

FUZZY SOFT HERMITIAN OPERATORS

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ABSTRACT. In this paper, we define the fuzzy soft hermitian operator, which is a special type of fuzzy soft linear operators in fuzzy soft Hilbert spaces. Moreover, related theorems including fuzzy soft point spectrum theorem and more are introduced. Furthermore, an example in favor of the fuzzy soft hermitian operator and another one against it are investigated.

1. INTRODUCTION

In real world, the complexity generally arises from uncertainty in the form of ambiguity. So, we always have many complicated problems in the areas like economics, engineering, medical science, environmental science, sociology, business management and many other fields. We can't successfully use classical mathematical methods to overcome difficulties of uncertainties in those problems.

In 1965, Zadeh [11] proposed an extension of the set theory which is the theory of fuzzy sets to deal with uncertainty. Just as a crisp set on a universal set X is defined by its characteristic function from X to $\{0, 1\}$, a fuzzy set on a domain X is defined by its membership (characteristic) function from X to $[0, 1]$.

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2010 *Mathematics Subject Classification.* 46B99, 03E72, 46S40.

Key words and phrases. Fuzzy set, Fuzzy soft Hilbert space, Fuzzy soft linear operator, Fuzzy soft self-adjoint operator, Fuzzy soft set, Soft set.

In fact, the notion of a fuzzy set is completely non-statistical in nature. Fuzzy set theory is very useful mathematical tool to handle uncertainty, but this single value (membership degree) combines the evidence for element's belonging and the evidence against element's belonging without indicating how much there is of each, i.e., the single number tells us nothing about its accuracy.

In 1999, Molodtsov [7] introduced an extension of the set theory namely soft set theory to overcome uncertainties and solve complicated problems which can't be dealt with by classical methods in many areas such as Riemann integration, measure theory, environmental science, decision making, game theory, physics, engineering, computer science, medicine, economics and many other fields. The soft set is a mathematical tool for modeling uncertainty by associating a set with a set of parameters, i.e., it is a parameterized family of subsets of the universal set. After that, many researchers introduced new extended concepts based on soft sets, gave examples for them and studied their properties like soft point [1], soft metric spaces [5], soft normed spaces [10], soft inner product spaces [4] and soft Hilbert spaces [9], etc.

But almost all the time, although this progress, in real life problems and situations, we still have inexact information about our considered objects. So, to improve those two concepts; fuzzy set and soft set, Maji et al. [6] combined them together in one concept and called this new concept fuzzy soft set. This new concept widened the soft sets approach from crisp (ordinary) cases to fuzzy cases which is more general than any other. In recent years, many researchers applied this notion and gave some concepts such as fuzzy soft point [8], fuzzy soft metric spaces [2] and fuzzy soft normed spaces [3].

In present, Faried et al. introduced the fuzzy soft inner product spaces with studying its properties and some related results. Also, they gave the definition of the fuzzy soft Hilbert space with studying its properties and many more related results. In addition, they continued by defining the fuzzy soft linear operators in fuzzy soft Hilbert spaces with their related theorems including spectral theory and proving fuzzy soft Hilbert space's fuzzy soft self-duality. Finally, they defined the fuzzy soft symmetric operator and studied its properties.

In this work, we progress on these stated previous studies by introducing a special type of fuzzy soft linear operators in fuzzy soft Hilbert spaces, which is the fuzzy soft hermitian operator, establishing related theorems including fuzzy soft point spectrum theorem, an example in favor of the fuzzy soft hermitian operator and another one against it and more.

2. DEFINITIONS AND PRELIMINARIES

The aim of this section is to list some notations, definitions and preliminaries needed in the following discussion.

Definition 2.1. [6] Let U be a universal set, E be a set of parameters and $A \subseteq E$. A pair (G, A) is called a fuzzy soft set over U , where G is a mapping given by $G : A \rightarrow \mathcal{F}(U)$, $\mathcal{F}(U)$ is the family of all fuzzy subsets of U and the fuzzy subset of U is defined as a map f from U to $[0, 1]$. The family of all fuzzy soft sets (G, A) over a universal set U , in which all the parameter sets A are the same, is denoted by $FSS(U)_A = FSS(\tilde{U})$.

