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BLOW-UP FOR SEMIDISCRETE FORMS OF SOME NONLINEAR PARABOLIC EQUATIONS WITH CONVECTION

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ABSTRACT. This paper concerns the study of the numerical approximation for the following parabolic equations with a nonlinear convection term

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) - g(u(x,t))u_x(x,t) + f(u(x,t)), \quad 0 < x < 1,\ t > 0, \\ u_x(0,t) = 0, \quad u_x(1,t) = 0, \quad t > 0, \\ u(x,0) = u_0(x) > 0, \quad 0 \le x \le 1, \end{cases}$$

where $f : [0, +\infty) \to [0, +\infty)$ is C^3 convex, nondecreasing function, $g : [0, +\infty) \to [0, +\infty)$ is C^1 convex, nondecreasing function, $\lim_{s\to+\infty} f(s) = +\infty$, $\lim_{s\to+\infty} g(s) = +\infty$, $\lim_{s\to+\infty} \frac{f(s)}{g(s)} = +\infty$ and $\int_c^{+\infty} \frac{ds}{f(s)} < +\infty$ for $c > 0$. We obtain some conditions under which the solution of the semidiscrete form of the above problem blows up in a finite time and estimate its semidiscrete blow-up time. We also prove that the semidiscrete blow-up time converges to the real one, when the mesh size goes to zero. Finally, we give some numerical results to illustrate ours analysis.

1. INTRODUCTION

Consider the following boundary value problem

$$u_t = u_{xx} - g(u)u_x + f(u),\ 0 \le x \le 1,\ t > 0, \tag{1.1}$$

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$$u_x(0,t) = 0, \quad u_x(1,t) = 0, \quad t > 0, \tag{1.2}$$

$$u(x,0) = u_0(x) > 0, \quad 0 \le x \le 1, \tag{1.3}$$

where $f : [0,+\infty) \to [0,+\infty)$ is C^3 convex, nondecreasing function, $g : [0,+\infty) \to [0,+\infty)$ is C^1 convex, nondecreasing function, $\lim_{s\to+\infty} f(s) = +\infty$, $\lim_{s\to+\infty} g(s) = +\infty$, $\lim_{s\to+\infty} \dfrac{f(s)}{g(s)} = +\infty$, $\displaystyle\int_c^{+\infty} \frac{ds}{f(s)} < +\infty$ for $c > 0$, $u_0 \in C^2([0,1])$, u_0 is nonincreasing on (0,1) and verifies

$$u_0'(0) = 0, \quad u_0'(1) = 0,$$

$$u_0''(x) - g(u_0(x))u_0'(x) + f(u_0(x)) \ge 0, \quad 0 \le x \le 1\,.$$

Definition 1.1. *We say that the solution u of* (1.1)–(1.3) *blows up in a finite time if there exists a finite time T_b such that $\|u(.,t)\|_\infty < +\infty$ for $t \in [0,T_b)$ but*

$$\lim_{t\to T_b^-} \|u(.,t)\|_\infty = +\infty.$$

The time T_b is called the blow-up time of the solution u.

These equations arise in the theory of fluid convection. Convection refers to the transfer of thermal energy within a moving fluid or between a moving fluid and a solid wall. This energy transfer is carried out by two combined modes which are advection and diffusion. The first equations is a heat equation including a nonlinear convection term $g(u)u_x$ and a nonlinear source $f(u)$. It is the term of convection which ensures the movement, generates instability and is also responsible of the turbulent appearance (here we'll refer to it as intermittent since we are in one dimension) when it happens (see [9], [10], [11], [13], [17]).

The theoretical study of blow-up solutions for the reaction-diffusion equations with a nonlinear convection term has been the subject of investigations of many authors (see [2], [5], [6], [7], [8], [13], [14], [15] and the references cited therein). Local in time existence and uniqueness of the solution have been proved(see [3], [4], [12], [18] and the references cited therein). Here, we are interesting in the numerical study using a semidiscrete form of (1.1)–(1.3). We give some assumptions under which the solution of a semidiscrete form of (1.1)–(1.3) blows up in a finite time and estimate its semidiscrete blow-up time. We also show that the semidiscrete blow-up time converges to the theoretical one when the mesh size goes to zero. A similar study has been undertaken (see [1], [10], [13]).

The paper is organized as follows. In the next section, we present a semidiscrete scheme of (1.1)–(1.3) and give some lemmas which will be used throughout the paper. In section 3, under some conditions, we prove that the solution of the semidiscrete form of (1.1)–(1.3) blows up in a finite time. In section 4, we study the convergence of the semidiscrete blow-up time. Finally, in last section, taking some discrete forms of (1.1)–(1.3), we give some numerical experiments.

2. Properties of the semidiscrete scheme

In this section, we give some lemmas which will be used later. We start by the construction of the semidiscrete scheme. Let I be a positive integer and let $h = 1/I$. Define the grid $x_i = ih$, $0 \leq i \leq I$. Approximate the solution u of (1.1)–(1.3) by the solution $U_h = (U_0, \dots, U_i, \dots, U_I)^T$ and approximate the initial condition u_0 of (1.1)–(1.3) by the initial condition $\varphi_h = (\varphi_0, \dots, \varphi_i, \dots, \varphi_I)^T$ of the following semidiscrete equations

$$\frac{dU_i}{dt} = \delta^2 U_i - g(U_i)\delta^0 U_i + f(U_i),\ 1 \leq i \leq I-1,\ t \in [0, T_b^h), \tag{2.1}$$

$$\frac{dU_0(t)}{dt} = \delta^2 U_0(t) + f(U_0(t)), \qquad t \in [0, T_b^h), \tag{2.2}$$

$$\frac{dU_I(t)}{dt} = \delta^2 U_I(t) + f(U_I(t)), \qquad t \in [0, T_b^h), \tag{2.3}$$

$$U_i(0) = \varphi_i > 0, \quad 0 \leq i \leq I, \tag{2.4}$$

where

$$\delta^2 U_i(t) = \frac{U_{i+1}(t) - 2U_i(t) + U_{i-1}(t)}{h^2}, \quad 1 \leq i \leq I-1,$$

$$\delta^2 U_0(t) = \frac{2U_1(t) - 2U_0(t)}{h^2}, \quad \delta^2 U_I(t) = \frac{2U_{I-1}(t) - 2U_I(t)}{h^2},$$

