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$B\gamma$-OPEN SETS IN TOPOLOGICAL SPACES

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ABSTRACT. The aim of this paper is to introduce and study the notion of $B\gamma$-open, B pre open, B semi open and $B\beta$-open sets. Some characterization of these notions are presented.

1. INTRODUCTION

Levine [7] in 1963, started the study of generalized open sets with the introduction of semi-open sets. And in the year 1970, Levine [7] introduced g-closed sets in topological spaces (briefly, ts's) as a generalization of closed sets. Levine [8], defined the notion of B-open sets in a ts and he obtained various properties. The purpose of this paper is to introduce and study the notion of $B\gamma$-open, B pre open, B semi open and $B\beta$-open sets. Also, some characterizations of these notions are presented. Throughout this paper (X,τ) and (Y,σ) (simply X and Y) represent nonempty ts's on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$, $int(A)$ and $X\backslash A$ denote the closure of A, the interior of A and the complement of A respectively.

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2. Preliminaries

Since, we shall require the following known definitions, notations and some properties, we recall them in this section. A subset A of a space (X,τ) is called semiopen [10] (resp. β-open [1] or semi-preopen [2], b-open [3] or γ-open [4] or sp-open [5] and preopen [11] set) if $A\subseteq cl(int(A))$ (resp. , $A\subseteq cl(int(cl(A)))$ $A\subseteq int(cl(A))\cup cl(int(A))$ and $A\subseteq int(cl(A))$). The complement of a semi open (resp.β open, γ open and pre open) set is called semi-closed(resp. β-closed, γ-closed and pre closed) set. The intersection of all semi closed (resp. β-closed, γ-closed and pre closed) sets containing A is called the semi closue (resp. β-closure, γ-closure and pre closure) of A and is denoted by $scl(A)$ (resp. $\beta cl(A)$,$\gamma cl(A)$and $pcl(A)$). The union of all semi open (resp. β-open, γ-open and pre-open) sets contained in A is called the semi-interior (resp. β-interior, γ-interior and pre-interior) of A and is denoted by $sint(A)$ (resp. $\beta int(A)$, $\gamma int(A)$ and $pint(A)$. The family of all semi-open (resp. β-open, γ-open and pre-open) is denoted by SO(X) (resp. βO(X), γO(X) and PO(X)).

Definition 2.1. *A subset A of a ts $(X,\ \tau)$ is called g-closed [9] if $cl(A)\subseteq G$ whenever $A\subseteq G$ and G is open in $(X,\ \tau)$. The complement of g-closed set is called g-open.*

Definition 2.2. *Levine [8], defined $\tau(B)=\{O\cup(O'\cap B): O,\ O'\in\tau\}$ and called it simple expansion of τ by B, where $B\notin\tau$.*

Lemma 2.1. *[8] Let A & B be subsets of a space (X,τ). Then*

(i) *A is B-open (briefly, Bo) iff $A=Bint(A)$.*
(ii) *$Bint(A)$ is the union of all open sets of X whose closures are contained in A.*
(iii) *For any subset A of X, $A\subseteq cl(A)\subseteq Bcl(A)$ (resp. $Bint(A)\subseteq int(A)\subseteq A$.)*
(iv) *$Bint(A\cap C)=Bint(A)\cap Bint(C)$ and $Bint(A)\cup Bint(C)\subset Bint(A\cup C)$.*
(v) *$Bcl(A\cup C)=Bcl(A)\cup Bcl(C)$ and $Bcl(A\cap C)=Bcl(A)\cap Bcl(C)$.*

Definition 2.3.

(i) *A subset A of X is called a locally closed set [6] if $A=U\cap F$, where $U\in\tau$, F is closed in X.*

(ii) *A space (X,τ) is called extremely disconnected [12] if the closure of every open set of X is open.*

3. $B\gamma$-OPEN SETS

Definition 3.1. *Let (X,τ) be a ts. Then a subset A of X is said to be*

(i) *$B\gamma$-open (briefly, $B\gamma o$) set if $A \subseteq Bint(Bcl(A)) \cup Bcl(Bint(A))$.*
(ii) *B-semi open (briefly, Bso) set if $A \supseteq Bcl(Bint(A))$.*
(iii) *B-pre open (briefly, Bpo) set if $A \supseteq Bint(Bcl(A))$.*
(iv) *$B\beta$ open (briefly, $B\beta o$) set $A \subseteq Bcl(Bint(Bcl(A)))$.*

