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SUBDIVISION VERTEX AND SUBDIVISION VERTEX NEIGHBOURHOOD CORONA OF CYCLIC GRAPHS

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ABSTRACT. Let us newly define the Subdivision vertex corona and subdivision vertex neighbourhood corona product of the Cycle graph C_m, for any $m \geq 3$ with empty graph $\overline{K_n}$, for any $n \geq 1$. Additionally we will give generalization result based on the graph $S(C_m \odot \overline{K_n})$ and $S_N(C_m \boxdot \overline{K_n})$ for any, $m \geq 3, n \geq 1$. Then we shall calculate the topological indices of the different types of Zagreb index for our newly defined graphs. The molecular graph of Subdivision vertex corona product of C_3 and $\overline{K_2}$ i.e., $S(C_3 \odot \overline{K_2})$ can be represented in the form of tri-thioacetane.

1. INTRODUCTION

Let G_1 and G_2 be the two simple graphs with the vertex set $V(G_1) = \{u_1, u_2, u_3, \ldots, u_x\}$ and $V(G_2) = \{v_1, v_2, v_3, \ldots, v_y\}$ respectively. Then, the corona of G_1 and G_2, denoted by $G_1 \odot G_2$ is defined as take one copy of G_1 (center graph) and x copies of G_2(external graph) by adjoining x^{th} vertex of G_1 to each vertex G_2 of y^{th} copy. Several authors motivated about the different graph operations such as cartesian product, corona product, join graph, vertex and edge contraction of graphs etc. Recently, people identified the other special versions

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of corona product of graphs such as subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood corona, subdivision-edge neighborhood corona and vertex edge corona etc. and investigated them (see [**?**]).

In this paper, we proceed to generalize about our newly defined graphs, particularly the subdivision vertex corona and sub division vertex neighborhood corona of cycle graphs with empty graphs $S(C_m \odot \overline{K_n})$ and $S_N(C_m \boxdot \overline{K_n})$ respectively. Then we can derive a few types of Zagreb indices for our newly defined Subdivision vertex corona and Sub division vertex neighborhood corona of our graphs. Recall that, the subdivision graph $S = S(G_1)$ is the graph obtained from G_1 by replacing each of its edges by a path of length two, or equivalently, by inserting an additional vertex into each edge of G_1. The main intuition of subdivision vertex corona and subdivision vertex neighborhood corona graph representing the chemical compounds in drug discovery, network navigation, designing network tools etc.

The Subdivision vertex corona and Sub division vertex neighborhood corona of graph may represent the molecular structure of a certain chemical compound and it is associated with the different physic-chemical properties which are found in molecular graph connectivity. The molecular descriptors are based on the sense of ultimate outcome of logic and the numerical procedure which converts the preset chemical information into a symbolic representation of molecule will reach a practical number. Therefore, the topological indices(see [4, 6,7])are very essential tool in the field of nanotechnology, drug discovery, computer networks, designing tool, information technology etc., It is mainly motivated to explore classification purpose, such as Quantitative Structure Activity Relationship (QSAR) and Quantitative Structure Property Relation (QSPR) and Fuzzy Lattice Neural Networks (FLNN) (see [5]) etc.

2. Preliminary results

Some of the well-known topological indices and different graph operations related to corona product are defined earlier for any simple graph as follows

Definition 2.1. *The First Zagreb Index [2] of a graph G, denoted by $M_1(G)$ is defined as*

$$M_1(G) = \sum_{u,v \in E(G)} [d(u) + d(v)].$$

Definition 2.2. *The Second Zagreb Index [2] of a graph G, denoted by $M_2(G)$ is defined as*

$$M_2(G) = \sum_{u,v \in E(G)} [d(u)d(v)].$$

Definition 2.3. *The Third Zagreb Index [3] of a graph G, denoted by $M_3(G)$ is defined as*

$$M_3(G) = \sum_{u,v \in E(G)} |d(u) - d(v)|.$$

Definition 2.4. *The First Multiple Zagreb Index [8] of a graph G, denoted by $PM_1(G)$ is defined as*