$$\delta^0 U_i(t) = \frac{U_{i+1}(t) - U_{i-1}(t)}{2h}, \quad 1 \leq i \leq I-1,$$

$$\delta^0 U_0(t) = 0, \qquad \delta^0 U_I(t) = 0,$$

$$\delta^+\varphi_i = \frac{\varphi_{i+1}-\varphi_i}{h}, \quad 0 \le i \le I-1,$$

$$\delta^+\varphi_i \le 0, \quad 0 \le i \le I-1\,.$$

Here, $[0, T_b^h)$ is the maximal time interval on which $\|U_h(t)\|_\infty < +\infty$, where

$$\|U_h(t)\|_\infty = \max_{0\le i\le I} |U_i(t)|.$$

The time T_b^h can be finite or infinite. When the time T_b^h is finite, we say that the solution U_h of (2.1)–(2.4) blows up in finite time, and the time T_b^h is called the blow-up time of the solution U_h. When the time T_b^h is infinite, we say that the solution U_h of (2.1)–(2.4) blows up globally or exist globally,

Lemma 2.1. *Let* $a_h(t), b_h(t) \in C^0([0,T], \mathbb{R}^{I+1})$ *and let* $V_h(t) \in C^1([0,T], \mathbb{R}^{I+1})$ *where* $b_h(t)\delta^0 V_h(t) \le 0$ *and* $a_h(t) \le 0$*, such that*

$$\frac{d}{dt}V_i - \delta^2 V_i + b_i\delta^0 V_i + a_i V_i \ge 0,\ 0 \le i \le I,\ t \in (0,T), \tag{2.5}$$

$$V_i(0) \ge 0, \quad 0 \le i \le I.$$

Then we have

$$V_i(t) \ge 0, \quad 0 \le i \le I, \quad t \in (0,T).$$

Proof. Let T_0 be any quantity satisfying the inequality $T_0 < T$ and define the vector $Z_h(t) = e^{\alpha t}V_h(t)$ where α is such that

$$a_i(t) - \alpha > 0 \quad \text{for} \quad 0 \le i \le I, \quad t \in [0, T_0].$$

Let $m = \min\limits_{0\le i\le I, 0\le t\le T_0} Z_i(t)$. Since, for $i \in \{0, ..., I\}$, $Z_i(t)$ is a continuous function on the compact $[0, T_0]$, there exists $i_0 \in \{0, ..., I\}$ and $t_0 \in [0, T_0]$ such that $m = Z_{i_0}(t_0)$. We observe that

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k\to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0-k)}{k} \le 0, \quad 0 \le i_0 \le I, \tag{2.6}$$

$$\delta^2 Z_{i_0}(t_0) = \frac{Z_{i_0+1} - 2Z_{i_0} + Z_{i_0-1}}{h^2} \ge 0, 1 \le i_0 \le I-1, \tag{2.7}$$

$$\delta^2 Z_{i_0}(t_0) = \frac{2Z_1(t_0) - 2Z_0(t_0)}{h^2} \ge 0 \quad if \quad i_0 = 0, \tag{2.8}$$

$$\delta^2 Z_{i_0}(t_0) = \frac{2Z_{I-1}(t_0) - 2Z_I(t_0)}{h^2} \ge 0 \quad if \quad i_0 = I. \tag{2.9}$$

From (2.5), we obtain the following inequality

$$\frac{dZ_{i_0}}{dt} - \delta^2 Z_{i_0} + b_{i_0}\delta^0 Z_{i_0} + (a_{i_0} - \alpha)Z_{i_0} \geq 0.$$

It follows from (2.6)–(2.9) that

$$(a_{i_0}(t_0) - \alpha)Z_{i_0}(t_0) \geq 0,$$

which implies that $Z_{i_0}(t_0) \geq 0$ because $a_{i_0}(t_0) - \alpha > 0$. We deduce that $V_h(t) \geq 0$ for $t \in [0, T_0]$ and the proof is complete. □

Lemma 2.2. *Let* $V_h(t)$*,* $W_h(t) \in C^1([0,T], \mathbb{R}^{I+1})$ *and* $f, g \in C^1(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ *such that*

$$\frac{dV_i}{dt} - \delta^2 V_i + g(V_i)\delta^0 V_i + f(V_i) < \frac{dW_i}{dt} -$$

$$\delta^2 W_i + g(V_i)\delta^0 W_i + f(W_i), \quad 0 \leq i \leq I, \ t \in (0,T),$$

$$V_i(0) < W_i(0), \quad 0 \leq i \leq I.$$

Then we have

$$V_i(t) < W_i(t), \quad 0 \leq i \leq I, \quad t \in (0,T).$$

Proof. Define the vector $Z_h(t) = W_h(t) - V_h(t)$. Let t_0 be the first $t > 0$ such that $Z_i(t) > 0$ for $t \in [0, t_0)$, $0 \leq i \leq I$, but $Z_{i_0}(t_0) = 0$ for a certain $i_0 \in \{0, ..., I\}$. We remark that

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k \to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0 - k)}{k} \leq 0, \ 0 \leq i_0 \leq I,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{Z_{i_0+1}(t_0) - 2Z_{i_0}(t_0) + Z_{i_0-1}(t_0)}{h^2} \geq 0, \ 1 \leq i_0 \leq I-1,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{2Z_1(t_0) - 2Z_0(t_0)}{h^2} \geq 0 \quad if \quad i_0 = 0,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{2Z_{I-1}(t_0) - 2Z_I(t_0)}{h^2} \geq 0 \quad if \quad i_0 = I.$$

Therefore, we have

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) + g(Wi_0(t_0))\delta^0 Z_{i_0}(t_0) + g^{'}(\mu_{i_0}(t_0))\delta^0 V_{i_0}(t_0))Z_{i_0}(t_0)$$
$$-f^{'}(\beta_{i_0}(t_0))Z_{i_0}(t_0) \leq 0,$$

where $\mu_{i_0}(t_0), \beta_{i_0}(t_0) \in (V_{i_0}(t_0), W_{i_0}(t_0))$, which contradicts the first strict inequality of the lemma and this ends the proof. □

