = \omega^n - \omega + h$ is absolutely continuous with respect to $\mathbb{P}$.

As in [7], notice that the density of the set of bounded functions in $\mathbb{L}^2$ and the continuity of the application $(h, \psi, \chi) \mapsto S(h, \psi, \chi) \in B_M^{1/2}$ also allow us to show that the adhesions of the sets $\{S(h, \psi, \chi), h \in \mathcal{H}, h \in \mathbb{L}^\infty, \psi \in \text{supp} Y \text{ and } \chi \in \text{supp} Z\}$ and $\mathbb{S}$ coincide in $B_M^{1/2, 0}$.

In Varadhan-Stroock [11], the support Theorem was first proved for the case $Y \equiv 0$ and $Z \equiv 0$ for the equation (1.1) in finite dimensional state spaces and with finite dimensional Wiener processes. Many authors have tried to extend their results for the same case but by different methods that we have also adopted. Mellouk [7] have used also approximation methods of the stochastics differential equation to prove the support Theorem in Besov-Orlicz spaces see [3, 4, 8] for the Hölder norm see also [1] in separable Hilbert space driven by Wiener processes without infinitesimal generators.

We know that the norm of $B_M^{1/2}$ is stronger than the function höderian space order $\alpha < 1/2$. This result makes it possible to generalize this case and the support Theorem in Andriatahina and al. [2].

The rest of this paper is organized as follows. In Section 2, we introduce some notions on the Besov-Orlicz spaces and we will show that the trajectory of X is almost surely in $B_M^{1/2, 0}$. In Section 3, we will give our main result and some approximations in general of X solution of (1.1). Finally, Section 4 will be devoted to the proof of the main result.

Throughout this paper, C_p is a positive constant depending on some parameter p , and C is a constant depending on no specific parameter (except x_0 and K), whose value may be different from line to line by convention.

2. REGULARITY OF THE SOLUTIONS IN THE BESOV-ORLICZ SPACE

In this section, we give some notions on the Besov-Orlicz space. For more details of this space, the reader may consult [5, 9, 10]. Let $M(x) = e^{x^2} - 1$ and for all continuous function $f : [0, 1] \rightarrow \mathbb{R}^d$, the Orlicz's norm is defined by

$$\|f\|_M = \inf \left\{ \theta > 0, \frac{1}{\theta} \left[1 + \int_0^1 M(\theta |f(t)|) dt \right] \right\}.$$

The modulus of continuity of f in Orlicz norm is

$$\omega_M(f, \delta) = \sup_{0 \leq h \leq \delta} \|\Delta_h f\|_M,$$

where

$$\Delta_h f(x) = 1_{[0, 1-\delta]}(x)[f(x+h) - f(x)], \forall h \in [0, 1].$$

Let $\omega_{1/2}(t) = \sqrt{t(1 + \log(1/t))}$, for all $t \in [0, 1]$. Note that $B_M^{1/2}$ as the space of continuous functions $f : [0, 1] \rightarrow \mathbb{R}^d$ such that

$$\|f\|_{\omega_{1/2}, M, \infty} = \|f\|_M + \sup_{0 \leq t \leq 1} \frac{w_M(f, t)}{\omega_{1/2}(t)} < \infty.$$

Notice that there is a isomorphism between $B_M^{1/2}$ and some spaces of the sequences, see [5]. Let $f_0, f_1 = f(1) - f(0)$, and for $0 \leq j, 1 \leq k \leq 2^j$,

$$f_{j,k} = 2^{j/2} \left[f\left(\frac{2k-1}{2^{j+1}}\right) - \frac{1}{2} \left(f\left(\frac{2k}{2^{j+1}}\right) + f\left(\frac{2k-2}{2^{j+1}}\right) \right) \right].$$

Let $P_0(t) = 1, P_1(t) = t$ and $P_{j,k} = \int_0^1 \chi_{j,k}(s) ds$ be the basic function of Schauder where $\{\chi_{j,k}, j \geq 0, 1 \leq k \leq 2^j\}$ is Harr's system defined by $\chi_1(t) = 1$ and $\chi_{j,k} = 2^{j/2} (1_{[\frac{k-1}{2^j}, \frac{2k-1}{2^{j+1}}]} - 1_{[\frac{2k-1}{2^{j+1}}, \frac{k}{2^j}]})$. Then $f(t) = f_0 P_0(t) + f_1 P_1(t) + \sum_{j,k} f_{j,k} P_{j,k}(t)$.

Theorem 2.1. [5]

i) For $p_0 \geq 1, f$ belongs to $B_M^{1/2}$ if and only if the norm

$$\max(|f_0|, |f_1|, \sup_{p \geq p_0} \sup_{j \geq 0} \frac{2^{-\frac{j}{p}}}{\sqrt{p}} (j \vee 1)^{-\alpha} \|f_{j,\cdot}\|_p) < \infty$$

ii) f belongs at $B_M^{1/2,0}$ (separable subspace of $B_M^{1/2}$) if and only if

$$\lim_{p \vee j \rightarrow \infty} \frac{2^{-\frac{j}{p}}}{\sqrt{p}} j^{-\frac{1}{2}} \|f_{j,\cdot}\|_p = 0, \text{ where } \|f_{j,\cdot}\|_p = \left(\sum_{k=1}^{2^j} |f_{j,k}|^p \right)^{\frac{1}{p}}.$$

Throughout, we need the following inequality

$$(2.1) \quad \sup_p 2^{-\frac{j}{p}} \|f_{j,\cdot}\|_p \leq \sup_k |f_{j,k}|.$$

The proof of (2.1) can be found in [5].