The complement of a $B\gamma o$ set (resp. Bso, Bpo and $B\beta o$) is called $B\beta$-closed set (resp. B-semi closed, B-pre closed and $B\beta$ closed) (briefly, $B\gamma c$ (resp. Bsc, Bpc, $B\beta c$)). The family of all $B\gamma o$, Bso, Bpo and $B\beta o$ (resp. $B\gamma c$, Bsc, Bpc and $B\beta c$) subsets of a space (X,τ) will be as always denoted by $B\gamma O(X)$, $BSO(X)$, $BPO(X)$ and $B\beta O(X)$ (resp. $B\gamma C(X)$, $BSC(X)$, $BPC(X)$ and $B\beta C(X)$).

Proposition 3.1. *Let A be a subset of a space (X,τ). Then*

(i) *Every Bo set is Bso (resp. Bpo).*
(ii) *Every Bso set is $B\gamma o$.*
(iii) *Every Bpo set is $B\gamma o$.*
(iv) *Every $B\gamma o$ set is $B\beta o$.*

Remark 3.1. *The converse of the above proposition is not necessarily true as shown by the following examples.*

Example 1. *Let $X = \{a,b,c,d,e\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$, a non open set $B = \{c,d\}$. Then the subset*

(i) *$\{a,e\}$ of X is Bso but not Bo.*
(ii) *$\{c\}$ of X is Bpo (resp. $B\gamma o$) but not Bo (resp. Bso).*
(iii) *A subset $\{a,b\}$ of X is $B\gamma o$ but not Bpo.*
(iv) *A subset $\{c,e\}$ of X is $B\beta$-open but not $B\gamma o$.*

Remark 3.2. *According to Definition 3.1 and Proposition 3.1, the following diagram holds for a subset A of a space X:*

$$
\begin{tikzcd}
Bo \arrow[r] \arrow[d] & Bso \arrow[d] & \\
Bpo \arrow[r] & B\gamma o \arrow[r] & B\beta o
\end{tikzcd}
$$

In the following, we present some properties on the notion of $B\gamma o$.

Proposition 3.2.

(i) *If A is a $B\gamma o$ subset of a ts (X, τ) and $Bint(A) = \phi$, then A is Bpo.*
(ii) *If A is a Bso subset of a ts (X, τ) and $Bint(A) = \phi$, then A is Bo.*
(iii) *If A is a Bpo subset of a ts (X, τ) and $Bint(A) = \phi$, then A is Bo.*
(iv) *If A is a $B\beta o$ subset of a ts (X, τ) and $Bint(A) = \phi$, then A is $B\gamma o$.*

Lemma 3.1. *Let (X, τ) be a ts. Then the following statements are hold.*

(i) *The union of arbitrary $B\gamma o$ sets (resp. Bso, Bpo and $B\beta o$) is $B\gamma o$ (resp. Bso, Bpo and $B\beta o$).*
(ii) *The intersection of arbitrary $B\gamma c$ sets (resp. Bsc, Bpc and $B\beta c$) is $B\gamma c$ (resp. Bsc, Bpc and $B\beta c$).*

Proof. (i) Let $\{A_i \in I\}$ be a family of $B\gamma o$ sets. Then $A_i \subseteq Bint(Bcl(A_i)) \cup Bcl(Bint(A_i))$. Hence $\bigcup\limits_i A_i \subseteq \bigcup\limits_i (Bint(Bcl(A_i)) \cup Bcl(Bint(A_i))) \subset (Bint(Bcl(\bigcup\limits_i A_i)) \cup Bcl(Bint(\bigcup\limits_i A_i))$, for all $i \in I$. Thus $\bigcup\limits_i A_i$ is $B\gamma o$.