$$PM_1(G) = \prod_{u,v \in E(G)} [d(u) + d(v)].$$

Definition 2.5. *The Second Multiple Zagreb Index [8] of a graph G, denoted by $PM_2(G)$ is defined as*

$$PM_2(G) = \prod_{u,v \in E(G)} [d(u)d(v)].$$

Definition 2.6. *The First Zagreb polynomial Index of a graph G, denoted by $M_1(G,x)$ is defined as*

$$M_1(G,x) = \sum_{u,v \in E(G)} x^{[d(u)+d(v)]}.$$

Definition 2.7. *The Second Zagreb polynomial Index of a graph G, denoted by $M_2(G,x)$ is defined as*

$$M_2(G,x) = \sum_{u,v \in E(G)} x^{[d(u)d(v)]},$$

where $d(u)$ and $d(v)$ represent the degrees of vertices of u and v in the graph G respectively.

Definition 2.8. *Let G_1 and G_2 be two vertex disjoint graphs. The Subdivision-vertex Corona of two vertex-disjoint graphs G_1 and G_2 is denoted by $G_1 \odot G_2$ is the graph obtained from $S(G_1)$ and $|V(G_1)|$ copies of G_2, all vertex-disjoint, by joining the x^{th} vertex of $V(G_1)$ to every vertex in the x^{th} copy of G_2.*

Definition 2.9. *The Subdivision-edge Corona of two vertex-disjoint graphs* G_1 *and* G_2 *is denoted by* $G_1 \ominus G_2$ *is the graph obtained from* $S(G_1)$ *and* $|I(G_1)|$ *copies of* G_2*, all vertex-disjoint, by joining the* x^{th} *vertex of* $I(G_1)$ *to every vertex in the* x^{th} *copy of* G_2*, where* $I(G_1)$ *is the set of inserted vertices of* $S(G_1)$.

Definition 2.10. *The Subdivision-vertex neighborhood corona of* G_1 *and* G_2 *is denoted by* $G_1 \boxdot G_2$ *and obtained from* $S(G_1)$ *and* $|V(G_1)|$ *copies of* G_2*, all vertex-disjoint, by joining the neighbors of the* i^{th} *vertex of* $V(G_1)$ *to every vertex in the* i^{th} *copy of* G_2 *.*

Definition 2.11. *The Subdivision-edge neighborhood corona of* G_1 *and* G_2 *is denoted by* $G_1 \boxminus G_2$ *and obtained from* $S(G_1)$ *and* $|I(G_1)|$ *copies of* G_2*, all vertex-disjoint, by joining the neighbors of the* i^{th} *vertex of* $V(G_1)$ *to every vertex in the* i^{th} *copy of* G_2 *.*

Definition 2.12. *A graph* $G = (V(G), E(G))$ *is a null graph [1], if there are no edges in the graph i.e.,* $|E(G)| = 0$. *It is denoted by* $\overline{k_n}$ *of order* $n \geq 1$ *is the edgeless graph (or empty graph). The notation* $\overline{k_n}$ *arises from the fact, that the n-vertex edgeless graph is the complement of the Complete Graph* k_n.

3. Main results

3.1. **Subdivision vertex corona of cycle with empty graphs.** In this section, we determine the generalization result based on subdivision vertex corona of cycle graphs with edgeless graphs. The subdivision vertex corona of C_m where $m \geq 3$, with empty graph $\overline{k}_n$, $n \geq 1$ is obtained by $S(C_m)$ and $V(C_m)$ copies of n, all vertex disjoint, by making the m^{th} vertex of $V(C_m)$ is adjacent to every vertex in the m^{th} copy of $\overline{k}_n$, where $1 \leq i \leq V(G)$. By the definition of the subdivision vertex corona of C_m and $\overline{k}_n$, we get $m(n+2)$ vertices and Edges. The subdivision vertex of corona of cycle with null graphs