Lemma 2.3. *Let U_h be the solution of* (2.1)–(2.4). *Then we have*

$$U_i(t) > 0 \quad for \quad 0 \leq i \leq I, \quad t \in (0, T_b^h).$$

Proof. Assume that there exists a time $t_0 \in (0, T_b^h)$ such that $U_{i_0}(t_0) = 0$ for a certain $i_0 \in \{0, ..., I\}$. We observe that

$$\frac{dU_{i_0}(t_0)}{dt} = \lim_{k \to 0} \frac{U_{i_0}(t_0) - U_{i_0}(t_0 - k)}{k} \leq 0, \ 0 \leq i_0 \leq I,$$

$$\delta^2 U_{i_0}(t_0) = \frac{U_{i_0+1}(t_0) - 2U_{i_0}(t_0) + U_{i_0-1}(t_0)}{h^2} > 0, \ 1 \leq i_0 \leq I - 1,$$

$$\delta^2 U_{i_0}(t_0) = \frac{2U_1(t_0) - 2U_0(t_0)}{h^2} > 0 \quad if \quad i_0 = 0,$$

$$\delta^2 U_{i_0}(t_0) = \frac{2U_{I-1}(t_0) - 2U_I(t_0)}{h^2} > 0 \quad if \quad i_0 = I,$$

which implies that

$$\frac{dU_{i_0}(t_0)}{dt} - \delta^2 U_{i_0}(t_0) + g(U_{i_0}(t_0))\delta^0 U_{i_0}(t_0) - f(U_{i_0}(t_0)) < 0, \ 1 \leq i_0 \leq I - 1,$$

$$\frac{dU_0(t_0)}{dt} - \delta^2 U_0(t_0) - f(U_0(t_0)) < 0,$$

$$\frac{dU_I(t_0)}{dt} - \delta^2 U_I(t_0) - f(U_I(t_0)) < 0.$$

But these inequalities contradict (2.1)–(2.3) and we obtain the desired result.
□

Lemma 2.4. *Let U_h be the solution of* (2.1)–(2.4). *Then we have*

$$U_{i+1}(t) < U_i(t) \quad for \quad 0 \leq i \leq I - 1, \quad t \in (0, T_b^h).$$

Proof. Introduce the vector $Z_h(t)$ defined as follows $Z_i(t) = U_{i+1}(t) - U_i(t)$ for $0 \leq i \leq I - 1$. Let t_0 be the first t>0 such that $Z_i(t) < 0$ for $t \in [0, t_0)$ but $Z_{i_0}(t_0) = 0$ for a certain $i_0 \in \{0, ..., I - 1\}$. Without loss of generality, we may suppose that i_0 is the smallest integer which satisfies the above equality. It follows that

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k\to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0 - k)}{k} \geq 0,\ 0 \leq i_0 \leq I - 1,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{Z_{i_0+1}(t_0) - 2Z_{i_0}(t_0) + Z_{i_0-1}(t_0)}{h^2} < 0,\ 1 \leq i_0 \leq I - 1,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{2Z_1(t_0) - 2Z_0(t_0)}{h^2} < 0 \quad if \quad i_0 = 0,$$

which implies that

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) + g(U_{i_0+1}(t_0))\delta^0 Z_{i_0}(t_0) +$$

$$(g^{'}(\mu_{i_0}(t_0))\delta^0 U_{i_0}(t_0) - f^{'}(\beta_{i_0}(t_0)))Z_{i_0}(t_0) > 0,\ 1 \leq i_0 \leq I - 1,$$

$$\frac{dZ_0(t_0)}{dt} - \delta^2 Z_0(t_0) + g(U_1(t_0))\delta^0 Z_0(t_0) - f^{'}(\beta_0(t_0))Z_0(t_0) > 0.$$

where $\beta_0(t_0) \in (U_1(t_0), U_0(t_0))$ and $\mu_{i_0}(t_0), \beta_{i_0}(t_0) \in (U_{i_0+1}(t_0), U_{i_0}(t_0))$. Therefore, we have a contradiction because of (2.1)–(2.2). This ends the proof. $\square$

Lemma 2.5. *Let U_h be the solution of* (2.1)–(2.4). *Then we have*

$$\frac{dU_i(t)}{dt} > 0 \quad for \quad 0 \leq i \leq I, \quad t \in (0, T_b^h).$$

Proof. Consider the vector $Z_h(t)$ with $Z_i(t) = \frac{d}{dt}U_i(t)$, $0 \leq i \leq I$. Let t_0 be the first $t > 0$ such that $Z_i(t) > 0$ for $t \in [0, t_0)$ but $Z_{i_0}(t_0) = 0$ for a certain $i_0 \in \{1, ..., I\}$. Without loss of generality, we may suppose that i_0 is the smallest integer which satisfies the above equality. We get

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k\to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0 - k)}{k} \leq 0,\ 0 \leq i_0 \leq I,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{Z_{i_0+1}(t_0) - 2Z_{i_0}(t_0) + Z_{i_0-1}(t_0)}{h^2} > 0,\ 1 \leq i_0 \leq I - 1,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{2Z_1(t_0) - 2Z_0(t_0)}{h^2} > 0 \quad if \quad i_0 = 0,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{2Z_{I-1}(t_0) - 2Z_I(t_0)}{h^2} > 0 \quad if \quad i_0 = I,$$

which implies that

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) + g(U_{i_0}(t_0))\delta^0 Z_{i_0}(t_0) +$$
$$(g^{'}(U_{i_0}(t_0))\delta^0 U_{i_0}(t_0) - f^{'}(U_{i_0}(t_0)))Z_{i_0}(t_0) < 0, if \quad 1 \leq i_0 \leq I-1,$$

$$\frac{dZ_0(t_0)}{dt} - \delta^2 Z_0(t_0) + g(U_0(t_0))\delta^0 Z_0(t_0) - f^{'}(U_0(t_0))Z_0(t_0) < 0,$$

$$\frac{dZ_I(t_0)}{dt} - \delta^2 Z_I(t_0) + g(U_I(t_0))\delta^0 Z_I(t_0) - f^{'}(U_I(t_0))Z_I(t_0) < 0.$$