In order to prove Theorem 2.2 by using the similar arguments in [7] we also need the following lemmas.

Lemma 2.1. Under the assumptions $(H_0) - (H_2)$, there exists an arbitrary constant $M > 0$ which depends on x_0, K and p such that for all $p \geq 2$,

$$(2.2) \quad \mathbb{E}\|X\|^p \leq M.$$

Proof. For $p = 2$ and $p = 3$, Itô's formula applied respectively to the functions x^2 and x^3 , and Gronwall's inequality imply that:

$$\mathbb{E}||X||^2 \leq (||x_0||^2 + 3K)e^{3K} \quad \text{and} \quad \mathbb{E}||X||^3 \leq (||x_0||^3 + 3K + 1)e^{9K/2}.$$

Suppose that for all integer $n \geq 1$, we have the recurrence relations

$$(2.3) \quad \mathbb{E}||X||^{2n} \leq Ce^{\frac{3n(n+1)}{2}K}$$

and

$$(2.4) \quad \mathbb{E}||X||^{2n+1} \leq Ce^{\frac{3n(n+2)}{2}K}.$$

By Itô's formula applied to the function $f(x, y, z) = ||x||^{p+2}$, for all p and $x \in \mathbb{R}^d$, $y \in \mathbb{R}^m$ and $z \in \mathbb{R}^l$, we have:

$$\begin{aligned} df(X_t, Y_t, Z_t) &= \langle \nabla f(X_t, Y_t, Z_t), \sigma(X_t, Z_t)dW_t \rangle + \langle b(X_t, Y_t), \nabla f(X_t, Y_t, Z_t) \rangle dt \\ &+ \frac{1}{2} \sum_{i,j} (\sigma\sigma^*)_{ij}(X_t, Z_t) \frac{\partial^2}{\partial x_i \partial x_j} f(X_t, Y_t, Z_t) dt \\ &= \langle \nabla f(X_t, Y_t, Z_t), \sigma(X_t^\varepsilon, Z_t)dW_t \rangle + \mathcal{L}(X_t, Y_t, Z_t)dt, \end{aligned}$$

$\mathcal{L}$ is the infinitesimal generator of X_t .

By the condition $(\mathbf{H}_2)$,

$$\begin{aligned} \mathcal{L}(x, y, z) &= (p+2)||x||^p \langle x, b(x, y) \rangle + (p+2)||x||^p \text{tr}(\sigma\sigma^*)(x, z) \\ &+ p(p+2)||x||^{p-2} \sum_i (\sigma\sigma^*)_{ii} x_i^2 \\ &\leq (p+2)||x||^p [p + K(1 + ||x||^2)]. \end{aligned}$$

As $\mathbb{E}(\int_0^t \langle \nabla f(X_s, Y_s, Z_s), \sigma(X_s, Z_s)dW_s \rangle) = 0$, and Fubini's Theorem, we have that:

$$\mathbb{E}(|X_t|^{p+2}) \leq ||x_0||^p + (p+2)(p+K) \int_0^t \mathbb{E}||X_s||^p ds + (p+2)K \int_0^t \mathbb{E}||X_s||^{p+2} ds.$$

By (2.3), (2.4) and Gronwall's inequality, there exists a constant $C > 0$ depending on x_0, p, K such that we have respectively:

$$\mathbb{E}||X||^{2n+2} \leq Ce^{\frac{3(n+1)(n+2)}{2}K} \quad \text{and} \quad \mathbb{E}||X||^{2n+3} \leq Ce^{\frac{3(n+1)(n+3)}{2}K}.$$

□

Lemma 2.2. *Under the assumptions $(\mathbf{H}_0) - (\mathbf{H}_1)$ and (2.2), there exists a constant $M > 0$ such that for all $s, t \in [0, 1]$ and $p \geq 2$,*

$$(2.5) \quad \mathbb{E}(\|X_t - X_s\|^p) \leq M(2K)^p p^{p/2} |t - s|^{p/2}.$$

The proof of Lemma 2.2 is given in [7] by using the Burkholder's inequality and the isometry for $p \in \{2, 3\}$. For $2 \leq p \leq n-1$ ($n \geq 4$), Itô's formula and the recurrence hypothesis (2.5) allow us to show the result. But, in our situation, we must add Lemma 2.1 because of the linear growth of b in the assumption $(\mathbf{H}_0)$.

Now, we have the following theorem.

Theorem 2.2. *Suppose that the assumptions $(\mathbf{H}_0) - (\mathbf{H}_2)$ are satisfied. Let X be the solution of (1.1). Then*

$$\mathbb{P}(X \in B_M^{\frac{1}{2}, 0}) = 1.$$

Proof. The proof is to show that the solution set (1.1) satisfying the conditions i) and ii) of the Theorem 2.1.

