

ANALYTIC FUNCTION OF COMPLEX ORDER INVOLVING CERTAIN MULTIPLIER TRANSFORM

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ABSTRACT. In this paper we introduce and investigate the unification of starlike and convex subclasses of normalized analytic functions given by $S^*C(\delta, \mu, \alpha, \beta, \gamma; b)$ and $TS^*C(\delta, \mu, \alpha, \beta, \gamma; b) = T \cap S^*C(\delta, \mu, \alpha, \beta, \gamma; b)$.

We obtain the initial coefficient bounds for the subclasses so obtained and also examine some relationship of these subclasses with certain existing results in the literature. It was noted that the linear transformation employed in this paper extend some existing transformation and the results generalize some of the results in the existing literature.

1. INTRODUCTION AND PRELIMINARIES

We denote by A the class of normalized analytic functions f in the open unit disc $U = \{z \in C : |z| < 1\}$ with $f(0) = f'(0) = 0$ which is of the form

$$(1.1) \quad f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad a_n \in C,$$

and S the class of all functions in A that are univalent in U . Also, we denote by T the subclass of functions in A and of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0.$$

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The subclasses $S^*(\delta)$, $C(\delta)$ are given respectively by

$$S^*(\delta) = \left\{ f \in S : \operatorname{Re} \left(\frac{zf'(z)}{f(z)} \right) > \delta, z \in U \right\}$$

$$C(\delta) = \left\{ f \in S : \operatorname{Re} \left(1 + \frac{zf''(z)}{f'(z)} \right) > \delta, z \in U, 0 \leq \delta < 1 \right\}.$$

Also, the class $TS^*(\delta)$ given by $T \cap S^*(\delta)$ and the class $TC(\delta)$ are the subclasses of function $f \in T$ such that f is starlike of order δ and respectively f is convex of order δ . The unification of the classes $S^*(\delta)$ and $C(\delta)$ is given by $S^*C(\delta, \tau)$ and satisfies the condition

$$(1.2) \quad \operatorname{Re} \left\{ \frac{zf'(z) + \tau z^2 f''(z)}{\tau z f'(z) + (1 - \tau)f(z)} \right\} > \delta \quad 0 \leq \delta < 1, z \in U.$$

This unification has been studied extensively by various researchers, see [6] and [1, 2, 3]. The classes $S^*(\delta)$ and $C(\delta)$ are the special cases of the unification for $\tau = 0, 1$ respectively.

Moreover, Altintas et al. and other researchers studied the class $TS^*C(\delta, \tau)$ see [3, 5, 6] and using the unification in (1.2), Nizami Mustafa [7] introduced and investigated the classes $S^*C(\delta, \tau; \eta)$ and $TS^*C(\delta, \tau; \eta)$, $0 \leq \delta < 1; \tau \in [0, 1]; \eta \in C$ which he defined as follows:

A function $f \in S$ given by (1.1) is said to belong to the class $S^*C(\delta, \tau; \eta)$ if the following condition is satisfied:

$$\operatorname{Re} \left\{ 1 + \frac{1}{\eta} \left[\frac{zf'(z) + \tau z^2 f''(z)}{\tau z f'(z) + (1 - \tau)f(z)} - 1 \right] \right\} > \delta, \\ 0 \leq \delta < 1; \tau \in [0, 1]; \eta \in C - \{0\}, z \in U.$$

Furthermore, the author in [4] defined a linear transformation $D_{\alpha, \beta, \gamma}^s f$ by

$$(1.3) \quad D_{\alpha, \beta, \gamma}^s f(z) = z + \sum_{n=2}^{\infty} \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s a_n^i z^n, \\ \beta, \gamma \geq 0; \alpha \geq 1; s \in \mathbb{N} \cup 0, i(1 \leq i \leq k).$$

where $k \in \mathbb{N}$.

Motivated by the work of Mustafa, Makinde and Najafzadeh in [7, 4], we study effect of the application of the linear operator $D_{\alpha, \beta, \gamma}^s f$ on the unification of the classes of the functions $S^*C(\gamma, \beta; \tau)$.