$$\begin{aligned} S(C_m \odot \overline{K}_n) &= [\text{ Total number of vertices in } \overline{k}_n \times \text{ Total no of vertices in} C_m] \\ &\quad + [\text{ Total number of vertices of } S(C_m)] \\ &= [\text{ Total number of vertices in } \overline{k}_n \\ &\quad \times \text{ Total no of vertices Adjacent to Pendant vertices}] \\ &\quad + 2 \times [\text{ Total no of vertices Non Adjacent to Pendant vertices}] \end{aligned}$$

$$= [\text{ Total number of vertices in } C_m]$$
$$\times [\text{ Total number of vertices of } \overline{k}_n + 2]$$
$$= m(n+2).$$

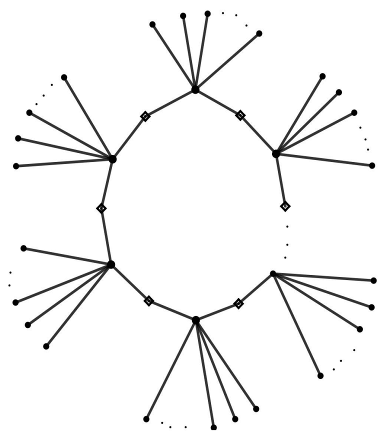

FIGURE 1. $S(C_m \odot \overline{K}_n)$

Theorem 3.1. *Let $S(C_m \odot \overline{K}_n)$ be the subdivision-vertex corona prodcut of a graph. Then its Zagreb indices are given by*

(i) $M_1(S(C_m \odot \overline{K}_n)) = m(n(m+1) + 2(m+2))$
(ii) $M_2(S(C_m \odot \overline{K}_n)) = m^2(n+4)$
(iii) $M_3(S(C_m \odot \overline{K}_n)) = m^2(n+2) - m(m+4)$
(iv) $PM_1(S(C_m \odot \overline{K}_n)) = (1+m)^{mn} \times (2+m)^{2m}$
(v) $PM_2(S(C_m \odot \overline{K}_n)) = (m)^{mn} \times (2m)^{2m}$
(vi) $M_1(S(C_m \odot \overline{K}_n), x) = mnx^{(1+m)} + 2mx^{(2+m)}$
(vii) $M_2(S(C_m \odot \overline{K}_n), x) = mnx^{m} + 2mx^{2m}$.

Proof. Consider the subdivision vertex corona product of a graph $S(C_m \odot \overline{K}_n)$, where $m \geq 3, n \geq 1$. The number of vertices and edge set of our graph is defined as follows.

TABLE 1. The number of edges of each type

Types of Edges	Number of Edges
$(1, m)$	mn
$(2, m)$	$2m$

(i) The First Zagreb Index

$$M_1(S(C_m \odot \overline{K}_n)) = \sum_{uv\in E(G)} [d(u) + d(v)] = (1 + m)mn + (2 + m)2m$$
$$= m(n(m + 1) + 2(m + 2)) .$$

(ii) The Second Zagreb index

$$M_2(S(C_m \odot \overline{K}_n)) = \sum_{uv\in E(G)} d(u)d(v) = (1 \times m)mn + (2 \times m)2m$$
$$= m^2n + 4m^2 = m^2(n + 2) - m(m + 4) .$$

(iii) The Third Zagreb index

$$M_3(S(C_m \odot \overline{K}_n)) = \sum_{uv\in E(G)} |d(u) - d(v)| = |m - 1|mn + |m - 2|2m$$
$$= m^2n - mn + 2m^2 - 4m = m^2(n + 2) - m(m + 4) .$$

(iv) The First Multiple Zagreb index

$$PM_1(S(C_m \odot \overline{K}_n)) = \prod_{uv\in E(G)} [d(u) + d(v)] = (1 + m)^{mn} \times (2 + m)^{2m} .$$

(v) The Second Multiple Zagreb index

$$PM_2(S(C_m \odot \overline{K}_n)) = \prod_{uv\in E(G)} d(u)d(v) = (m)^{mn} \times (2m)^{2m} .$$