Let $\alpha < \frac{1}{2}$, for all $j \geq 0$ and for all $p \geq p_0$, note that

$$\delta_{j,p}^\alpha(X) := \frac{2^{-\frac{j}{p}}}{\sqrt{p}} (j \vee 1)^{-\alpha} \|X_{j,\cdot}\|_p \quad \text{and} \quad \theta_{j,p}(X) := \frac{2^{-\frac{j}{p}}}{\sqrt{p}} (j \vee 1)^{-1/2} \|X_{j,\cdot}\|_p.$$

We will show that for all p_0 that we will specify later and for all $\alpha < \frac{1}{2}$, we have

$$(2.6) \quad \sup_{j \geq 0} \sup_{p \geq p_0} \delta_{j,p}^\alpha(X) < \infty \quad p.s.$$

$$(2.7) \quad \lim_{j \vee p \rightarrow \infty} \theta_{j,p} = 0 \quad p.s.$$

Let $\lambda > 0$, and note that

$$S_\lambda(\alpha) = \sum_{j \geq 0} \sum_{p \geq p_0} \mathbb{P}\left\{\delta_{j,p}^\alpha(X) > \lambda\right\}.$$

Using Tchebychev inequality and Lemma 2.2, we have:

$$\begin{aligned} \mathbb{P}\left\{\delta_{j,p}^\alpha(X) > \lambda\right\} &\leq \frac{2^{-j}}{p^{p/2} \lambda^p (j \vee 1)^{\alpha p}} \sum_{k=1}^{2^j} \mathbb{E} |X_{j,k}|^p \\ &\leq \frac{2^{-j} 2^{jp/2} 2^p}{\lambda^p p^{p/2} (j \vee 1)^{\alpha p}} \sum_{k=1}^{2^j} \sup_{|t-s| \leq 2^{-(j+1)}} \mathbb{E} |X_t - X_s|^p \end{aligned}$$

$$\leq \left(\frac{M}{\lambda}\right)^p \frac{1}{(j \vee 1)^{\alpha p}}.$$

Choosing $p_0 \geq \frac{1}{\alpha}$ and $\lambda > 0$ large enough, we deduce the convergence of the series $S_\lambda(\alpha)$ and Borel-Cantelli's Lemma leads to the validity of (2.6).

By (2.1), we have $\sup_{j \geq 0} \theta_{j,p}(X) \leq \frac{1}{\sqrt{p}} \sup_{j \geq 0} \sup_k |X_{j,k}|$ and by Borel-Cantelli's Lemma, it suffices to show the convergence of the series $R_\lambda = \sum_j \mathbb{P}\left\{(j \vee 1)^{-1/2} \sup_k |X_{j,k}| > \lambda\right\}$ to prove (2.7). Indeed, the exponential inequality on the stochastic integrals leads to the existence of the positive constants C' and C'' such that for all $\lambda > 0$ large enough,

$$\begin{aligned} \mathbb{P}\left\{(j \vee 1)^{-1/2} \sup_k |X_{j,k}| > \lambda\right\} &= \mathbb{P}\left\{\sup_k |X_{j,k}| > \lambda(j \vee 1)^{1/2}\right\} \\ &= \mathbb{P}\left\{\left|X_{\frac{k}{2j+1}} - X_{\frac{k-1}{2j+1}}\right| > \lambda(j \vee 1)^{1/2} 2^{-(j/2)-1}\right\} \\ &\leq C' \exp\left(-C'' \frac{\lambda^2(j \vee 1)}{K^2}\right). \end{aligned}$$

So, $\sup_{j \geq 0} \theta_{j,p}(X) < \infty$. This proves relation (2.7), completing the proof of the Theorem. $\square$

3. MAIN RESULT AND APPROXIMATIONS

We want to show that the following Theorem characterizes the support of the law of X solution of (1.1) in $B_M^{1/2,0}$.

Theorem 3.1. *If σ is of class C^2 , bounded together with its partial derivatives of order one and two, and b is globally Lipschitz, the support of the law of X in $B_M^{\frac{1}{2},0}$ is the closure of the set $\mathcal{S} = \{S(h, \psi, \chi), h \in \mathcal{H}, \psi \in \text{supp} Y, \chi \in \text{supp} Z\}$ where $S(h, \psi, \chi)$ is given by (1.2).*

(L) : Given an integer $n > 0$, let $D_n = \{i2^{-n}; 0 \leq i \leq 2^n\}$ be the set of n -dyadic points. ω^n is the linear interpolation adapted to ω defined by $\dot{\omega}_t^n = 2^n[\omega(k2^{-n}) - \omega((k+1)2^{-n})]$ for $t \in [k2^{-n}, (k+1)2^{-n}[$, $k \geq 1$ and 0 otherwise. The approximation of stochastic integrals by the Riemann sum's imply that

$X^n(\omega)$ is solution of the Itô's stochastic differential equation(SDE)

$$(3.1) \quad X_t^n = x_0 + \int_0^t \sigma(X_s^n, Z_s) dW_s - \int_0^t \sigma(X_s^n, Y_s) \dot{\omega}_s^n ds + \int_0^t \sigma(X_s^n, Z_s) \dot{h}_s ds + \int_0^t b(X_s^n, Y_s) ds.$$

Let X^n be the function $X^n : \omega^n \mapsto X(\omega^n)$ and $X(\omega^n)$ is the function $X(\omega^n) : (Y, Z) \mapsto X(\omega^n)(Y, Z) = X^n$ solution of (3.1). Let $S(\omega^n, \psi, \chi)$ satisfies:

$$\begin{aligned} S(\omega^n, \psi, \chi)_t &= S(\omega^n, \psi, \chi)_0 + \int_0^t \sigma(S(\omega^n, \psi, \chi)_s, \chi_s) \dot{\omega}_s^n ds + \int_0^t (b(S(\omega^n, \psi, \chi)_s, \psi_s) \\ &\quad - \frac{1}{2}(\nabla_x \sigma) \sigma(S(\omega^n, \psi, \chi)_s, \chi_s)) ds. \end{aligned}$$