Now, we define $S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$ to be class of functions $f \in S$ satisfying the condition

$$(1.4) \quad \operatorname{Re} \left\{ 1 + \frac{1}{b} \left[\frac{z(D_{\alpha, \beta, \gamma}^s f)'(z) + \mu z^2(D_{\alpha, \beta, \gamma}^s f)''(z)}{\mu z(D_{\alpha, \beta, \gamma}^s f)'(z) + (1 - \mu)(D_{\alpha, \beta, \gamma}^s f)(z)} - 1 \right] \right\} > \delta,$$

$$0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; \alpha \geq 1; s \in \mathbb{N} \cup 0.$$

In addition, we denote by D_T the subclass of the class of functions f in (1.3) which is of the form

$$(1.5) \quad D_{\alpha, \beta, \gamma}^s f(z) = z - \sum_{n=2}^{\infty} \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s a_n^i z^n,$$

$$\beta, \gamma \geq 0; \alpha \geq 1; s \in \mathbb{N} \cup 0, i(1 \leq i \leq k).$$

and denote by $TS^*C(\delta, \mu, \alpha, \gamma, \beta; b) = T \cap S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$ the class of functions f in (1.5) such that f belong to the class $S^*C(\delta, \mu, \alpha, \gamma, \beta; b) = T \cap S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$.

In this paper, we investigate the subclasses $S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$ and $TS^*C(\delta, \mu, \alpha, \gamma, \beta; b) = T \cap S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$ which is presented in the next section.

2. COEFFICIENT BOUNDS FOR THE CLASSES $S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$ AND $TS^*C(\delta, \mu, \alpha, \gamma, \beta; b)$

Theorem 2.1. *Let f be as defined in (1.1). Then the function $D_{\alpha, \beta, \gamma}^s f$ belongs to the class $S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$, $0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; \alpha \geq 1; s \in \mathbb{N} \cup 0$*

if

$$\sum_{n=2}^{\infty} \left[\left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s [1 + \mu(n-1)][n + |b|(1-\delta) - 1] \right] |a_n^i| \leq |b|(1-\delta).$$

The result is sharp for the function

$$D_{\alpha, \beta, \gamma}^s f(z) = z + \frac{|b|(1-\delta)(\alpha + \beta + \gamma)^s}{[1 + \mu(n-1)][n + |b|(1-\delta)](\alpha + n\beta + n^2\gamma)^s} z^n \quad n \geq 2.$$

Proof. By (1.4), f belong to the class $S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$ if

$$\operatorname{Re} \left\{ 1 + \frac{1}{b} \left[\frac{z(D_{\alpha, \beta, \gamma}^s f)'(z) + \mu z^2(D_{\alpha, \beta, \gamma}^s f)''(z)}{\mu z(D_{\alpha, \beta, \gamma}^s f)'(z) + (1 - \mu)(D_{\alpha, \beta, \gamma}^s f)(z)} - 1 \right] \right\} > \delta$$

It suffices to show that:

$$(2.1) \quad \left| \frac{1}{b} \left[\frac{z(D_{\alpha,\beta,\gamma}^s f)'(z) + \mu z^2(D_{\alpha,\beta,\gamma}^s f)''(z)}{\mu z(D_{\alpha,\beta,\gamma}^s f)'(z) + (1-\mu)(D_{\alpha,\beta,\gamma}^s f)(z)} - 1 \right] \right| < 1 - \delta$$

Simple computation in (2.1), using (1.3), we have:

$$\begin{aligned} & \left| \frac{1}{b} \left[\frac{z(D_{\alpha,\beta,\gamma}^s f)'(z) + \mu z^2(D_{\alpha,\beta,\gamma}^s f)''(z)}{\mu z(D_{\alpha,\beta,\gamma}^s f)'(z) + (1-\mu)(D_{\alpha,\beta,\gamma}^s f)(z)} - 1 \right] \right| \\ &= \left| \frac{1}{b} \left[\frac{z + \sum_{n=2}^{\infty} n \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s a_n^i z^n + \mu \sum_{n=2}^{\infty} n(n-1) \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s a_n^i z^n}{\mu z + \sum_{n=2}^{\infty} \mu n \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s a_n^i z^n + (1-\mu) \left(z + \sum_{n=2}^{\infty} \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s a_n^i z^n \right)} - 1 \right] \right| \\ &= \left| \frac{1}{b} \left[\frac{z + \sum_{n=2}^{\infty} n[1 + \mu(n-1)] \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s a_n^i z^n}{z + \sum_{n=2}^{\infty} (1 + \mu(n-1)) \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s a_n^i z^n} - 1 \right] \right| \\ &\leq \frac{1}{|b|} \left[\frac{\sum_{n=2}^{\infty} (n-1)[1 + \mu(n-1)] \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s |a_n^i|}{1 - \sum_{n=2}^{\infty} (1 + \mu(n-1)) \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s |a_n^i|} \right] \end{aligned}$$

which is bounded by $1 - \delta$ if

$$\begin{aligned} & \sum_{n=2}^{\infty} (n-1)[1 + \mu(n-1)] \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s |a_n^i| \leq \\ & \leq |b|(1-\delta)1 - \sum_{n=2}^{\infty} (1 + \mu(n-1)) \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s |a_n^i|, \end{aligned}$$

which is equivalent to:

$$\begin{aligned} & \sum_{n=2}^{\infty} \left[(n-1)[1 + \mu(n-1)] \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s + \right. \\ & \left. + |b|(1-\delta)(1 + \mu(n-1)) \left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s \right] |a_n^i| \leq |b|(1-\delta). \end{aligned}$$

This implies that:

$$(2.2) \quad \sum_{n=2}^{\infty} \left[\left(\frac{\alpha+n\beta+n^2\gamma}{\alpha+\beta+\gamma} \right)^s [1 + \mu(n-1)][n + |b|(1-\delta) - 1] \right] |a_n^i| \leq |b|(1-\delta).$$

Thus, (2.1) is satisfied if (2.2) is satisfied. $\square$

Corollary 2.1. *Let f be as defined in (1.1) and the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, \mu, \alpha, \gamma, \beta; b)$,*

$0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; \alpha \geq 1; s \in \mathbb{N} \cup 0$. Then:

$$|a_n^i| \leq \frac{|b|(1-\delta)(\alpha + \beta + \gamma)^s}{[1 + \mu(n-1)][n + |b|(1-\delta) - 1](\alpha + n\beta + n^2\gamma)^s}.$$

Corollary 2.2. *Let f be as defined in (1.1). Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, \mu, 1, \gamma, \beta, s; b)$,*

$0 \geq \delta < 1, z \in U; \gamma, \beta \geq 0, \mu \leq 1; s \in \mathbb{N} \cup 0$ if

$$\sum_{n=2}^{\infty} \left(\frac{1 + n\beta + n^2\gamma}{1 + \beta + \gamma} \right)^s [1 + \mu(n-1)] [n + |b|(1-\delta) - 1] |a_n^i| \leq |b|(1-\delta).$$

The result is sharp for the function

$$D_{1,\beta,\gamma}^s f(z) = z + \frac{|b|(1-\delta)(1 + \beta + \gamma)^s}{[1 + \mu(n-1)][n + |b|(1-\delta)](1 + n\beta + n^2\gamma)^s} z^n, \quad n \geq 2.$$

Corollary 2.3. *Let f be as defined in (1.1). Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, \mu, 1, \beta, \gamma, 1; b)$,*

$0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; s \in \mathbb{N} \cup 0$ if

$$\sum_{n=2}^{\infty} \left(\frac{1 + n\beta + n^2\gamma}{1 + \beta + \gamma} \right) [1 + \mu(n-1)] [n + |b|(1-\delta) - 1] |a_n^i| \leq |b|(1-\delta).$$

The result is sharp for the function

$$D_{1,\beta,\gamma}^1 f(z) = z + \frac{|b|(1-\delta)(1 + \beta + \gamma)}{[1 + \mu(n-1)][n + |b|(1-\delta)](1 + n\beta + n^2\gamma)} z^n, \quad n \geq 2.$$