(vi) The First Zagreb Polynomial index

$$M_1(S(C_m \odot \overline{K}_n), x) = \sum_{uv\in E(G)} x^{[d(u)+d(v)]} = mnx^{(1+m)} + 2mx^{(2+m)}$$
$$= m(nx^{(1+m)} + 2x^{(2+m)}) .$$

(vii) The Second Zagreb Polynomial index

$$M_2(S(C_m \odot \overline{K}_n), x) = \sum_{uv \in E(G)} x^{d(u)d(v)} = mnx^m + 2mx^{2m}$$
$$= mx^m(n + 2x^m)\,.$$

□

3.2. **Subdivision vertex neighborhood corona product of cycle with empty graphs.** The subdivision vertex neighbourhood corona of C_m where $m \geq 3$, with empty graph $\overline{k}_n, n \geq 1$ is obtained by $S(C_m)$ and $V(C_m)$ copies of n, all vertex disjoint, by making the m^{th} vertex of $V(C_m)$ is adjacent to every vertex in the m^{th} copy of $\overline{k}_n$, where $1 \leq i \leq V(G)$. By the definition of the subdivision vertex corona of C_m and $\overline{k}_n$ has $m(n+2)$ vertices and $2m(n+1)$ Edges. The

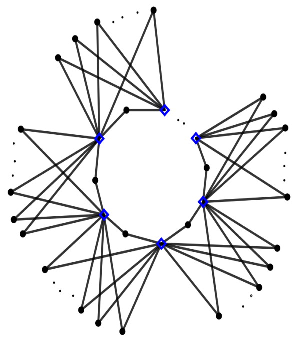

FIGURE 2. $S_N(C_m \odot \overline{K}_m)$

subdivision vertex neighbourhood corona of

$$\begin{aligned}
S_N(C_m \boxdot \overline{K}_n) &= [\text{ Total number of vertices in } \overline{k}_n \times \text{ Total no. of vertices in } C_m] \\
&\quad + [\text{ Total number of vertices of } S(C_m)] \\
&= [\text{ Total number of vertices in } \overline{k}_n \times \text{ Total no of vertices } C_m] \\
&\quad + 2 \times [\text{ Total no of vertices } C_m] \\
&= [\text{ Total number of vertices in } C_m] \\
&\quad \times [\text{ Total number of vertices of } \overline{k}_n + 2] = 2m(n+1)\,.
\end{aligned}$$

The edge set of the subdivision vertex neighbourhood corona of

$$\begin{aligned} S_N(C_m \boxdot \overline{K}_n) = &\text{ Subdivision of } C_m[\text{ Total number of vertices in } \overline{k}_n + 1] \\ &= 2[\text{ Total number of vertices in } C_m] \\ &\quad \times [\text{ Total number of vertices of } \overline{k}_n + 1] \\ &= 2m(n+1)\,. \end{aligned}$$

Theorem 3.2. *Let $S_N(C_m \boxdot \overline{K}_n)$ be the subdivisionâĂŞvertex neighborhood corona of graph. Then its Zagreb indices are given by*

(i) $M_1(S_N(C_m \boxdot \overline{K}_n)) = M_1(C_m)(n+2)(n+1)$
(ii) $M_2(S_N(C_m \boxdot \overline{K}_n)) = 2M_2(C_m)(n+1)^2$
(iii) $M_3(S_N(C_m \boxdot \overline{K}_n)) = M_1(C_m)(n(n+1))$
(iv) $PM_1(S_N(C_m \boxdot \overline{K}_n)) = (2(n+2))^{2m(n+1)}$
(v) $PM_2(S_N(C_m \boxdot \overline{K}_n)) = (4(n+1))^{2m(n+1)}$
(vi) $M_1(S_N(C_m \boxdot \overline{K}_n), x) = 2m(n+1)x^{2(n+2)}$
(vii) $M_2(S_N(C_m \boxdot \overline{K}_n), x) = 2m(n+1)x^{4(n+1)}$.