Definition 2.2. [8] The fuzzy soft set $(G, A) \in FSS(\tilde{U})$ is called a fuzzy soft point over U , denoted by $(u_{f_{G(e)}}, A)$ (briefly denoted by $\tilde{u}_{f_{G(e)}}$), if for the element $e \in A$ and $u \in U$,

$$f_{G(e)}(u) = \begin{cases} \alpha, & \text{if } u = u_0 \in U \text{ and } e = e_0 \in A, \\ 0, & \text{if } u \in U - \{u_0\} \text{ or } e \in A - \{e_0\} \end{cases},$$

where $\alpha \in (0, 1]$ is the value of the membership degree.

It should be noted that $\mathbb{C}(A)$ and $\mathbb{R}(A)$ denote the set of all fuzzy soft complex numbers and the set of all fuzzy soft real numbers, respectively.

Definition 2.3. Let $(\tilde{U}, \widetilde{\langle \cdot, \cdot \rangle})$ be a fuzzy soft inner product space and

$$\tilde{v}_{f_{1G(e_1)}}^1, \tilde{v}_{f_{2G(e_2)}}^2 \in \tilde{H}.$$

$\tilde{v}_{f_{1G(e_1)}}^1$ is said to be fuzzy soft orthogonal to $\tilde{v}_{f_{2G(e_2)}}^2$ ($\tilde{v}_{f_{1G(e_1)}}^1 \perp \tilde{v}_{f_{2G(e_2)}}^2$) if

$$\widetilde{\langle \tilde{v}_{f_{1G(e_1)}}^1, \tilde{v}_{f_{2G(e_2)}}^2 \rangle} \approx \tilde{0}.$$

Definition 2.4. Let $(\tilde{U}, \widetilde{\langle \cdot, \cdot \rangle})$ be a fuzzy soft inner product space. Then, this space, which is fuzzy soft complete in the induced fuzzy soft norm is called a fuzzy soft Hilbert space, denoted by $(\tilde{H}, \widetilde{\langle \cdot, \cdot \rangle})$ (shortly $\tilde{H}$). It is clear that every fuzzy soft Hilbert space is a fuzzy soft Banach space.

Theorem 2.1. Let $\tilde{T}$ be a fuzzy soft linear operator on $\tilde{H}$ and $\tilde{\mathbb{B}}(\tilde{H})$ be the set of all fuzzy soft bounded linear operators on $\tilde{H}$.

If $\tilde{T}, \tilde{T}_1, \tilde{T}_2 \in \tilde{\mathbb{B}}(\tilde{H})$ and $\tilde{\lambda} \in \mathbb{C}(A)$, then:

$$\tilde{T}^{*\sim} \simeq \tilde{T}, \quad (\tilde{\lambda}\tilde{T})^* \simeq \tilde{\lambda}\tilde{T}^*, \quad (\tilde{T}_1 + \tilde{T}_2)^* \simeq \tilde{T}_1^* + \tilde{T}_2^*, \quad (\tilde{T}_1\tilde{T}_2)^* \simeq \tilde{T}_2^*\tilde{T}_1^*,$$

and if $\tilde{T}^{-1}$ exists, then $(\tilde{T}^*)^{-1}$ exists and $(\tilde{T}^{-1})^* \simeq (\tilde{T}^*)^{-1}$.

Definition 2.5. $\tilde{\lambda} \in \mathbb{C}(A)$ is said to be a fuzzy soft eigenvalue of $\tilde{T}$ if there exists $\tilde{\theta} \neq \tilde{v}_{f_{G(e)}} \in \tilde{H}$ such that $\tilde{T}\tilde{v}_{f_{G(e)}} \simeq \tilde{\lambda}\tilde{v}_{f_{G(e)}}$ and $\tilde{v}_{f_{G(e)}} \neq \tilde{\theta}$ is called the fuzzy soft eigenvector of $\tilde{T}$ corresponding to $\tilde{\lambda}$. The set of all such $\tilde{\lambda}$ is called the fuzzy soft point spectrum of $\tilde{T}$, denoted by $\tilde{\sigma}_p(\tilde{T})$.