But these inequalities contradict (2.1)–(2.3) and lead to the desired result. □

Lemma 2.6. *Let U_h be the solution of* (2.1)–(2.4). *Then we have*

$$g^{'}(U_i(t))f(U_i(t)) > -h(\delta^0 U_i(t))g(U_i(t))f^{''}(U_{i-1}(t)),\ 1 \leq i \leq I-1,\ t \in (0, T_b^h).$$

Proof. Define the vectors $Z_h(t)$, $K_h(t)$ and $V_h(t)$ such that $Z_i(t) = K_i(t) - V_i(t)$ with $K_i(t) = g^{'}(U_i(t))f(U_i(t))$ and $V_i(t) = -h(\delta^0 U_i(t))g(U_i(t))f^{''}(U_{i-1}(t))$ for $1 \leq i \leq I-1$. Let t_0 be the first $t > 0$ such that $Z_i(t) > 0$ for $t \in [0, t_0)$ but $Z_{i_0}(t_0) = 0$ for a certain $i_0 \in \{0, ..., I\}$. We may suppose that i_0 is the smallest integer which satisfies the above equality. It follows that

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k\to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0 - k)}{k} \leq 0,\ 0 \leq i_0 \leq I,$$

$$\delta^2 Z_{i_0}(t_0) = \frac{Z_{i_0+1}(t_0) - 2Z_{i_0}(t_0) + Z_{i_0-1}(t_0)}{h^2} > 0,\ 1 \leq i_0 \leq I-1,$$

which implies that

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) + g(K_{i_0}(t_0))\delta^0 Z_{i_0}(t_0) +$$

$$(g^{'}(\mu_{i_0}(t_0))\delta^0 V_{i_0}(t_0) - f^{'}(\beta_{i_0}(t_0)))Z_{i_0}(t_0) < 0, \quad 1 \leq i_0 \leq I-1,$$

where $\mu_{i_0}(t_0), \beta_{i_0}(t_0) \in (V_{i_0}(t_0), K_{i_0}(t_0))$.
But this inequalities contradict (2.1)–(2.3) and we obtain the desired result. □

Lemma 2.7. *Let $U_h \in C^1([0,T], R^{I+1})$ such that $U_h > 0$. Then we have*

$$\delta^2 f(U_i) \geq f^{'}(U_i)\delta^2 U_i \quad for \quad 0 \leq i \leq I.$$

Proof. Apply Taylor's expansion to obtain

$$f(U_1) = f(U_0) + (U_1 - U_0)f^{'}(U_0) + \frac{(U_1 - U_0)^2}{2} f^{''}(\eta_0),$$

$$f(U_{i+1}) = f(U_i) + (U_{i+1} - U_i)f^{'}(U_i) + \frac{(U_{i+1} - U_i)^2}{2} f^{''}(\theta_i), \quad 1 \le i \le I-1,$$

$$f(U_{i-1}) = f(U_i) + (U_{i-1} - U_i)f^{'}(U_i) + \frac{(U_{i-1} - U_i)^2}{2} f^{''}(\eta_i), \quad 1 \le i \le I-1,$$

$$f(U_{I-1}) = f(U_I) + (U_{I-1} - U_I)f^{'}(U_I) + \frac{(U_{I-1} - U_I)^2}{2} f^{''}(\eta_I),$$

where θ_i is an intermediate between U_i and U_{i+1} and η_i the one between U_{i-1} and U_i. The first and last equalities imply that

$$\delta^2 f(U_0) = f^{'}(U_0)\delta^2 U_0 + \frac{(U_1 - U_0)^2}{h^2} f^{''}(\eta_0),$$

$$\delta^2 f(U_I) = f^{'}(U_I)\delta^2 U_I + \frac{(U_{I-1} - U_I)^2}{h^2} f^{''}(\eta_I).$$

Combining the second and third equalities, we see that

$$\delta^2 f(U_i) = f^{'}(U_i)\delta^2 U_i + \frac{(U_{i+1} - U_i)^2}{2h^2} f^{''}(\theta_i) + \frac{(U_{i-1} - U_i)^2}{2h^2} f^{''}(\eta_i), \quad 1 \le i \le I-1.$$

Use the fact that $f(s)$ is a convex function and $U_h > 0$ to complete the rest of the proof. □

Lemma 2.8. *Let $U_h \in C^1([0,T], R^{I+1})$ such that $U_h > 0$. Then we have*

$$\delta^0 f(U_i) \le f^{'}(U_i)\delta^0 U_i + h(\delta^0 U_i)^2 f^{''}(U_{i-1}), \quad 0 \le i \le I.$$

Proof. Using Taylor's expansion, we obtain

$$\delta^0 f(U_i) = f^{'}(U_{i-1})\delta^0 U_i + \frac{(U_{i+1} - U_{i-1})^2}{4h} f^{''}(U_{i-1}) + \frac{(U_{i+1} - U_{i-1})^3}{12h} f^{'''}(\zeta_i), \quad 0 \le i \le I,$$

where $\zeta_i \in (U_{i+1}, U_{i-1})$.
Using Lemma 2.4 and $U_h > 0$, we have the desired result. □

3. SEMIDISCRETE BLOW-UP SOLUTIONS

In this section under some assumptions, we show that the solution U_h of (2.1)–(2.4) blows up in a finite time and estimate its semidiscrete blow-up time.

Theorem 3.1. *Let U_h be the solution of* (2.1)–(2.4)*, then the solution U_h blows up in a finite time T_b^h with following estimate*

$$T_b^h \leq \int_{\varphi_{hmin}}^{+\infty} \frac{d\sigma}{f(\sigma)},$$

where $\varphi_{hmin} = \min\limits_{0\leq i\leq I} \{\varphi_i\}$.