By Proposition 2.1 in [8], Theorem 2.2 will be obtained by results of the following convergences. For all $\delta > 0$,

$$(3.2) \quad \lim_n \mathbb{P}(\|S(\omega^n, \psi, \chi) - X(\omega)\|_{\omega_{1/2}, M, \infty} > \delta) = 0,$$

$$(3.3) \quad \lim_n \mathbb{P}(\|X(\omega - \omega^n + h) - S(h, \psi, \chi)\|_{\omega_{1/2}, M, \infty} > \delta) = 0.$$

The sequences of the processes (X^n) and $(S(\omega^n, \psi, \chi))$ are respectively the special cases of (K^n) and $(\tilde{K}^n)$, solutions of the following stochastic differential equations

$$\begin{aligned} K_t^n &= x + \int_0^t F(K_s^n, Z_s) dW_s + \int_0^t G(K_s^n, Z_s) \dot{\omega}_s^n ds + \int_0^t H(K_s^n, Z_s) \dot{h}_s ds \\ (3.4) \quad &+ \int_0^t I(K_s^n, Z_s) ds + \int_0^t B(K_s^n, Y_s) ds, \end{aligned}$$

and

$$\begin{aligned} \tilde{K}_t^n &= x + \int_0^t F(\tilde{K}_s^n, \chi_s) dW_s + \int_0^t G(\tilde{K}_s^n, \chi_s) \dot{\omega}_s^n ds + \int_0^t H(\tilde{K}_s^n, \chi_s) \dot{h}_s ds \\ (3.5) \quad &+ \int_0^t I(\tilde{K}_s^n, \chi_s) ds + \int_0^t B(\tilde{K}_s^n, \psi_s), \end{aligned}$$

where $F, G, H, I : \mathbb{R}^d \times \mathbb{R}^l \longrightarrow \mathbb{R}^d \otimes \mathbb{R}^k$ and $B : \mathbb{R}^d \times \mathbb{R}^m \longrightarrow \mathbb{R}^d$ are Lipschitz functions, G of class $\mathcal{C}^2$, bounded together with its partial derivatives of order

one and two. Let (L_t) be the solution of the stochastic differential equation:

$$(3.6) \quad L_t = x + \int_0^t [F(L_s, Z_s) + G(L_s, Z_s)] dW_s + \int_0^t H(L_s, Z_s) \dot{h}_s ds + \int_0^t [B(L_s, Y_s) + I(L_s, Z_s)] ds + \int_0^t \nabla G(L_s, Z_s) [F(L_s, Z_s) + \frac{1}{2} G(L_s, Z_s)] ds.$$

We consider the following convergences which have the particular cases of (3.2) for $F = 0; G = \sigma; H = 0; I = -\frac{1}{2}(\nabla \sigma)\sigma$ and $B = b$, and (3.3) for $F = \sigma; G = -\sigma; H = \sigma, I = 0$ and $B = b$. For all $\varepsilon > 0$,

$$(3.7) \quad \lim_n \mathbb{P}(\|K^n - L\|_{\omega_{1/2}, M, \infty} > \varepsilon) = 0$$

and

$$(3.8) \quad \lim_n \mathbb{P}(\|\tilde{K}^n - L\|_{\omega_{1/2}, M, \infty} > \varepsilon) = 0.$$

Using a similar estimate to Lemma 2.2, there exist a constant $C_p > 0$ such that for all $s, t \in [0, 1]$ and $p \geq 2$,

$$(3.9) \quad \mathbb{E}(|L_t - L_s|^{2p}) \leq C_p |t - s|^p.$$

We suppose that for $t \in [0, 1]$ and $p \geq 2$,

$$(3.10) \quad \mathbb{E}|\psi_t - Y_t|^{2p} < \infty,$$

$$(3.11) \quad \mathbb{E}|\chi_t - Z_t|^{2p} < \infty.$$

Furthermore, for all $t \in [0, 1]$ and let $\bar{t}_n = \frac{k}{2^n}$, $\underline{t}_n = \frac{k-1}{2^n} \vee 0$, we have

$$(3.12) \quad \lim_{n \rightarrow \infty} \sup_{t \in [0, 1]} \mathbb{E}|Z_t - Z_{\underline{t}_n}|^{2p} = 0.$$

By triangular inequality and (3.11), we have

$$(3.13) \quad \sup_{t \in [0, 1]} |\chi_t - \chi_{\underline{t}_n}|^{2p} < \infty.$$

4. PROOF OF THE MAIN RESULT

The following Proposition is derived from what we said previously.

Proposition 4.1. *Let (K_t^n) , $(\tilde{K}_t^n)$ and (L_t) be defined by (3.4), (3.5) and (3.6) respectively with a function h such that $\dot{h}$ is bounded. For any $\varepsilon > 0$, $\lim_n \mathbb{P}(\|K^n - L\|_{\omega_{1/2}, M, \infty} > \varepsilon) = 0$ and $\lim_n \mathbb{P}(\|\tilde{K}^n - L\|_{\omega_{1/2}, M, \infty} > \varepsilon) = 0$.*

Proof. The proof is done in two cases. We suppose that we have (L). Let F, G, H, I, B be the bounded functions and B satisfy the linear growth condition. Let $\varepsilon > 0$, $\gamma < 1$ and $p \geq 1$, note that

$$R_n := \mathbb{P}(\sup_{j \geq n\gamma} \sup_p \theta_{j,p}(K^n - L) > \varepsilon)$$

and

$$T_n := \mathbb{P}(\sup_{j \geq n\gamma} \sup_p \theta_{j,p}(\tilde{K}^n - L) > \varepsilon).$$