Corollary 2.4. *Let f be as defined in (1.1). Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, \mu, 1, 1, \beta, 1; b)$,*

$0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; s \in \mathbb{N} \cup 0$ if

$$\sum_{n=2}^{\infty} \left(\frac{1 + n(\beta + n)}{2 + \beta} \right) [1 + \mu(n-1)] [n + |b|(1-\delta) - 1] |a_n^i| \leq |b|(1-\delta).$$

The result is sharp for the function

$$D_{1,\beta,1}^1 f(z) = z + \frac{|b|(1-\delta)(2 + \beta)}{[1 + \mu(n-1)][n + |b|(1-\delta)](1 + n(\beta + n))} z^n, \quad n \geq 2.$$

Corollary 2.5. *Let f be as defined in (1.1). Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, \mu, 1, 0, 0, 1; b)$, $0 \leq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; s \in \mathbb{N} \cup 0, i = 1$. if*

$$\sum_{n=2}^{\infty} [[1 + \mu(n-1)][n + |b|(1-\delta) - 1]] |a_n| \leq |b|(1-\delta).$$

The result is sharp for the function

$$D_{1,0,0}^1 f(z) = z + \frac{|b|(1-\delta)}{[1 + \mu(n-1)][n + |b|(1-\delta) - 1]} z^n, \quad n \geq 2.$$

Remark 2.1. *This result agrees with the Theorem 2.1 in [6].*

Corollary 2.6. *Let f be as defined in (1.1). Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, \mu, \alpha, \beta, \gamma, 0; b)$, $0 \leq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; s \in \mathbb{N} \cup 0, i = 1$. if*

$$\sum_{n=2}^{\infty} [[1 + \mu(n-1)][n + |b|(1-\delta) - 1]] |a_n| \leq |b|(1-\delta).$$

The result is sharp for the function

$$D_{\alpha,\beta,\gamma}^0 f(z) = z + \frac{|b|(1-\delta)}{[1 + \mu(n-1)][n + |b|(1-\delta) - 1]} z^n, \quad n \geq 2.$$

Remark 2.2. *This result agrees with the Corollary 1 in [5] with $\lambda = 0, m = 1, \alpha = 1$ and $\delta = \gamma$.*

Corollary 2.7. *Let f be as defined in (1.1). Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, \mu, \alpha, \beta, \gamma, 0; 1)$, $0 \leq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; s \in \mathbb{N} \cup 0, i = 1$. if*

$$\sum_{n=2}^{\infty} [[1 + \mu(n-1)][n - \delta]] |a_n| \leq 1 - \delta.$$

The result is sharp for the function

$$D_{\alpha,\beta,\gamma}^0 f(z) = z + \frac{1-\delta}{[1 + \mu(n-1)][n - \delta]} z^n, \quad n \geq 2.$$

Remark 2.3. *This result agrees with the Corollary 2.1 in [7] and Corollary 6 in [5] with $\delta = \gamma$.*

Corollary 2.8. *Let f be as defined in (1.1). Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $S^*C(\delta, 0, \alpha, \beta, \gamma, 0; 1)$,*

$0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; s \in \mathbb{N} \cup 0, i = 1$. if

$$(2.3) \quad \sum_{n=2}^{\infty} (n - \delta) |a_n| \leq 1 - \delta$$

The result is sharp for the function

$$D_{\alpha,\beta,\gamma}^0 f(z) = z + \frac{1 - \delta}{n - \delta} z^n, \quad n \geq 2$$

Remark 2.4. *This result agrees with the Corollary 2.2 in [7] and Corollary 7 in [5] with $\delta = \gamma$.*

Theorem 2.2. *Let $f \in D_T$. Then the function $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $D_T S^*C(\delta, \mu, \alpha, \beta, \gamma; b)$,*

$0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; \alpha \geq 1; s \in \mathbb{N} \cup 0$ if and only if

$$\sum_{n=2}^{\infty} (n - 1) [1 + \mu(n - 1)] [n + b(1 - \delta)] \left(\frac{x}{y} \right)^s |a_n^i| \leq |b|(1 - \delta).$$

Proof. We shall prove only the necessity part of the Theorem 2.2 as the sufficiency proof is similar to the proof of Theorem 2.1.