Proof. Consider the following differential equation

$$\dot{\alpha}(t) = f(\alpha(t)), \quad t \in (0, T_\alpha),$$

$$\alpha(0) = \varphi_{hmin},$$

with $\quad T_\alpha = \displaystyle\int_{\varphi_{hmin}}^{+\infty} \frac{d\sigma}{f(\sigma)}.$

Introduce the vector $V_h(t)$ such that $V_i(t) = \alpha(t),\ 0 \leq i \leq I,\ t \in (0, T_\alpha)$.
Let the vector $Z_h(t)$ defined as follows $Z_h(t) = U_h(t) - V_h(t)$. It is not hard to see that, for $t \in (0, T_1)$,

$$\frac{dZ_i(t)}{dt} - \delta^2 Z_i(t) + g(U_i(t))\delta^0 Z_i(t) + (g^{'}(\mu_i(t))\delta^0 V_i(t) - f^{'}(\beta_i(t)))Z_i(t) \geq 0, \quad 0 \leq i \leq I,$$

$$\frac{dZ_0(t)}{dt} - \delta^2 Z_0(t) + g(U_0(t))\delta^0 Z_0(t) + (g^{'}(\mu_0(t))\delta^0 V_0(t) - f^{'}(\beta_0(t)))Z_0(t) \geq 0,$$

$$\frac{dZ_I(t)}{dt} - \delta^2 Z_I(t) + g(U_I(t))\delta^0 Z_I(t) + (g^{'}(\mu_I(t))\delta^0 V_I(t) - f^{'}(\beta_I(t)))Z_I(t) \geq 0,$$

$$Z_i(0) \geq 0,$$

where $\mu_i(t)$ and $\beta_i(t)$ are intermediate values between $V_i(t)$ and $U_i(t))$. $T_1 = \min\{T_\alpha, T_b^h\}$. Due to Lemma 2.1, we have $U_i(t) \geq V_i(t)$, $0 \leq i \leq I$, $t \in (0, T_1)$.
We deduce that

$$T_b^h \leq T_\alpha \leq \int_{\varphi_{hmin}}^{+\infty} \frac{d\sigma}{f(\sigma)}.$$

□

The following theorem gives a best result than the previous.

Theorem 3.2. *Let U_h be the solution of* (2.1)–(2.4). *Suppose that there exists a positive integer λ such that*

$$\delta^2\varphi_i - g(\varphi_i)\delta^0\varphi_i + f(\varphi_i) \geq \lambda f(\varphi_i), \quad 0 \leq i \leq I. \tag{3.1}$$

Then, the solution U_h blows up in a finite time T_b^h and we have the following estimate

$$T_b^h \leq \frac{1}{\lambda}\int_{\|\varphi_h\|_\infty}^{+\infty} \frac{d\sigma}{f(\sigma)}.$$

Proof. Let $[0, T_b^h)$ be the maximal time interval on which $\|U_h(t)\|_\infty < +\infty$. Our aim is to show that T_b^h is finite and satisfies the above inequality. Introduce the vector $J_h(t)$ such that

$$J_i(t) = \frac{dU_i(t)}{dt} - \lambda f(U_i(t)), \quad 0 \leq i \leq I.$$

A straightforward calculation gives

$$\frac{dJ_i}{dt} - \delta^2 J_i = \frac{d^2U_i}{dt^2} - \lambda f^{'}(U_i)\frac{dU_i}{dt} - \delta^2(\frac{dU_i}{dt}) + \lambda\delta^2 f(U_i), \quad 1 \leq i \leq I-1.$$

From Lemma 2.7, we have $\delta^2 f(U_i) \geq f^{'}(U_i)\delta^2 U_i$, which implies that

$$\frac{dJ_i}{dt} - \delta^2 J_i \geq \frac{d}{dt}(\frac{dU_i}{dt} - \delta^2 U_i) - \lambda f^{'}(U_i)(\frac{dU_i}{dt} - \delta^2 U_i), \quad 1 \leq i \leq I-1.$$

$$\frac{dJ_i}{dt} - \delta^2 J_i \geq \frac{d}{dt}(-g(U_i)\delta^0 U_i + f(U_i)) - \lambda f^{'}(U_i)(-g(U_i)\delta^0 U_i + f(U_i)),$$
$$1 \leq i \leq I-1.$$

We have

$$\frac{dJ_i}{dt} - \delta^2 J_i \geq -(g^{'}(U_i)\delta^0 U_i - f^{'}(U_i))J_i - g(U_i)\delta^0(\frac{dU_i}{dt}) + \lambda f^{'}(U_i)g(U_i)\delta^0 U_i$$
$$-\lambda g^{'}(U_i)f(U_i)\delta^0 U_i, \quad 1 \leq i \leq I-1.$$

By the Lemma 2.8, we obtain

$$\frac{dJ_i}{dt} - \delta^2 J_i + g(U_i)\delta^0 J_i + (g^{'}(U_i)\delta^0 U_i - f^{'}(U_i))J_i \geq -\lambda g^{'}(U_i)f(U_i)\delta^0 U_i$$
$$-\lambda h g(U_i)(\delta^0 U_i)^2 f^{''}(U_{i-1}), \quad 1 \leq i \leq I-1,$$

From Lemma 2.6, we get $-\lambda g^{'}(U_i)f(U_i)\delta^0 U_i - \lambda h g(U_i)(\delta^0 U_i)^2 f^{''}(U_{i-1}) > 0$. Using (2.2)–(2.4), we arrive at

$$\frac{dJ_i}{dt} - \delta^2 J_i + g(U_i)\delta^0 J_i + (g^{'}(U_i)\delta^0 U_i - f^{'}(U_i))J_i \geq 0, \quad 1 \leq i \leq I-1,$$

$$\frac{dJ_0}{dt} - \delta^2 J_0 + g(U_0)\delta^0 J_0 + (g^{'}(U_0)\delta^0 U_0 - f^{'}(U_0))J_0 \geq 0,$$

$$\frac{dJ_I}{dt} - \delta^2 J_I + g(U_I)\delta^0 J_I + (g^{'}(U_I)\delta^0 U_I - f^{'}(U_I))J_I \geq 0.$$

From (3.1), we observe that

$$J_i(0) = \delta^2 \varphi_i - g(\varphi_i)\delta^0 \varphi_i + f(\varphi_i) - \lambda f(\varphi_i) \geq 0, \quad 0 \leq i \leq I.$$

We deduce from Lemma 2.1 that $J_h(t) \geq 0$ for $t \in (0, T_b^h)$, which implies that

$$\frac{dU_i(t)}{dt} \geq \lambda f(U_i(t)), \quad 0 \leq i \leq I, \quad t \in (0, T_b^h).$$