To show (3.7) and (3.8), it suffices to prove the convergences to 0 of R_n and T_n . Indeed, by (2.1) and Tchebychev's inequality,

$$\begin{aligned} R_n &\leq \sum_{j \leq n\gamma} \sum_{1 \leq k \leq 2^j} \mathbb{P}(|(K^n - L)_{j,k}| > \varepsilon(j \wedge 1)^{1/2}) \\ (4.1) \quad &\leq \frac{1}{\varepsilon^{2p}} \sum_{j \leq n\gamma} \frac{2^j 2^{jp} 2^{2p}}{j^p} \sup_{|t-s| \leq 2^{-(j+1)}} \mathbb{E}(|(K_t^n - L_t) - (K_s^n - L_s)|^{2p}). \end{aligned}$$

For all $s, t \in [0, 1]$,

$$\mathbb{E}(|(K_t^n - L_t) - (K_s^n - L_s)|^{2p}) \leq C_p \sum_{i=0}^4 R_i$$

where

$$\begin{aligned} R_0 &= \mathbb{E}\left(\left|\int_s^t [F(K_u^n, Z_u) - F(L_u, Z_u)] dW_u\right|^{2p}\right), \\ R_1 &= \mathbb{E}\left(\left|\int_s^t G(K_u^n, Z_u) \dot{\omega}_u^n - G(L_u, Z_u) dW_u\right|^{2p}\right) \\ R_2 &= \mathbb{E}\left(\left|\int_s^t [H(K_u^n, Z_u) - H(L_u, Z_u)] \dot{h}_u du\right|^{2p}\right) \\ R_3 &= \mathbb{E}\left(\left|\int_s^t [(I(K_u^n, Z_u) + B(K_u^n, Y_u)) - (I(L_u, Z_u) + I(L_u, Y_u))] du\right|^{2p}\right) \\ R_4 &= \mathbb{E}\left(\left|\int_s^t \nabla G(L_u, Z_u) [F(L_u, Z_u) + \frac{1}{2} G(L_u, Z_u)] du\right|^{2p}\right). \end{aligned}$$

By Burkholder's inequality, Schwartz's inequality and Fubini's Theorem together, we have

$$R_0 + R_2 + R_3 \leq C_p K^{2p} (|t - s|^p + 1) \left(\int_s^t \mathbb{E} |K_u^n - L_u|^2 du \right)^p.$$

By Proposition 3.3 in [2], for large enough n and the hypothesis (L), there exists an positive constant C''' such that $\mathbb{E} |K_u^n - L_u|^2 \leq C'''$. Thus,

$$R_0 + R_2 + R_3 \leq C_p K^{2p} (|t - s|^p + |t - s|^{2p}).$$

Since $(\nabla G)F$ and $(\nabla G)G$ are Lipschitzian,

$$R_4 \leq C_p K^{2p} |t - s|^{2p}.$$

We have

$$R_1 \leq C_p (R_{1,0} + R_{1,1}),$$

where

$$\begin{aligned} R_{1,0} &= \mathbb{E} \left(\left| \int_s^t |G(K_u^n, Z_u) - G(K_{\underline{u}_n}^n, Z_{\underline{u}_n})| |\dot{\omega}_u^n| du \right|^{2p} \right) \\ R_{1,1} &= \mathbb{E} \left(\left| \int_s^t G(K_{\underline{u}_n}^n, Z_{\underline{u}_n}) \dot{\omega}_s^n du - G(L_u, Z_u) dW_u \right|^{2p} \right). \end{aligned}$$

Let $a > 1$, $b > 1$ such that $\frac{1}{a} + \frac{1}{b} = 1$. By Hölder's inequality and Fubini's Theorem, we have

$$\begin{aligned} R_{1,0} &\leq |t - s|^{2p-1} \int_s^t \left\{ \mathbb{E} |G(K_u^n, Z_u) - G(K_{\underline{u}_n}^n, Z_{\underline{u}_n})|^{2pa} \right\}^{\frac{1}{a}} \left\{ \mathbb{E} |\dot{\omega}_u^n|^{2pb} \right\}^{\frac{1}{b}} du \\ &\leq K^{2p} |t - s|^{2p-1} 2^{np} \int_s^t \left\{ \mathbb{E} (|K_u^n - K_{\underline{u}_n}^n| + |Z_u - Z_{\underline{u}_n}|)^{2pa} \right\}^{\frac{1}{a}} du \\ &\leq C_p K^{2p} |t - s|^{2p-1} 2^{np} \int_s^t \left\{ \mathbb{E} |K_u^n - K_{\underline{u}_n}^n|^{2pa} + \mathbb{E} |Z_u - Z_{\underline{u}_n}|^{2pa} \right\}^{\frac{1}{a}} du. \end{aligned}$$

By (4.7) and (3.12), we have

$$R_{1,0} \leq C_p K^{2p} |t - s|^{2p}.$$

By (3.9), we obtain

$$R_{1,1} \leq C_p K^{2p} |t - s|^p.$$

Therefore,

$$R_1 \leq C_p K^{2p} (|t - s|^p + |t - s|^{2p}).$$

Finally, we have for all $m \in]\frac{1}{2}, 1[$

$$\mathbb{E}(|(K_t^n - L_t) - (K_s^n - L_s)|^{2p}) \leq CC_p K^{2p} (2^{-(2m-1)np} |t - s|^{p(1-m)}).$$

This last inequality and (4.1) imply

$$R_n \leq C \frac{C_p K^{2p}}{\varepsilon^{2p}} \sum_{j \leq n\gamma} j^{-p} (2^{j-(n-j)p(2m-1)})$$

and finish the proof of (3.7) when $p(2m-1) > \frac{\gamma}{1-\gamma}$ and n going to ∞ .