(ii) Similar to (i). $\square$

Remark 3.3. *The intersection of any two $B\gamma o$ sets is not $B\gamma o$. Let $X = \{a, b, c, d, e\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}, B = \{c, d\}$. Then $A = \{a, e\}$ & $B = \{b, e\}$ are $B\gamma o$ sets, but $A \cap B = \{e\}$ is not $B\gamma o$.*

Definition 3.2. *Let (X, τ) be a ts. Then*

(i) *The union of all $B\gamma o$ sets (resp. Bso, Bpo and $B\beta o$) of X contained in A is called the $B\gamma$-interior (resp. B-semi interior, B-pre interior and $B\beta$-interior) of A and is denoted by $B\gamma int(A)$ (resp. $Bsint(A)$, $Bpint(A)$ and $B\beta int(A)$).*
(ii) *The intersection of all $B\gamma c$ sets (resp. Bsc, Bpc and $B\beta c$) of X contained in A is called the $B\gamma$-(resp. B-semi , B-pre and $B\beta$) closure of A and is denoted by $B\gamma cl(A)$ (resp. $Bscl(A)$, $Bpcl(A)$ and $B\beta cl(A)$).*

Proposition 3.3. For subsets A, B of a space (X, τ), the following statements hold:

(i) $Bscl(A) = A \cup Bint(Bcl(A))$, $Bint(A) = A \cap Bcl(Bint(A))$,
(ii) $Bpcl(A) = A \cup Bcl(Bint(A))$, $Bpint(A) = A \cap Bint(Bcl(A))$,
(iii) $Bscl(X \backslash A) = X \backslash Bsint(A)$, $Bscl(A \cup C) \subset Bscl(A) \cup Bscl(C)$,
(iv) $Bpcl(X \backslash A) = X \backslash Bpint(A)$, $Bpcl(A \cup C) \subset Bpcl(A) \cup Bpcl(C)$,
(v) $X \backslash (Bint(A)) = Bcl(X \backslash A)$ and $Bint(X \backslash A) = X \backslash Bcl(A)$.

Lemma 3.2. *The following hold for a subset H of a space (X, τ).*

(i) $Bpcl(H) = H \cup Bcl(Bint(H))$ *and* $Bpint(H) = H \cap Bint(Bcl(H))$,
(ii) $Bpcl(Bpint(H)) = Bpint(H) \cup Bcl(Bint(H))$ *and* $Bpint(Bpcl(H)) = Bpcl(H) \cap Bint(Bcl(H))$,
(iii) $Bsint(H) = H \cap Bcl(Bint(H))$ *and* $Bscl(H) = H \cup Bint(Bcl(H))$.

Lemma 3.3. *The following hold for a subsets H of a space (X, τ),*

(i) $Bcl(Bint(H)) = Bcl(int(H))$, *and*
(ii) $Bint(Bcl(H)) = Bint(Bcl(H))$.

Theorem 3.1. *Let (X, τ) be a ts and $A \subset X$. Then the following statements are equivalent*

(i) *A is a $B\gamma o$ set,*
(ii) $A = Bsint(A) \cup Bpint(A)$.

Proof. (i) $\Rightarrow$ (ii): Let A be an $B\gamma o$ set. Then $A \subseteq Bint(Bcl(A)) \cup Bcl(Bint(A))$, hence by Proposition 3.3 and Lemma 3.2

$$\begin{aligned} Bsint(A) \cup Bpint(A) &= (A \cap Bint(Bcl(A))) \cup (A \cup Bcl(Bint(A))) \\ &= A \cap (Bint(Bcl(A)) \cup Bcl(Bint(A)) = A. \end{aligned}$$

(ii) $\Rightarrow$ (i): Suppose that $A = Bsint(A) \cup Bpint(A)$. Then by Proposition 3.3 and Lemma 3.2
$A = (A \cap Bint(Bcl(A))) \cup (A \cup Bcl(Bint(A))) \subset Bint(Bcl(A) \cup Bcl\ (Bint(A)))$. Therefore, A is $B\gamma o$. ☐

Proposition 3.4. *Let (X, τ) be a ts and $A \subset X$. Then the following statements are equivalent:*

(i) *A is an $B\gamma c$ set,*
(ii) $A = Bscl(A) \cap Bpcl(A)$.

Theorem 3.2. *Let A be a subset of a space (X, τ). Then*

(i) $B\gamma cl(A) = Bscl(A) \cap Bpcl(A)$,
(ii) $B\gamma int(A) = Bsint(A) \cup Bpint(A)$.