These estimates may be rewritten in the following form

$$\frac{dU_i(t)}{f(U_i(t))} \geq \lambda dt, \quad 0 \leq i \leq I.$$

Integrating the above inequalities over (t, T_b^h), we arrive at

$$T_b^h - t \leq \frac{1}{\lambda} \int_t^{T_b^h} \frac{dU_i(t)}{f(U_i(t))}, \quad 0 \leq i \leq I. \tag{3.2}$$

Using the fact that $\|U_h(0)\|_\infty = \|\varphi_h\|_\infty$ and taking $t = 0$ in (3.2), we get

$$T_b^h \leq \frac{1}{\lambda} \int_{\|\varphi_h\|_\infty}^{+\infty} \frac{d\sigma}{f(\sigma)}.$$

□

Remark 3.1. *The inequalities* (3.2) *imply that*

$$T_b^h - t_0 \leq \frac{1}{\lambda} \int_{\|U_h(t_0)\|_\infty}^{+\infty} \frac{d\sigma}{f(\sigma)} \quad for \quad t_0 \in [0, T_b^h).$$

4. Convergence of the semidiscrete blow-up time

In this section, under some assumptions, we show that the semidiscrete blow-up time converges to the real one when the mesh size goes to zero. In order to obtain the convergence of semidiscrete blow-up time, we firstly prove the following theorem about the convergence of the semidiscrete scheme.

Theorem 4.1. *Assume that* (1.1)–(1.3) *has a solution* $u \in C^{4,1}([0,1] \times [0,T])$ *and the initial condition at* (2.4) *satisfies*

$$\|\varphi_h - u_h(0)\|_\infty = o(1) \quad as \quad h \to 0, \tag{4.1}$$

where $u_h(t) = (u(x_0,t),...,u(x_I,t))^T$*. Then, for* h *sufficiently small, the problem* (2.1)–(2.4) *has a unique solution* $U_h \in C^1([0,T],\mathbb{R}^{I+1})$ *such that*

$$\max_{0\leq t\leq T} \|U_h(t) - u_h(t)\|_\infty = O(\|\varphi_h - u_h(0)\|_\infty + h^2) \quad as \quad h \to 0.$$

Proof. Let $K > 0$ be such that

$$\|u\|_\infty \leq K\,.$$

The problem (2.1)–(2.4) has for each h, a unique solution $U_h \in C^1([0,T_q^h),\mathbb{R}^{I+1})$. Let $t(h)$ the greatest value of $t > 0$ such that

$$\|U_h(t) - u_h(t)\|_\infty < 1 \quad for \quad t \in (0,t(h)). \tag{4.2}$$

The relation (4.1) implies that $t(h) > 0$ for h sufficiently small. Let $t^*(h) = \min\{t(h),T\}$. By the triangular inequality, we obtain

$$\|U_h(t)\|_\infty \leq \|u(x,t)\|_\infty + \|U_h(t) - u_h(t)\|_\infty \quad for \quad t \in (0,t^*(h)),$$

which implies that

$$\|U_h(t)\|_\infty \leq 1 + K, \quad for \quad t \in (0,t^*(h)). \tag{4.3}$$

Let $e_h(t) = U_h(t) - u_h(t)$ be the error of discretization.
Using Taylor's expansion, we have for $t \in (0,t^*(h))$,

$$\frac{d}{dt}e_i(t) - \delta^2 e_i(t) + g(u(x_i,t))\delta^0 e_i(t) = (f'(\beta_i(t)) - g'(\mu_i(t))\delta^0 u(x_i,t))e_i(t) - \frac{h^2}{6}g(u(x_i,t))u_{xxx}(\widetilde{x}_i,t),$$

$$\frac{d}{dt}e_0(t) - \delta^2 e_0(t) = f'(\beta_0(t))e_0(t) + \frac{h^2}{12}u_{xxxx}(\widetilde{x}_0,t),$$

$$\frac{d}{dt}e_I(t) - \delta^2 e_I(t) = f^{'}(\beta_I(t))e_I(t) + \frac{h^2}{12}u_{xxxx}(\widetilde{x}_I, t),$$

where $\beta_i(t)$ is an intermediate value between $u(x_i, t)$ and $U_i(t)$ for $i \in \{0, ..., I\}$.
Using (4.3), there exists a constant $M > 0$ such that

$$\frac{d}{dt}e_i - \delta^2 e_i + g(u)\delta^0 e_i \leq M|e_i| + Mh^2,\ 1 \leq i \leq I-1,$$

$$\frac{d}{dt}e_0(t) - \delta^2 e_0(t) \leq M|e_0(t)| + Mh^2,$$

$$\frac{d}{dt}e_I(t) - \delta^2 e_I(t) \leq M|e_I(t)| + Mh^2\,.$$

Consider the vector $W_h(t)$ such that

$$W_i(t) = e^{(M+1)t}(\|\varphi_h - u_h(0)\|_\infty + Mh^2), \quad 0 \leq i \leq I.$$

A direct calculation yields

$$\frac{d}{dt}W_i - \delta^2 W_i + g(u)\delta^0 W_i > M|W_i| + Mh^2, 1 \leq i \leq I-1,$$

$$\frac{d}{dt}W_0(t) - \delta^2 W_0(t) > M|W_0(t)| + Mh^2,$$

$$\frac{d}{dt}W_I(t) - \delta^2 W_I(t)\delta^0 > M|W_I(t)| + Mh^2,$$

$$W_i(0) > e_i(0), \quad 0 \leq i \leq I.$$

It follows from Lemma 2.2 that

$$W_i(t) > e_i(t) \quad for \quad t \in (0, t^*(h)), \quad 0 \leq i \leq I.$$

By the same way, we also prove that

$$W_i(t) > -e_i(t) \quad for \quad t \in (0, t^*(h)), \quad 0 \leq i \leq I,$$

which implies that

$$W_i(t) > |e_i(t)| \quad for \quad t \in (0, t^*(h)), \quad 0 \leq i \leq I.$$

We deduce that

$$\|U_h(t) - u_h(t)\|_\infty \leq e^{(M+1)T}(\|\varphi_h - u_h(0)\|_\infty + Mh^2), \quad t \in (0, t^*(h)).$$