By (2.1) and Tchebychev's inequality, we have

$$(4.2) \quad T_n \leq \frac{1}{\varepsilon^{2p}} \sum_{j \leq n\gamma} \frac{2^j 2^{jp} 2^{2p}}{j^p} \sup_{|t-s| \leq 2^{-(j+1)}} \mathbb{E}(|(\tilde{K}_t^n - L_t) - (\tilde{K}_s^n - L_s)|^{2p}).$$

For all $s, t \in [0, 1]$,

$$\mathbb{E}(|(\tilde{K}_t^n - L_t) - (\tilde{K}_s^n - L_s)|^{2p}) \leq C_p \sum_{i=0}^4 T_i$$

where

$$\begin{aligned} T_0 &= \mathbb{E}\left(\left|\int_s^t [F(\tilde{K}_u^n, \chi_u) - F(L_u, Z_u)] dW_u\right|^{2p}\right), \\ T_1 &= \mathbb{E}\left(\left|\int_s^t G(\tilde{K}_u^n, \chi_u) \dot{\omega}_u^n - G(L_u, Z_u) dW_u\right|^{2p}\right) \\ T_2 &= \mathbb{E}\left(\left|\int_s^t [H(\tilde{K}_u^n, \chi_u) - H(L_u, Z_u)] \dot{h}_u du\right|^{2p}\right) \\ T_3 &= \mathbb{E}\left(\left|\int_s^t [(I(\tilde{K}_u^n, \chi_u) + B(\tilde{K}_u^n, \psi_u)) - (I(L_u, Z_u) + I(L_u, Y_u))] du\right|^{2p}\right) \\ T_4 &= \mathbb{E}\left(\left|\int_s^t \nabla G(L_u, Z_u) [F(L_u, Z_u) + \frac{1}{2} G(L_u, Z_u)] du\right|^{2p}\right). \end{aligned}$$

Burkholder's inequality, Schwartz's inequality and Fubini Theorem together imply

$$T_0 + T_2 \leq C_p K^{2p} \left(\int_s^t \mathbb{E}(|(\tilde{K}_u^n - L_u) + (\chi_u - Z_u)|^2 du) \right)^p.$$

By Schwartz's inequality and Fubini Theorem,

$$T_3 \leq C_p K^{2p} |t - s|^p \left(\int_s^t \mathbb{E}(|(\tilde{K}_u^n - L_u) + (\chi_u - Z_u) + (Y_u - \psi_u)|^2 du) \right)^p.$$

The estimate T_4 is none other than R_4 we have already seen above.

We have

$$T_1 \leq C_p (T_{1,0} + T_{1,1}),$$

where

$$\begin{aligned} T_{1,0} &= \mathbb{E} \left(\left| \int_s^t |G(\tilde{K}_u^n, \chi_u) - G(\tilde{K}_{\underline{u}_n}^n, \chi_{\underline{u}_n})| |\dot{\omega}_u^n| du \right|^{2p} \right) \\ T_{1,1} &= \mathbb{E} \left(\left| \int_s^t G(\tilde{K}_{\underline{u}_n}^n, \chi_{\underline{u}_n}) \dot{\omega}_s^n du - G(L_u, Z_u) dW_u \right|^{2p} \right). \end{aligned}$$

Let $a > 1$, $b > 1$ satisfying $\frac{1}{a} + \frac{1}{b} = 1$. By Hölder's inequality and Fubini's Theorem, we have

$$\begin{aligned} T_{1,0} &\leq |t - s|^{2p-1} \int_s^t \left\{ \mathbb{E} |G(\tilde{K}_u^n, \chi_{\underline{u}_n}) - G(\tilde{K}_{\underline{u}_n}^n, \chi_u)|^{2pa} \right\}^{\frac{1}{a}} \left\{ \mathbb{E} |\dot{\omega}_u^n|^{2pb} \right\}^{\frac{1}{b}} du \\ &\leq K^{2p} |t - s|^{2p-1} 2^{np} \int_s^t \left\{ \mathbb{E} (|\tilde{K}_u^n - \tilde{K}_{\underline{u}_n}^n| + |\chi_u - \chi_{\underline{u}_n}|)^{2pa} \right\}^{\frac{1}{a}} du \\ &\leq C_p K^{2p} |t - s|^{2p-1} 2^{np} \int_s^t \left\{ \mathbb{E} |\tilde{K}_u^n - \tilde{K}_{\underline{u}_n}^n|^{2pa} + |\chi_u - \chi_{\underline{u}_n}|^{2pa} \right\}^{\frac{1}{a}} du. \end{aligned}$$

By the same reasoning as (4.7), we obtain

$$\mathbb{E}(|\tilde{K}_u^n - \tilde{K}_{\underline{u}_n}^n|^p) \leq C_p K^p p^{p/2} 2^{-n(p/2)}.$$

Therefore, (3.13) and the last inequality imply

$$T_{1,0} \leq C_p K^{2p} |t - s|^{2p}.$$

By (3.9), $T_{1,1}$ and $R_{1,1}$ have the same estimate, and the Proposition 3.4 in [2] and the hypothesis (L) prove that there exists a positive constant C'''' such that $\mathbb{E}|\tilde{K}_u^n - L_u|^2 \leq C''''$ and more (3.10) and (3.11), for all $\frac{1}{2} < m < 1$

$$\mathbb{E}(|(\tilde{K}_t^n - L_t) - (\tilde{K}_s^n - L_s)|^{2p}) \leq C C_p K^{2p} (2^{-(2m-1)np} |t - s|^{p(1-m)}).$$

Finally, taking this last inequality in (4.2), for a large enough n , we obtain the convergence to 0 of T_n .