Proof. (i) It is easy to see that $B\gamma cl(A) = Bscl(A) \cap Bpcl(A)$. Also, $Bscl(A) \cap Bpcl(A) = (A \cup Bint(Bcl(A)) \cap A \cup Bcl(Bint(A))) = A \cup (Bint\ (Bcl(A)) \cap Bcl(Bint(A)))$. But, $B\gamma cl(A)$ is $B\gamma c$, hence $B\gamma cl(A) \supset Bint(Bcl(B\gamma cl(A))) \cap Bcl(Bint(B\gamma cl(A))) \supset Bint(Bcl(A)) \cap Bcl(Bint(A))$. Thus $A \cup (Bint(Bcl(A)) \cap Bcl(Bint(A))) \subset A \cup B\gamma cl(A) = B\gamma cl(A)$, therefore, $Bscl(A) \cap Bpcl(A) \subset B\gamma cl(A)$. So, $B\gamma cl(A) = Bscl(A) \cap Bpcl(A)$.
(ii) Similar to (i). □

Theorem 3.3. *Let A be a subset of a space (x, τ). Then*

(i) *A is an $B\gamma o$ set (resp. Bso set, Bpo set and $B\beta o$ set) iff $A = B\gamma int(A)$ (resp. $A = Bsint(A)$, $A = Bpint(A)$ and $A = B\beta int(A)$),*
(ii) *A is an $B\gamma c$ set (resp. Bsc set, Bpc set and $B\beta c$ set) iff $A = B\gamma cl(A)$ (resp. $A = Bscl(A)$, $A = Bpcl(A)$ and $A = B\beta cl(A)$).*

Proof. (i) Let A be an $B\gamma o$ set. Then by Theorem 3.1, $A = Bsint(A) \cup Bpint(A)$ and by Theorem 3.2, we have $A = B\gamma int(A)$. Conversely, let $A = B\gamma int(A)$. Then by Theorem 3.2, $A = Bsint(A) \cup Bpint(A)$ and by Theorem 3.1, A is $B\gamma o$.
(ii) Similar to (i). □

Theorem 3.4. *Let A and B be a subsets of a space (X, τ). Then the following are hold*

(i) $B\gamma cl(X \backslash A) = X \backslash B\gamma int(A)$.
(ii) $B\gamma int(X \backslash A) = X \backslash B\gamma cl(A)$.
(iii) *If $A \subset B$, then $B\gamma cl(A) \subseteq B\gamma cl(B)$ and $B\gamma int(A) \subseteq B\gamma int(B)$.*
(iv) *$x \in B\gamma cl(A)$ iff there exists an $B\gamma o$ set U and $x \in U$ such that $U \cap A \neq \phi$.*
(v) *$B\gamma cl(B\gamma cl(A)) = B\gamma cl(A)$ and $B\gamma int(B\gamma int(A)) = B\gamma int(A)$.*
(vi) *$B\gamma cl(A) \cup B\gamma cl(B) \subset B\gamma cl(A \cup B)$ and $B\gamma int(A) \cup B\gamma int(B) \subset B\gamma int(A \cup B)$.*
(vii) *$B\gamma int(A \cap B) \subset B\gamma int(A) \cap B\gamma int(B)$ and $B\gamma cl(A \cap B) \subset v - cl(A) \cap B\gamma cl(B)$.*

Proof. (i) Since $(X\backslash A) \subseteq X$, by Theorem 3.3, $B\gamma cl(X\backslash A) = Bscl(X\backslash A) \cap Bpcl(X\backslash A)$ and by Proposition 3.3,

$$B\gamma cl(X\backslash A) = (X\backslash Bsint(A)) \cap (X\backslash Bpint(A)) = X\backslash(Bsint(A) \cup Bpint(A)),$$

hence by Theorem 3.3, $B\gamma cl(X\backslash A) = X\backslash B\gamma int(A)$.

(ii) Similar to (i).

(iii) Since, $B\gamma cl(A) = Bscl(A) \cap Bpcl(A)$ and $A \subseteq B$,
$B\gamma cl(A) = Bscl(A) \cap Bpcl(A) \subseteq Bscl(B) \cap Bpcl(B) = B\gamma cl(B)$.

(v) Since, $B\gamma cl(B\gamma cl(A)) = Bscl(B\gamma cl(A)) \cap Bpcl(B\gamma cl(A))$, by Theorem 3.3 we have:

$$\begin{aligned} &Bscl(Bscl(A) \cap Bpcl(A)) \cap Bpcl(Bscl(A) \cap Bpcl(A)) \\ &\quad \subseteq (Bscl(A) \cap Bscl(Bpcl(A))) \cap (Bpcl(Bscl(A) \cap Bpcl(A)) \\ &\quad = Bscl(A) \cap Bpcl(A) = B\gamma cl(A), \end{aligned}$$

hence $B\gamma cl(B\gamma cl(A)) \subseteq B\gamma cl(A)$. But, $B\gamma cl(A) \subseteq B\gamma cl(B\gamma cl(A))$. Therefore, $B\gamma cl(B\gamma cl(A)) = B\gamma cl(A)$.