Proof. Consider the subdivision vertex neighborhood corona product of a graph i.e. $S_N(C_m \boxdot \overline{K}_n)$, where $m \geq 3, n \geq 1$. The number of vertices and edge set of our graph is defined as follows.

TABLE 2. The number of edges of each type

Types of Edges	Number of Edges
$2, 2(n+1)$	$2m(n+1)$

(i) The First Zagreb Index

$$\begin{aligned} M_1(S_N(C_m \boxdot \overline{K}_n)) &= \sum_{uv \in E(G)} [d(u) + d(v)] \\ &= (2 + 2(n+1))2m(n+1) \\ &= 4m(n+2)(n+1) \\ &= M_1(C_m)(n+2)(n+1)\,. \end{aligned}$$

(ii) The Second Zagreb index

$$\begin{aligned}M_2(S_N(C_m \boxdot \overline{K}_n)) &= \sum_{uv\in E(G)} d(u)d(v)\\ &= (2\times(2n+2))m(2n+2)\\ &= 8m(n+1)^2\\ &= 2M_2(C_m)(n+1)^2\,.\end{aligned}$$

(iii) The Third Zagreb index

$$\begin{aligned}M_3(S_N(C_m \boxdot \overline{K}_n)) &= \sum_{uv\in E(G)} |d(u)-d(v)|\\ &= |2n+2-2|m(2n+2)\\ &= 4mn(n+1)\\ &= M_1(C_m)n(n+1)\,.\end{aligned}$$

(iv) The First Multiple Zagreb index

$$\begin{aligned}PM_1(S_N(C_m \boxdot \overline{K}_n)) &= \prod_{uv\in E(G)} [d(u)+d(v)]\\ &= (2+(2n+2))^{2m(n+1)}\\ &= (2(1+n+1))^{2m(n+1)}\\ &= (2(n+2))^{2m(n+1)}\,.\end{aligned}$$

(v) The Second Multiple Zagreb index

$$\begin{aligned}PM_2(S_N(C_m \boxdot \overline{K}_n)) &= \prod_{uv\in E(G)} d(u)d(v)\\ &= (2\times(2n+2))^{2m(n+1)}\\ &= (4(n+1))^{2m(n+1)}\,.\end{aligned}$$

(vi) The First Zagreb Polynomial index

$$\begin{aligned}M_1(S_N(C_m \boxdot \overline{K}_n),x) &= \sum_{uv\in E(G)} x^{[d(u)+d(v)]}\\ &= 2m(n+1)x^{2+2n+2}\\ &= 2m(n+1)x^{2(n+2)}\,.\end{aligned}$$

(vii) The Second Zagreb Polynomial index

$$M_2(S_N(C_m \boxdot \overline{K}_n), x) = \sum_{uv \in E(G)} x^{d(u)d(v)}$$
$$= 2m(n+1)x^{2(2n+2)}$$
$$= 2m(n+1)x^{4(n+1)}.$$

□

Example 1. *Let us consider the subdivision vertex corona i.e.,* $S(C_3 \odot \overline{K}_2)$. *Then the First Zagreb Index is given by*

$$M_1(G) = 6(1+3) + 1(2+3) = 54.$$

It can be represented in the molecular structure called as tri-thioacetone.

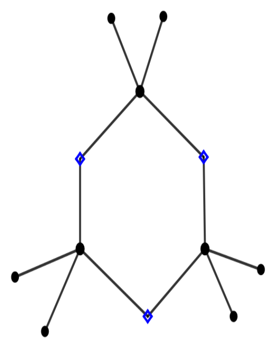

FIGURE 3. $S(C_3 \odot \overline{K}_2)$

4. CONCLUSION

In this paper, we newly defined the generalization result based on the subdivision vertex corona and subdivision vertex neighbourhood corona of cycle with empty graphs. Also, we determined some types of zagreb indices of subdivision vertex corona and subdivision vertex neighbourhood corona of cyclic graphs.

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