Now, we suppose the case where $D_n = \{\sup_n |\dot{\omega}_t^n| \leq 2^{5n/8}\}$. For all $p \geq 1$, $\mathbb{P}(D_n^c) \leq C_p 2^{-n((p/8)-1)}$. We can now study the convergences in probability on D_n . Thus, we will first show that there exist a constant $M > 0$ such that for all $p \geq 2$ and $0 \leq s < t \leq 1$, we have the following estimates

$$(4.3) \quad \mathbb{E}(\mathbf{1}_{D_n} |K_t^n - K_s^n|^p) \leq (MK)^p p^{p/2} [|t - s|^{p/2} + |t - s|^p 2^{np/8}]$$

$$(4.4) \quad \mathbb{E}(\mathbf{1}_{D_n} |\tilde{K}_t^n - \tilde{K}_s^n|^p) \leq (MK)^p p^{p/2} [|t - s|^{p/2} + |t - s|^p 2^{np/8}].$$

Indeed,

$$\begin{aligned}
\mathbb{E}(\mathbf{1}_{D_n} |K_t^n - K_s^n|^p) &\leq C_p \left(\mathbb{E} \left(\left| \int_s^t F(K_u^n, Z_u) dW_u \right|^p \right) + \mathbb{E} \left(\mathbf{1}_{D_n} \left| \int_s^t G(K_u^n, Z_u) \dot{\omega}_s^n du \right|^p \right) \right. \\
&\quad \left. + \mathbb{E} \left(\left| \int_s^t H(K_u^n, Z_u) \dot{h}_u du \right|^p \right) + \mathbb{E} \left(\left| \int_s^t [I(K_u^n, Z_u) + B(K_u^n, Y_u)] du \right|^p \right) \right) \\
&\leq C_p \sum_{i=0}^3 N_i.
\end{aligned}$$

By the Lemma 2.2, Schwartz's inequality, Hölder's inequality and Fubini's Theorem together, we have

$$N_0 + N_2 + N_3 \leq C_p K^p p^{p/2} |t - s|^{p/2}.$$

$$\begin{aligned}
N_1 &= \mathbb{E} \left(\mathbf{1}_{D_n} \left| \int_s^t G(K_u^n, Z_u) \dot{\omega}_s^n du \right|^p \right) \\
&\leq \mathbb{E} \left(\left| \int_s^t |G(K_u^n, Z_u) - G(K_{\underline{u}_n}^n, Z_{\underline{u}_n})| |\dot{\omega}_u^n| du \right|^p \right) \\
&\quad + \mathbb{E} \left(\mathbf{1}_{D_n} \left| \int_s^t G(K_{\underline{u}_n}^n, Z_{\underline{u}_n}) \dot{\omega}_u^n du - G(K_{\underline{u}_n}^n, Z_{\bar{u}_n}) dW_u \right|^p \right) + \mathbb{E} \left(\left| \int_s^t G(K_{\bar{u}_n}^n, Z_{\bar{u}_n}) dW_u \right|^p \right) \\
&= N_{1,0} + N_{1,1} + N_{1,2}.
\end{aligned}$$

Let $a > 1$, $b > 1$ such that $\frac{1}{a} + \frac{1}{b} = 1$. Hölder's inequality gives

$$\begin{aligned}
N_{1,0} &\leq |t - s|^{p-1} \int_s^t \left\{ \mathbb{E} |G(K_u^n, Z_{\underline{u}_n}) - G(K_{\underline{u}_n}^n, Z_u)|^{pa} \right\}^{\frac{1}{a}} \left\{ \mathbb{E} |\dot{\omega}_u^n|^{pb} \right\}^{\frac{1}{b}} du \\
&\leq K^p |t - s|^{p-1} 2^{5np/8} \int_s^t \left\{ \mathbb{E} (|K_u^n - K_{\underline{u}_n}^n| + |Z_u - Z_{\underline{u}_n}|)^{pa} \right\}^{\frac{1}{a}} du \\
&\leq C_p K^p |t - s|^{p-1} 2^{5np/8} \int_s^t \left\{ \mathbb{E} |K_u^n - K_{\underline{u}_n}^n|^{pa} + \mathbb{E} |Z_u - Z_{\underline{u}_n}|^{pa} \right\}^{\frac{1}{a}} du.
\end{aligned}$$

We show (4.3) for the particular cases $s = \underline{u}_n$ and $t = u$ by following the same arguments of the previous proof. Therefore,

$$\begin{aligned}
\sup_s \mathbb{E} \left(\left| \int_{\underline{s}_n}^s \{F(K_u^n, Z_u) dW_u + H(K_u^n, Z_u) \dot{h}_u du + [I(K_u^n, Z_u) + B(K_u^n, Y_u)] du\} \right|^p \right) \\
(4.5) \qquad \qquad \qquad \leq C_p K^p p^{p/2} 2^{-np/2}.
\end{aligned}$$