Definition 2.6. $\tilde{T} \in \tilde{\mathbb{B}}(\tilde{H})$ is fuzzy soft symmetric operator if for all

$$\tilde{v}_{f_{1G(e_1)}}^1, \tilde{v}_{f_{2G(e_2)}}^2 \in \tilde{H},$$

$$(2.1) \quad \widetilde{\langle \tilde{T}\tilde{v}_{f_{1G(e_1)}}^1, \tilde{v}_{f_{2G(e_2)}}^2 \rangle} \simeq \widetilde{\langle \tilde{v}_{f_{1G(e_1)}}^1, \tilde{T}\tilde{v}_{f_{2G(e_2)}}^2 \rangle},$$

3. MAIN RESULTS

The aim of this section is to introduce the fuzzy soft self-adjoint operators or the fuzzy soft hermitian operators in fuzzy soft Hilbert spaces. In addition, a study on related theorems involving their fuzzy soft eigenvalues and fuzzy soft eigenvectors and many more results are introduced.

Finally, an example in favor of the fuzzy soft hermitian operator and another one against it are established.

Definition 3.1. Fuzzy soft hermitian (fuzzy soft self-adjoint) operator

Let $\tilde{H}$ be a fuzzy soft Hilbert space. $\tilde{T} \in \tilde{\mathbb{B}}(\tilde{H})$ is called fuzzy soft hermitian operator (or, fuzzy soft self-adjoint operator) if

$$(3.1) \quad \tilde{T} \simeq \tilde{T}^*.$$

Remark 3.1. Every fuzzy soft hermitian operator is fuzzy soft symmetric.

Proof. By using (3.1) from Definition 3.1 of fuzzy soft hermitian operator, we get:

$$\begin{aligned} \langle \widetilde{T\tilde{v}_{f_1G(e_1)}^1}, \widetilde{\tilde{v}_{f_2G(e_2)}^2} \rangle &\stackrel{\sim}{=} \langle \widetilde{\tilde{v}_{f_1G(e_1)}^1}, \widetilde{T^*\tilde{v}_{f_2G(e_2)}^2} \rangle \\ &\stackrel{\sim}{=} \langle \widetilde{\tilde{v}_{f_1G(e_1)}^1}, \widetilde{T\tilde{v}_{f_2G(e_2)}^2} \rangle, \end{aligned}$$

for each $\tilde{v}_{f_1G(e_1)}^1, \tilde{v}_{f_2G(e_2)}^2 \in \tilde{H}$, i.e., the fuzzy soft hermitian operator satisfies the condition (2.1) stated in Definition 2.6 of fuzzy soft symmetric operator and this completes the proof. $\square$

Lemma 3.1. Let $\tilde{T} \in \tilde{\mathbb{B}}(\tilde{H})$ be fuzzy soft hermitian. Then, for

$\tilde{v}_{fG(e)} \in \tilde{H}$, $\langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle$ is fuzzy soft real and

$$\inf_{\|\widetilde{\tilde{v}_{fG(e)}}\| \stackrel{\sim}{=} \tilde{1}} \langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle \stackrel{\sim}{\leq} \langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle \stackrel{\sim}{\leq} \sup_{\|\widetilde{\tilde{v}_{fG(e)}}\| \stackrel{\sim}{=} \tilde{1}} \langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle.$$

Proof. Since every fuzzy soft hermitian operator is fuzzy soft symmetric from Remark 3.1 and by using relation (2.1) from Definition 2.6, we have:

$$\langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle \stackrel{\sim}{=} \overline{\langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle}.$$

Hence, $\langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle$ is fuzzy soft real for $\tilde{v}_{fG(e)} \in \tilde{H}$. For the second part, we can define (write, shortly):

$$\tilde{m}(\tilde{T}) \stackrel{\sim}{=} \inf_{\|\widetilde{\tilde{v}_{fG(e)}}\| \stackrel{\sim}{=} \tilde{1}} \langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle$$

and

$$\tilde{M}(\tilde{T}) \stackrel{\sim}{=} \sup_{\|\widetilde{\tilde{v}_{fG(e)}}\| \stackrel{\sim}{=} \tilde{1}} \langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle.$$

