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LARGE DEVIATIONS FOR PERTURBED REFLECTED DIFFUSION PROCESSES DRIVEN BY A FRACTIONAL BROWNIAN MOTION IN HÖLDERIAN NORM

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ABSTRACT. In this paper we establish a large deviations for perturbed reflected diffusion processes driven by a fractional brownian motion for any Hurst parameter $H \in (0,1)$ using the method of Azencott in Hölderian norm.

1. INTRODUCTION

In recent years, many results on the fractional brownian motion (Fbm for short) have been obtained. This process was introduced by Kolmogorov [22] and studied by Mandelbrot and Van Ness [24], where a stochastic integral representation in terms of a standard Brownian motion was obtained [8]. Actually, many phenomena in telecommunication network, mathematical finance, filtering theory, biology, etc., are modeled by fractional Brownian motion [12]. It is well known that the fBm is not Markovian, nor a semi-martingale, but has a long-range dependence and it is self-similar.

There are several attempts to construct a stochastic calculus with respect to fBm, such as: sample paths theory, semimartingale approach and the Skorohod integral. SDE with respect to fBm are considered in [13], [4]. Such stochastic

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equations are of Volterra type with a weakly regular kernel and are not covered by standard stochastic Volterra equations as treated for example in Berger and Mizel [6], Protter [31].

Large deviations for stochastic equations driven by semi-martingales are by now well known (see Azencot [2], Baldi and Chaleyat-Maurel [3], Cutland [14], Dozzi [18], Freidlin and Wentzell [19], Liptser and Pukhalskii [23], Nualart and Rovira [28], Perez-Abreu and Tudor [30], Priouret [17], Rovira and Sanz-Solé [33]).

Doney and Zhang [16], obtained the existence and uniqueness of the solutions for the following perturbed diffusion and perturbed reflected diffusion equations:

$$\begin{aligned} X_t &= x_0 + \int_0^t b(X_s)\, ds + \sqrt{\varepsilon}\ \int_0^t \sigma(X_s)\, dW_s^H + \beta \sup_{0\le s\le t} X_s \\ & H \in (0,1), t \in [0,1]\,. \end{aligned} \tag{1.1}$$

Let $T = (T_t), t \geq 0$ solution of stochastic differential equation

$$T_t = y + \int_0^t \varsigma(T_s)\, dW_s^H + \beta \sup_{0\le s\le t} T_s + L_t, H \in (0,1), t \in [0,1]\,, \tag{1.2}$$

where $\alpha \in [0,1], x \in \mathbb{R}, y \in \mathbb{R}_+$ are deterministic, $b, \sigma : \mathbb{R} \longrightarrow \mathbb{R}$ and $\varsigma : \mathbb{R}_+ \longrightarrow \mathbb{R}$ are bounded Lipschitz continuous function, $\{L_t, t \in [0,1]\}$ is non-decreasing with L_0 and

$$\int_0^t \chi_{\{y_s=0\}}\, dL_s = L_t\,.$$

We may think of $\{L_t, t \in [0,1]\}$ the local time of the semi-martingale $\{T_t, t \in [0,1]\}$ at point zero. $\{W_t^H, t \in [0,1]\}$ is a fractional Brownian motion (fBm) of Hurst parameter $H \in$ (0,1) and $\{W_t, t \in [0,1]\}$ 1-dimensional standard Brownian motion on a completed probability space $(\Omega, \mathcal{F}, (\mathcal{F})_{t\geq 0}, \mathbb{P})$.