(iv), (vi) and (vii) are obvious. □

Remark 3.4. *The inclusion relation in part (vi) and (vii) of the above theorem cannot be replaced by equality as shown by the following example.*

Example 2. *Let $X = \{a, b, c, d, e\}$, with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ a non-open set $B = \{c, d\}$. Then*

(i) *If $A = \{a, b\}, B = \{a, c\}$ and $A \cup B = \{a, b, c\}$, then $B\gamma cl(A) = A$, $B\gamma cl(B) = B$ and $B\gamma cl(A \cup B) = X$. Hence, $B\gamma cl(A \cup B) \subset B\gamma cl(A) \cup B\gamma cl(B)$.*

(ii) *If $A = \{a, b\}$, $C = \{a, c\}$ and $A \cap B = \{b\}$, then $B\gamma cl(A) = A$, $B\gamma cl(C) = C$ and $B\gamma cl(A \cap C) = X$. Therefore, $B\gamma cl(A \cap B) \subset B\gamma cl(A) \cap B\gamma cl(C)$.*

(iii) *If $D = \{a, d\}$, $C = \{b, d\}$ and $D \cup C = \{a, b, d\}$, then $B\gamma int(D) = \phi$, $B\gamma int(C) = \{B\}$ and $B\gamma int(D \cup C) = \{a, b, d\}$. So, $B\gamma int(D \cup C) \subset B\gamma int(D) \cup B\gamma int(C)$.*

Lemma 3.4. *Let A be a subset of a space (X, τ). Then following statement hold: $Bpint(Bpcl(A)) = Bpcl(A) \cap Bint(Bcl(A))$ and $Bpcl(Bpint(A)) = Bpint(A) \cup Bcl(Bint(A))$.*

Proposition 3.5. *Let A be a subset of a space (X, τ). Then:*

(i) $B\gamma cl(A) = A \cup Bpint(Bpcl(A))$.
(ii) $B\gamma int(A) = A \cap Bpcl(Bpint(A))$.

Proof. (i) By Lemma 3.2,

$$\begin{aligned} A \cup Bpint(Bpcl(A)) &= A \cup (Bpcl(A) \cap Bint(Bcl(A))) \\ &= (A \cup Bpcl(A)) \cap (A \cup Bint(Bcl(A))) \\ &= Bpcl(A) \cap Bscl(A) \\ &= B\gamma cl(A). \end{aligned}$$

(ii) Similar to (i). □

Theorem 3.5. *Let A be a subset of a space (X, τ). Then the following are equivalent:*

(i) *A is an $B\gamma o$ set,*
(ii) *$A \subseteq Bpcl(Bpint(A))$,*
(iii) *$Bpcl(A) = Bpcl(Bpint(A))$.*

Proof. (i)⇒(ii): Let A be an $B\gamma o$ set. Then by Theorem 3.3, $A = B\gamma int(A)$ and by Proposition 3.5, $A = A \cap Bpcl(Bpint(A))$ and hence, $A \subseteq Bpcl(Bpint(A))$.

(ii)⇒(i): Let $A \subseteq Bpcl(Bpint(A))$. Then by Proposition 3.5, $A \subseteq A \cap Bpcl(Bpint(A)) = B\gamma int(A)$. So, $A \subseteq B\gamma int(A)$. Then $A = B\gamma int(A)$ and hence, A is $B\gamma o$.

(ii)⇒(iii): Let $A \subseteq Bpcl(Bpint(A))$. Then $Bpcl(A) \subseteq Bpcl(Bpint(A))$ and hence, $Bpcl(A) = Bpcl(Bpint(A))$.

(iii)⇒(ii): Obvious. □

Theorem 3.6. *Let A be a subset of a ts X. Then the following are equivalent:*

(i) *A is an $B\gamma c$ set,*
(ii) *$Bpint(Bpcl(A)) \subseteq A$,*
(iii) *$Bpint(A) = Bpint(Bpcl(A))$.*

Theorem 3.7. *If A is a subset of an extremally disconnected space (X, τ). Then the following are equivalent:*

(i) *A is an open set.*
(ii) *A is $B\gamma o$ and locally closed.*

Proof. (i)⇒(ii): Obvious from Definitions 2.2 and 3.1.