Let f be as defined in (1.1) and $D_{\alpha,\beta,\gamma}^s f$ belongs to the class $TS^*C(\delta, \mu, \alpha, \beta, \gamma; b)$, $0 \geq \delta < 1, z \in U; \beta, \gamma \geq 0, \mu \leq 1; \alpha \geq 1; s \in \mathbb{N} \cup 0; i(1 \leq i \leq k), b \in \mathbb{R} - \{0\}$, we have

$$(2.4) \quad \operatorname{Re} \left\{ 1 + \frac{1}{b} \left[\frac{z(D_{\alpha,\beta,\gamma}^s f)'(z) + \mu z^2 (D_{\alpha,\beta,\gamma}^s f)''(z)}{\mu z (D_{\alpha,\beta,\gamma}^s f)'(z) + (1 - \mu)(D_{\alpha,\beta,\gamma}^s f)(z)} - 1 \right] \right\} > \delta$$

Using (2.4) in (2.3) and by algebraic simplification, we have:

$$\operatorname{Re} \left\{ \frac{-\sum_{n=2}^{\infty} (n - 1) [1 + \mu(n - 1)] \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s a_n^i z^n}{b \left\{ z - \sum_{n=2}^{\infty} (1 + \mu(n - 1)) \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s a_n^i z^n \right\}} \right\} \geq \delta - 1.$$

Choosing z to be real and $z \rightarrow 1$, we have

$$(2.5) \quad \frac{-\sum_{n=2}^{\infty} (n - 1) [1 + \mu(n - 1)] \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s a_n^i}{b \left\{ 1 - \sum_{n=2}^{\infty} (1 + \mu(n - 1)) \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma} \right)^s a_n^i \right\}} \geq \delta - 1$$

$b \in \mathbb{R} - \{0\}$ implies that b could be greater or less than zero.

Let $b > 0$ in (2.4), we have

$$-\sum_{n=2}^{\infty} (n-1)[1+\mu(n-1)] \left(\frac{u}{v}\right)^s a_n^i \geq (\delta-1)b \left\{ 1 - \sum_{n=2}^{\infty} (1+\mu(n-1)) \left(\frac{u}{v}\right)^s a_n^i \right\}$$

where $u = \alpha + n\beta + n^2\gamma$ and $v = \alpha + \beta + \gamma$ From (2.5), we have

$$(2.6) \quad \sum_{n=2}^{\infty} (n-1)[1+\mu(n-1)][n+b(1-\delta)] \left(\frac{u}{v}\right)^s |a_n^i| \geq b(1-\delta)$$

Now suppose $b < 0$, which implies that $b = -|b|$ and substituting $b = -|b|$ in (1.5), we have

$$(2.7) \quad \frac{\sum_{n=2}^{\infty} (n-1)[1+\mu(n-1)] \left(\frac{u}{v}\right)^s a_n^i}{|b| \left\{ 1 - \sum_{n=2}^{\infty} (1+\mu(n-1)) \left(\frac{u}{v}\right)^s a_n^i \right\}} \geq$$

$$\sum_{n=2}^{\infty} (n-1)[1+\mu(n-1)] \left(\frac{u}{v}\right)^s |a_n^i| \geq (\delta-1)|b| \left\{ 1 - \sum_{n=2}^{\infty} (1+\mu(n-1)) \left(\frac{u}{v}\right)^s a_n^i \right\}$$

which implies

$$\sum_{n=2}^{\infty} (n-1)[1+\mu(n-1)][n+b(1-\delta)] \left(\frac{\alpha + n\beta + n^2\gamma}{\alpha + \beta + \gamma}\right)^s |a_n^i| \geq -|b|(1-\delta)$$

From (2.6) and (2.7), the proof of the necessity is completed. $\square$

3. CONCLUSIONS

In this study, we apply the generalized multiplier transform of Makinde *et-al* [4], in the method of Mustafa [7]. The results in this paper generalize the work of Irmak *et. al*, Mustafa and Makinde and Najafzadeh as it is shown in some of the corollaries.

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