Otherwise,

$$\begin{aligned}
 \mathbb{E} \left(\int_{\underline{s}_n}^s |G(K_u^n, Z_u) \dot{\omega}_s^n du|^p \right) &\leq C_p \mathbb{E} \left(\left(2^n \int_{\underline{s}_n}^{\bar{s}_n} |G(K_u^n, Z_u)| du \right)^p |W_{\underline{s}_n} - W_{\underline{s}_n - 2^{-n} \vee 0}|^p \right) \\
 &+ C_p \mathbb{E} \left(\left(2^n \int_{\bar{s}_n}^s |G(K_u^n, Z_u)| du \right)^p |W_{\bar{s}_n} - W_{\underline{s}_n}|^p \right) \\
 &\leq C_p K^p \left[\mathbb{E} (|W_{\underline{s}_n} - W_{\underline{s}_n - 2^{-n} \vee 0}|^p) + \mathbb{E} (|W_{\bar{s}_n} - W_{\underline{s}_n}|^p) \right] \\
 (4.6) \qquad \qquad \qquad &\leq C_p K^p 2^{-np/2}.
 \end{aligned}$$

The inequalities (4.5) and (4.6) imply

$$(4.7) \qquad \mathbb{E}(|K_u^n - K_{\underline{u}_n}^n|^p) \leq C_p K^p p^{p/2} 2^{-n(p/2)},$$

and (3.12) leads to the existence of a constant $C > 0$ such that

$$\mathbb{E}|Z_u - Z_{\underline{u}_n}|^p \leq C 2^{-np/2}.$$

Next,

$$N_{1,0} \leq C_p K^p p^{p/2} 2^{np/8} |t - s|^p.$$

By Lemma 2.2, we have

$$N_{1,1} \leq C_p K^p p^{p/2} |t - s|^{p/2}.$$

By Lemma 2.2 again, Hölder's inequality and Fubini's Theorem, we have

$$N_{1,2} \leq C_p K^p p^{p/2} |t - s|^{p/2}.$$

Finally,

$$N_1 \leq C_p K^p p^{p/2} 2^{np/8} |t - s|^p.$$

The proof of (4.3) is complete. The proof of (4.4) is similar through (3.13).

For $0 < \alpha \leq \frac{1}{2}, \gamma < 1$, let $f_n(\alpha) = \sup_{j > n\gamma} \sup_{p \geq p_0} 2^{-(j/p)} p^{-(p/2)} j^\alpha \| (K^n - L)_{j,\cdot} \|_p$ and $g_n(\alpha) = \sup_{j > n\gamma} \sup_{p \geq p_0} 2^{-(j/p)} p^{-(p/2)} j^\alpha \| (\tilde{K}^n - L)_{j,\cdot} \|_p$.

To finish the proof of the Theorem 2.1, by the Borel Cantelli's Lemma we show that the sequences $\mathbf{1}_{D_n} f_n(\alpha)$ and $\mathbf{1}_{D_n} g_n(\alpha)$ are bounded almost surely. We only deal with the boundedness of $\mathbf{1}_{D_n} f_n(\alpha)$ because the proof is similar.

Indeed,

$$\begin{aligned} \mathbb{P}(D_n \cap \{f_n(\alpha)\} \geq \delta) &\leq \sum_{j > n\gamma} \sum_{p \geq p_0} [\mathbb{P}(D_n \cap \{\|(K^n)_{j,\cdot}\|_p \geq \delta 2^{(j/p-1)} j^\alpha p^{1/2}\}) \\ &\quad + \mathbb{P}(\{\|(L_{j,\cdot}\|_p \geq \delta 2^{(j/p-1)} j^\alpha p^{1/2}\})] \\ &\leq \sum_{j > n\gamma} \sum_{p \geq p_0} \frac{2^{-j} 2^{jp/2} 2^p}{\delta^p p^{p/2} j^{\alpha p}} \sum_{k=1}^{2^j} \left(\sup_{|t-s| \leq 2^{-(j+1)}} \mathbb{E}(\mathbf{1}_{D_n} | K_t^n - K_s^n |^p) + \Gamma \right) \end{aligned}$$

where $\Gamma = \sup_{|t-s| \leq 2^{-(j+1)}} \mathbb{E}(|L_t - L_s|^p)$.

By (4.3) and (3.9), there exist a constants C and $C(K)$ such that for $p_0 > (1/\alpha) \vee 2$, $\delta > 2C(K)$ et $\gamma > 1/4$,

$$\begin{aligned} \mathbb{P}(D_n \cap \{f_n(\alpha)\} \geq \delta) &\leq C \sum_{j > n\gamma} \sum_{p \geq p_0} \left(\frac{2^{n/8} K M}{\delta} \right)^p \left(\frac{1}{j^{\alpha p}} \right) \\ &\leq C \left(\frac{C(K)}{\delta} \right)^{p_0} \sum_{j > n\gamma} (j^{-\alpha p_0}). \end{aligned}$$

We deduce the boundedness of the sequence $\mathbf{1}_{D_n} f_n(\alpha)$ in $\mathbb{L}^2$, therefore bounded almost surely by the Borel Cantelli's Lemma and gives its convergence to 0 almost surely and in $\mathbb{L}^1$. $\square$

Remark 4.1. For the case σ and b unbounded, we considere $\tau_N = \{t : |X_t| \geq N\} \wedge 1$ and we make the same idea in [7] to prove that $X_\cdot$ belongs to $B_M^{1/2,0}$. A classical truncation argument gives the support Theorem .

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