Let us show that $t^*(h) = T$. Suppose that $T > t(h)$. From (4.2), we obtain

$$1 = \|U_h(k) - u_h(k)\|_\infty \leq e^{(M+1)T}(\|\varphi_h - u_h(0)\|_\infty + Mh^2), \tag{4.4}$$

where k=t(h). Since $e^{(M+1)T}(\|\varphi_h - u_h(0)\|_\infty + Mh^2) \to 0$ when $h \to 0$, we deduce from (4.4) that $1 \leq 0$, which is impossible. Consequently $t^*(h) = T$, and we conclude the proof. □

Theorem 4.2. *Suppose that the solution u of* (1.1)–(1.3) *blows up in a finite time T_b such that $u \in C^{4,1}([0,1] \times [0,T_b), \mathbb{R})$ and the initial condition at* (2.4) *satisfies*

$$\|\varphi_h - u_h(0)\|_\infty = o(1) \quad as \quad h \to 0.$$

Assume that there exists a constant $\lambda > 0$ such that

$$\delta^2\varphi_i - g(\varphi_i)\delta^0\varphi_i + f(\varphi_i) \geq \lambda f(\varphi_i), \quad 0 \leq i \leq I.$$

Then the solution U_h of (2.1)–(2.4) *blows up in a finite time T_b^h and*

$$\lim_{h\to 0} T_b^h = T_b.$$

Proof. Let $\varepsilon > 0$. There exists a positive constant N such that

$$\frac{1}{\lambda}\int_y^{+\infty} \frac{d\sigma}{f(\sigma)} \leq \frac{\varepsilon}{2} < \infty \quad for \quad y \in [N, +\infty[. \tag{4.5}$$

Since u blows up at the time T_b. There exists $T_1 \in (T_b - \frac{\varepsilon}{2}, T_b)$ and $h_0(\varepsilon) > 0$ such that

$$\|u(.,t)\|_\infty \geq 2N \quad \text{for} \quad t \in [T_1, T_b), \quad h \leq h_0(\varepsilon).$$

Let $T_2 = \dfrac{T_1 + T_b}{2}$, then $\sup_{t\in[0,T_2]} |u(.,t)| < \infty$ for $h \leq h_0(\varepsilon)$. It follows from Theorem 4.1 that $\sup_{t\in[0,T_2]} \|U_h(t) - u_h(t)\|_\infty \leq N$ for $h \leq h_0(\varepsilon)$. Applying the triangular inequality, we get $\|U_h(t)\|_\infty \geq \|u_h(t)\|_\infty - \|U_h(t) - u_h(t)\|_\infty$ for $h \leq h_0(\varepsilon)$, which leads to $\|U_h(t)\|_\infty \geq N$ for $t \in [0, T_2]$, $h \leq h_0(\varepsilon)$. From Theorem 3.2, U_h blows up at the time T_b^h. We deduce from Remark 3.1 and (4.5) that for $h \leq h_0(\varepsilon)$

$$|T_b - T_b^h| \leq |T_b - T_2| + |T_2 - T_b^h| \leq \frac{\varepsilon}{2} + \frac{1}{\lambda}\int_{\|U_h(T_2)\|_\infty}^{+\infty} \frac{d\sigma}{f(\sigma)} \leq \varepsilon,$$

which leads us to the desired result. □

5. Numerical experiments

In this section, we present some numerical approximations to the blow-up time of (1.1)–(1.3). We use the following explicit scheme

$$\frac{U_i^{(n+1)} - U_i^{(n)}}{\Delta t_n} = \frac{U_{i+1}^{(n)} - 2U_i^{(n)} + U_{i-1}^{(n)}}{h^2} - g(U_i^{(n)})(\frac{U_{i+1}^{(n)} - U_{i-1}^{(n)}}{2h}) + f(U_i^{(n)}),$$
$$1 \leq i \leq I-1,$$

$$\frac{U_0^{(n+1)} - U_0^{(n)}}{\Delta t_n} = \frac{2U_1^{(n)} - 2U_0^{(n)}}{h^2} + f(U_0^{(n)}),$$

$$\frac{U_I^{(n+1)} - U_I^{(n)}}{\Delta t_n} = \frac{2U_{I-1}^{(n)} - 2U_I^{(n)}}{h^2} + f(U_I^{(n)}),$$

$$U_i^{(0)} = \varphi_i > 0, \quad 0 \leq i \leq I,$$

where $n \geq 0$, $g(s) = s^q$, $f(s) = s^p$, $s \geq 0$, $q \geq 1$, $p \geq q+1$, $\Delta t_n = \min(\frac{h^2}{2}, \tau \|U_h^{(n)}\|_\infty^{1-p})$, with $\tau = const \in (0,1)$ and $\varphi_i = \dfrac{1}{12} + (1-(ih)^2)^2$ for $0 \leq i \leq I$.
Also we use the implicit scheme

$$\frac{U_i^{(n+1)} - U_i^{(n)}}{\Delta t_n} = \frac{U_{i+1}^{(n+1)} - 2U_i^{(n+1)} + U_{i-1}^{(n+1)}}{h^2} - g(U_i^{(n)})(\frac{U_{i+1}^{(n+1)} - U_{i-1}^{(n+1)}}{2h}) + f(U_i^{(n)}),$$
$$1 \leq i \leq I-1,$$

$$\frac{U_0^{(n+1)} - U_0^{(n)}}{\Delta t_n} = \frac{2U_1^{(n+1)} - 2U_0^{(n+1)}}{h^2} + f(U_0^{(n)}),$$

$$\frac{U_I^{(n+1)} - U_I^{(n)}}{\Delta t_n} = \frac{2U_{I-1}^{(n+1)} - 2U_I^{(n+1)}}{h^2} + f(U_I^{(n)}),$$

$$U_i^{(0)} = \varphi_i > 0, \quad 0 \leq i \leq I,$$

where $n \geq 0$, $g(s) = s^q$, $f(s) = s^p$, $s \geq 0$, $q \geq 1$, $p \geq q+1$, $\Delta t_n = \tau \|U_h^{(n)}\|_\infty^{1-p}$ with $\tau = const \in (0,1)$ and $\varphi_i = \dfrac{1}{12} + (1-(ih)^2)^2$ for $0 \leq i \leq I$.
In the tables 1-6, in rows, we present the numerical blow-up times, numbers of iterations, the CPU times and the orders of the approximations corresponding to meshes of 16, 32, 64, 128, 256, 512, 1024, 2048. The numerical blow-up

time $T^n = \sum_{j=0}^{n-1} \Delta t_j$ is computed at the first time when $\Delta t_n = |T^{n+1} - T^n| \leq 10^{-16}$. The order(s) of the method is computed from

$$s = \frac{\log((T_{4h} - T_{2h})/(T_{2h} - T_h))}{\log(2)}.$$

First case: $(q, p) = (1, 4)$ and $\tau = \dfrac{h^2}{2}$.