(ii)$\Rightarrow$(i): Let A be an $B\gamma o$ and a locally closed subset of X. Then $A = U \cap cl(A)$ and $A \subseteq Bcl(Bint(A))UBint(Bcl(A))$, hence:

$$\begin{aligned} A &\subseteq U \cap [Bcl(Bint(A)) \cup Bint(Bcl(A))] \\ &\subseteq (U \cap Bcl(Bint(A))) \cup Bint(U \cap Bcl(A))) \\ &\subseteq (U \cap Bcl(Bint(A))) \cup Bint(U \cap Bcl(A)) \\ &\subseteq (U \cap Bint(Bcl(A))) \cup Bint(U \cap Bcl(A))(since X is E.D) \\ &\subseteq Bint(U \cap Bcl(A)) \cup Bint(U \cap Bcl(A)) = Bint(A) \cup Bint(A). \end{aligned}$$

Hence A is open. □

Definition 3.3. *A subset A of a ts (X, τ) is said to be*

(i) *locally $B\gamma c$ if $A = U \cap F$ for each $U \in \tau$ and $F \in B\gamma C(X)$.*
(ii) *locally Bsc if $A = U \cap F$ for each $U \in \tau$ and $F \in BSC(X)$.*
(iii) *locally Bpc if $A = U \cap F$ for each $U \in \tau$ and $F \in BPC(X)$.*
(iv) *locally $B\beta c$ if $A = U \cap F$ for each $U \in \tau$ and $F \in B\beta C(X)$.*

Theorem 3.8. *Let H be a subset of a space X. Then H is locally $B\gamma c$ (resp. locally Bsc, locally Bpc and locally $B\beta c$) iff $H = U \cap B\gamma cl(H)$ (resp.$H = U \cap BS$-$cl(H)$, $H = U \cap BP$-$cl(H)$, and $H = U \cap B\beta$-$cl(H)$).*

Proof. Since H is a locally $B\gamma c$ set, $H = U \cap F$, for each $U \in \tau$ and $F \in B\gamma C(X)$, hence $H \subseteq B\gamma cl(H) \subseteq B\gamma cl(F) = F$, thus $H \subseteq U \cap B\gamma cl(H) \subseteq U \cap B\gamma cl(F) = H$. Therefore $H = U \cap B\gamma cl(H)$. Conversely, since $B\gamma cl(H)$ is $B\gamma c$ and $H = U \cap B\gamma cl(H)$, then H is locally $B\gamma c$. □

Theorem 3.9. *Let A be a locally $B\gamma c$ subset of a space (X, τ). Then the following statements are hold:*

(i) *$B\gamma cl(A) \backslash A$ is an $B\gamma c$ set.*
(ii) *$A \cup (X \backslash B\gamma cl(A))$ is an $B\gamma o$.*
(iii) *$A \subseteq B\gamma int(A \cup (X \backslash B\gamma cl(A)))$.*

Proof. (i) If A is locally $B\gamma c$ set, then there exist an open set U such that $A = U \cap B\gamma cl(A)$. Hence,

$$\begin{aligned} B\gamma cl(A)\backslash A &= B\gamma cl(A)\backslash(U \cup B\gamma cl(A))) \\ &= B\gamma cl(A) \cap [X\backslash(U \cap B\gamma cl(A))] \\ &= B\gamma cl(A) \cap [(X\backslash U) \cup (X\backslash \cap B\gamma cl(A))] \\ &= B\gamma cl(A) \cap (X\backslash U) \end{aligned}$$

which is $B\gamma c$.

(ii) From (i), $B\gamma cl(A)\backslash A$ is $B\gamma c$, then $X\backslash[B\gamma cl(A)\backslash A]$ is an $B\gamma o$ set and $X\backslash[B\gamma cl(A)\backslash A] = X\backslash B\gamma cl(A) \cup (X \cap A) = A \cup (X\backslash B\gamma cl(A))$, hence $A \cup (X\backslash B\gamma cl(A))$ is $B\gamma o$.

(iii) It is clear that, $A \subseteq (A \cup (X\backslash B\gamma cl(A))) = B\gamma int(A \cup (X\backslash B\gamma cl(A)))$. □

CONCLUSION

In this paper is to introduced and studied the notion of $B\gamma$-open, B pre open, B semi open and $B\beta$-open sets. Some characterization of these notions are discussed.

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