It is clear from the definition of $\tilde{s}\tilde{u}\tilde{p}$ and $\tilde{i}\tilde{n}\tilde{f}$ that

$$\tilde{m}(\tilde{T}) \stackrel{\sim}{\leq} \langle \widetilde{T\tilde{v}_{fG(e)}}, \widetilde{\tilde{v}_{fG(e)}} \rangle \stackrel{\sim}{\leq} \tilde{M}(\tilde{T}).$$

$\square$

Theorem 3.1. *If $\tilde{T} \in \tilde{\mathbb{B}}(\tilde{H})$ is fuzzy soft hermitian operator and $\tilde{\lambda}$ is a fuzzy soft eigenvalue of $\tilde{T}$, then $\tilde{\lambda}$ is fuzzy soft real and $\tilde{m}(\tilde{T}) \leq \tilde{\lambda} \leq \tilde{M}(\tilde{T})$. Fuzzy soft eigenvectors corresponding to different fuzzy soft eigenvalues are fuzzy soft orthogonal.*

Proof. Let $\tilde{\lambda}$ be a fuzzy soft eigenvalue of $\tilde{T}$, then from Definition 2.5, there exists $\tilde{\theta} \neq \tilde{v}_{f_{G(e)}} \in \tilde{H}$ such that $\tilde{T}\tilde{v}_{f_{G(e)}} = \tilde{\lambda}\tilde{v}_{f_{G(e)}}$. Suppose that $\|\tilde{v}_{f_{G(e)}}\| = 1$, then:

$$\begin{aligned} \langle \tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}} \rangle &= \langle \tilde{\lambda}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}} \rangle \\ &= \tilde{\lambda} \langle \tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}} \rangle \\ &= \tilde{\lambda} \|\tilde{v}_{f_{G(e)}}\|^2 \\ &= \tilde{\lambda}. \end{aligned}$$

Now, we have $\tilde{\lambda} = \langle \tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}} \rangle$. Then, by using Lemma 3.1, we obtain that $\tilde{\lambda}$ is fuzzy soft real and $\tilde{m}(\tilde{T}) \leq \tilde{\lambda} \leq \tilde{M}(\tilde{T})$. To prove the second part, let $\tilde{T}\tilde{v}_{f_{G(e)}} = \tilde{\lambda}\tilde{v}_{f_{G(e)}}$ and $\tilde{T}\tilde{u}_{f_{G(e)}} = \tilde{\mu}\tilde{u}_{f_{G(e)}}$; $\tilde{\lambda} \neq \tilde{\mu}$, $\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \neq \tilde{\theta}$. Then, by using the first part of the Theorem 3.1. and since every fuzzy soft hermitian operator is fuzzy soft symmetric from Remark 3.1, we get:

$$\begin{aligned} \tilde{\lambda} \langle \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \rangle &= \langle \tilde{\lambda}\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \rangle \\ &= \langle \tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \rangle \\ &= \langle \tilde{v}_{f_{G(e)}}, \tilde{T}\tilde{u}_{f_{G(e)}} \rangle \\ &= \langle \tilde{v}_{f_{G(e)}}, \tilde{\mu}\tilde{u}_{f_{G(e)}} \rangle \\ &= \tilde{\mu} \langle \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \rangle. \end{aligned}$$

Therefore, $(\tilde{\lambda} - \tilde{\mu}) \langle \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \rangle = \tilde{0}$. Since $\tilde{\lambda} \neq \tilde{\mu}$, then $\tilde{\lambda} - \tilde{\mu} \neq \tilde{0}$ and thus $\langle \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \rangle = \tilde{0}$. Hence, $\tilde{v}_{f_{G(e)}}$ and $\tilde{u}_{f_{G(e)}}$ are fuzzy soft orthogonal. $\square$

Theorem 3.2. *If $\tilde{T}, \tilde{T}_1, \tilde{T}_2 \in \tilde{\mathbb{B}}(\tilde{H})$ are fuzzy soft hermitian operators and $\tilde{\lambda} \in \mathbb{R}(A)$, then*

- (1) $\tilde{T}_1 + \tilde{T}_2$ is fuzzy soft hermitian operator.
- (2) $\tilde{\lambda}\tilde{T}$ is fuzzy soft hermitian operator.