Table 1: Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the explicit Euler method

I	T^n	n	$CPU\ time$	s
16	0.95630084	5652	0.015	-
32	0.95322924	21640	0.093	-
64	0.95246070	82750	0.733	1.998
128	0.95226852	315844	5.351	1.999
256	0.95222047	1202803	38.969	1.999
512	0.95220846	4568988	301.440	2.000
1024	0.95220546	17307040	2129.460	2.000
2048	0.95220470	65351411	15709.004	2.000

Table 2: Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the implicit Euler method

I	T^n	n	$CPU\ time$	s
16	0.95486982	5564	0.046	-
32	0.95287040	21286	0.202	-
64	0.95237092	81338	1.326	2.001
128	0.95224607	310194	8.907	2.000
256	0.95221486	1180208	64.709	2.000
512	0.95220706	4478606	491.091	2.000
1024	0.95220510	16945512	3726.879	2.000
2048	0.95220462	63905292	28102.566	2.000

Second case: $(q,p)=(2,4)$ and $\tau=\dfrac{h^2}{2}$.

Table 3: Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the explicit Euler method

I	T^n	n	$CPU\ time$	s
16	1.03868306	5563	0.031	-
32	1.03585249	21302	0.109	-
64	1.03514423	81590	0.764	1.998
128	1.03496713	310812	5.382	1.999
256	1.03492285	1182920	36.785	1.999
512	1.03491178	4489383	273.048	2.000
1024	1.03490902	16988420	2054.393	2.000
2048	1.03490832	64077298	15534.408	2.000

Table 4: Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the implicit Euler method

I	T^n	n	$CPU\ time$	s
16	1.03693732	5448	0.031	-
32	1.03541307	20844	0.187	-
64	1.03503419	79755	1.373	2.008
128	1.03493961	303472	10.000	2.002
256	1.03491597	1153557	70.996	2.000
512	1.03491006	4371930	524.897	2.000
1024	1.03490859	16518615	9630.909	2.000
2048	1.03490822	62198077	44458.070	2.000

Third case: $(q,p)=(3,4)$ and $\tau=\dfrac{h^2}{2}$.

Table 5: Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the explicit Euler method

I	T^n	n	$CPU\ time$	s
16	1.10067937	5699	0.047	-
32	1.09816232	22460	0.141	-
64	1.09753226	82607	0.766	1.998
128	1.09737470	312896	5.344	1.999
256	1.09733531	1191036	39.125	1.999
512	1.09732547	4521852	292.969	2.000
1024	1.09732300	17118296	2204.078	2.000
2048	1.09732239	64596804	16831.391	2.000

Table 6: Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the implicit Euler method

I	T^n	n	$CPU\ time$	s
16	1.09876776	5486	0.032	-
32	1.09767989	21568	0.188	-
64	1.09741136	80444	1.266	2.018
128	1.09734446	304206	9.313	2.005
256	1.09732775	1156274	69.563	2.000
512	1.09732357	4382802	546.032	2.000
1024	1.09732253	16562097	13069.532	2.000
2048	1.09732227	62372008	39313.078	2.000

Remark 5.1. *We observe that, the solution of our problem blows up in a finite time and the convection term, responsible of the turbulence, delays the blow-up generated by the reaction term.*

In the following, we also give some plots to illustrate our analysis. For the different plots, we used both explicit and implicit schemes in the case where $I = 16$, $(q, p) = (1, 4)$, $(q, p) = (2, 4)$ and $(q, p) = (3, 4)$.

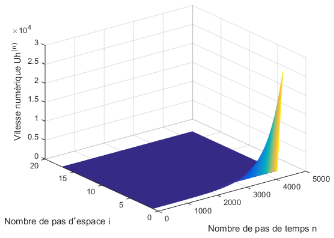

FIGURE 1. Evolution of the discrete solution with $(q,p) = (1,4)$ (explicit scheme).

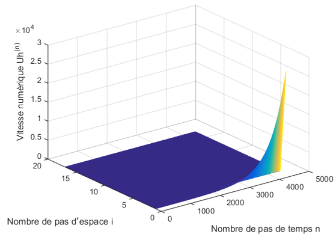

FIGURE 2. Evolution of the discrete solution with $(q,p) = (1,4)$ (implicit scheme).

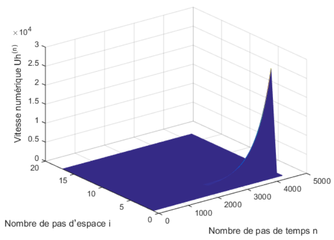

FIGURE 3. Evolution of the discrete solution with $(q,p) = (2,4)$ (explicit scheme).

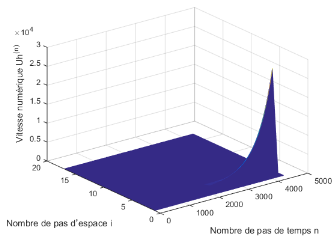

FIGURE 4. Evolution of the discrete solution with $(q,p) = (2,4)$ (implicit scheme).

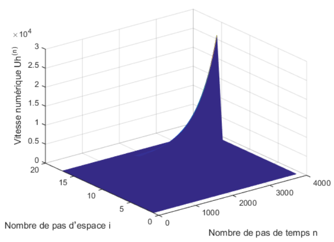

FIGURE 5. Evolution of the discrete solution $(q,p) = (3,4)$ (explicit scheme).

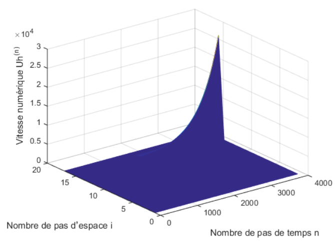

FIGURE 6. Evolution of the discrete solution with $(q,p) = (3,4)$ (implicit scheme).

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