Proof. (1) We have $(\tilde{T}_1 + \tilde{T}_2)^* \cong \tilde{T}_1^* + \tilde{T}_2^*$. Since $\tilde{T}_1$ and $\tilde{T}_2$ are fuzzy soft hermitian operators, then, from (3.1), we obtain that $\tilde{T}_1^* \cong \tilde{T}_1$ and $\tilde{T}_2^* \cong \tilde{T}_2$. Therefore, $(\tilde{T}_1 + \tilde{T}_2)^* \cong (\tilde{T}_1 + \tilde{T}_2)$. Hence, $(\tilde{T}_1 + \tilde{T}_2)$ is fuzzy soft hermitian operator.

(2) Since $(\tilde{\lambda}\tilde{T})^* \cong \tilde{\lambda}\tilde{T}^*$ and we have $\tilde{\lambda}$ is fuzzy soft real (i.e., $\tilde{\lambda} \cong \tilde{\lambda}$) and $\tilde{T}$ is fuzzy soft hermitian operator (i.e., $\tilde{T}^* \cong \tilde{T}$), then $(\tilde{\lambda}\tilde{T})^* \cong (\tilde{\lambda}\tilde{T})$. Hence, $(\tilde{\lambda}\tilde{T})$ is fuzzy soft hermitian operator. □

Theorem 3.3. If $\tilde{T} \in \tilde{\mathbb{B}}(\tilde{H})$, then, we have:

- (1) $\tilde{T}\tilde{T}^*$ and $\tilde{T}^*\tilde{T}$ are fuzzy soft hermitian operators.
- (2) $\tilde{T}\tilde{T}^* - \tilde{I}$ is fuzzy soft hermitian operator.
- (3) If $\tilde{T}$ is a fuzzy soft hermitian operator and $\tilde{T}^{-1}$ exists, then $\tilde{T}^{-1}$ is also a fuzzy soft hermitian operator.

Proof. First of all, we have $\tilde{I}$ is fuzzy soft hermitian operator, since $\tilde{I}^* \cong \tilde{I}$.

- (1) $(\tilde{T}\tilde{T}^*)^* \cong \tilde{T}^{**}\tilde{T}^*$. By using Theorem 2.1, we get $(\tilde{T}\tilde{T}^*)^* \cong \tilde{T}\tilde{T}^*$. Then, $\tilde{T}\tilde{T}^*$ is fuzzy soft hermitian operator.

Similarly, we have that $\tilde{T}^*\tilde{T}$ is fuzzy soft hermitian operator.

- (2) By using (1) from above, we get that:

$$(\tilde{T}\tilde{T}^* - \tilde{I})^* \cong (\tilde{T}\tilde{T}^*)^* - \tilde{I}^* \cong \tilde{T}\tilde{T}^* - \tilde{I}.$$

Hence, $\tilde{T}\tilde{T}^* - \tilde{I}$ is fuzzy soft hermitian operator.

- (3) If $\tilde{T}$ is fuzzy soft hermitian operator and $\tilde{T}^{-1}$ exists, then, by using Theorem 2.1, we have

$$(\tilde{T}^{-1})^* \cong (\tilde{T}^*)^{-1} \cong \tilde{T}^{-1}.$$

Therefore, $\tilde{T}^{-1}$ is fuzzy soft hermitian operator. □

Example 1. The fuzzy soft operator $\tilde{2}i\tilde{I}$ is not fuzzy soft hermitian.

Solution. Since we have:

$$(\tilde{2}i\tilde{I})^* \cong \tilde{2}i\tilde{I}^* \cong -\tilde{2}i\tilde{I}.$$

Then, $\tilde{2}i\tilde{I}$ is not fuzzy soft hermitian.

Example 2. If $\tilde{T} \in \tilde{\mathbb{B}}(\tilde{H})$, define

$$\begin{aligned}\tilde{T}_1 &\doteq \frac{\tilde{1}}{\tilde{2}}(\tilde{T} + \tilde{T}^*), \\ \tilde{T}_2 &\doteq \frac{-\tilde{i}}{\tilde{2}}(\tilde{T} - \tilde{T}^*).\end{aligned}$$

Then, $\tilde{T}_1$ and $\tilde{T}_2$ are fuzzy soft hermitian operators and $\tilde{T} \doteq \tilde{T}_1 + i\tilde{T}_2$.

Solution. For,

$$\begin{aligned}\tilde{T}_1^* &\doteq \frac{\tilde{1}}{\tilde{2}}(\tilde{T} + \tilde{T}^*)^* \\ &\doteq \frac{\tilde{1}}{\tilde{2}}(\tilde{T}^* + \tilde{T}^{**}) \\ &\doteq \frac{\tilde{1}}{\tilde{2}}(\tilde{T} + \tilde{T}^*)\end{aligned}$$

by using Theorem 2.1.

Also,

$$\begin{aligned}\tilde{T}_2^* &\doteq \left[\frac{-\tilde{i}}{\tilde{2}}(\tilde{T} - \tilde{T}^*) \right]^* \\ &\doteq \frac{\tilde{i}}{\tilde{2}}(\tilde{T} - \tilde{T}^*)^* \\ &\doteq \frac{\tilde{i}}{\tilde{2}}(\tilde{T}^* - \tilde{T}^{**}) \\ &\doteq \frac{-\tilde{i}}{\tilde{2}}(\tilde{T} - \tilde{T}^*)\end{aligned}$$

by using Theorem 2.1.

Therefore, $\tilde{T}_1$ and $\tilde{T}_2$ are fuzzy soft hermitian operators and it is clear that $\tilde{T} \doteq \tilde{T}_1 + i\tilde{T}_2$.

4. CONCLUSIONS

Introducing the fuzzy version or the soft version of topics like metric spaces, normed spaces and Hilbert spaces has been studied by many mathematicians. On the other hand, combining fuzzy and soft sets together gives us more extended, generalized and accurate results. Few researchers have studied some of those general extensions concepts. In our study, a special type of fuzzy soft linear operators, which is the fuzzy soft hermitian operator has been introduced. To make the picture complete, an example in favor of it and another one against it have been established. Finally, related theorems including fuzzy soft spectral theory and many more results are investigated.

REFERENCES

- [1] S. BAYRAMOV, C. GUNDUZ: *Soft locally compact spaces and soft paracompact spaces*, J. Math. Sys. Sci., **3**, (2013), 122 – 130.
- [2] T. BEAULA, C. GUNASEELI: *On fuzzy soft metric spaces*, Malaya J. Mat., **2**(3) (2014), 197 – 202.
- [3] T. BEAULA, M. M. PRIYANGA: *A new notion for fuzzy soft normed linear space*, Int. J. Fuzzy Math. Arch., **9** (1) (2015), 81 – 90.
- [4] S. DAS, S. K. SAMANTA: *On soft inner product spaces*, Ann. Fuzzy Math. Inform. **6** (1) (2013), 151 – 170.
- [5] S. DAS, S. K. SAMANTA: *Soft Metric*, Ann. Fuzzy Math. Inform., **6**(1) (2013), 77 – 94.
- [6] P. K. MAJI, R. BISWAS, A. R. ROY: *Fuzzy soft set*, J. Fuzzy Math., **9**(3) (2001), 677 – 692.
- [7] D. MOLODTSOV: *Soft set theory-First results*, Comput. Math. Appl., **37** (1999), 19 – 31.
- [8] T. J. NEOG, D.K. SUT, G. C. HAZARIKA: *Fuzzy soft topological spaces*, Int. J. Latest Trend Math., **2**(1) (2012), 54 – 67.
- [9] M. I. YAZAR, C. G. ARAS, S. BAYRAMOV: *Results on soft Hilbert spaces*, TWMS J. App. Eng. Math., **9**(1) (2019), 159 – 164.
- [10] M. I. YAZAR, T. BILGIN, S. BAYRAMOV, C. GUNDUZ: *A new view on soft normed spaces*, Int. Math. For., **9**(24) (2014), 1149 – 1159.
- [11] L. A. ZADEH: *Fuzzy sets*, Inf. Control, **8** (1965), 338 – 353.

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