For $\alpha \in]0, 1/2[$, let $C^\alpha([0,1], \mathbb{R})$ be the separable space of α-Hölder continuous functions from $[0,1]$ to $\mathbb{R}$. Consider the small noise perturbations of (1.1) and (1.2)

$$\begin{aligned} X_t^\varepsilon &= x_0 + \int_0^t b(X_s^\varepsilon)\, ds + \sqrt{\varepsilon}\ \int_0^t \sigma(X_s^\varepsilon)\, dW_s^H + \beta \sup_{0\le s\le t} X_t^\varepsilon \\ & H \in (0,1), t \in [0,1]\,. \end{aligned} \tag{1.3}$$

Let $T^\varepsilon = (T_t^\varepsilon), t \geq 0$ solution of stochastic differential equation

$$T_t^{\varepsilon} = y + \sqrt{\varepsilon}\int_0^t \varsigma(T_s^{\varepsilon})\, dW_s^H + \beta \sup_{0\le s\le t} T_t^{\varepsilon} + L_t\,, \tag{1.4}$$

where $H \in (0,1), t \in [0,1]$. The aim of this paper is to establish for $\alpha \in]0,1/2[$ and $\beta \in]0,1[$ (resp. $\alpha \in]0,1/2[$ and $\beta \in]0,1/2[$) a large deviation principle (LDP) for the laws of X^{ε} (resp. T^{ε}) solution of (1.3) (resp. (1.4)) in $C^{\alpha}([0,1],\mathbb{R})$ by using the classical Azencott's method.

The special case $\beta \equiv 0$ was studied by Freidlin and Wentzell [19] see also refered to Varadhan [35], Azencott [2] and Stroock [34] with the usual topology uniform, Ben Arous and Ledoux [5] have developed a large deviation principle(LDP) in $C^{\alpha}([0,1],\mathbb{R})$.

Doss and Priouret [17] considered the LDP for the small noise perturbations of reflected diffusions, through checking uniform Freidlin-Ventzell estimates. Further, Millet et al. [26] used the Freidlin-Ventzell estimates to obtain the LDP of a class of anticipating stochastic differential equations. Recently, Mohammed and Zhang [27] studied a LDP for small noise perturbed family of stochastic systems with memory.

When $H = 1/2$ the fBm becomes the standard Brownian motion, the perturbed reflected Brownian motion was first introduced by Le Gall and Yor [20], [21] and subsequently studied by Carmona et al. [9], [10], Perman and Werner [29] and Chaumont and Doney [11] and Bo and Zhang [7], L.I.Rajaonarison and Rabeherimanana [32] . The approach used here will be based on a classical result of Azencott [2] with uniform topology.

The rest of the paper is organized as follows. Section 2 contains some preliminary definitions and general results. Section 3 is for the LDP of perturbed diffusion processes. The large deviation estimates for the perturbed reflected diffusion processes are shown in Section 4.

2. Preliminary definitions and results

2.1. **Preliminary definitions.**

Definition 2.1. *A rate function is a function $I : \Xi \longrightarrow [0;+\infty]$ on a Hausdorff topological space Ξ which is lower semi-continuous, ie. where all the level set $\Gamma_{\lambda} = \{x \in \Xi, I(x) \le \lambda\}$ are closed in Ξ. A rate function $I : \Xi \longrightarrow [0;+\infty]$ is* *called a good rate function, if all the level set* $\{x \in \Xi, I(x) \leq \lambda\}$ *for* $\lambda \geq 0$ *are compact in* Ξ .

Definition 2.2. *A family* $\{P^{\varepsilon}\}_{\varepsilon>0}$ *of probabilities measures on Hausdorff topological space* Ξ *is satisfies the large deviation principle (or shorter LDP) with the rate function* $I : \Xi \longrightarrow [0; +\infty]$*, if the following two estimates hold:*

i) *(Lower bound.) For every open subset* $\mathcal{O}$ *of* Ξ

$$\liminf_{\varepsilon \longrightarrow 0} \varepsilon \log P_{\varepsilon}(\mathcal{O}) \geq -I(\mathcal{O}) .$$

ii) *(Upper bound.) For every closed subset* $\mathcal{F}$ *of* Ξ

$$\limsup_{\varepsilon \longrightarrow 0} \varepsilon \log P_{\varepsilon}(\mathcal{F}) \leq -I(\mathcal{F}) .$$

Now, we give a new formulation for the contraction principle which will be needed later.

Lemma 2.1. *Let* $(E_x, d_x), (E_y, d_y), (E_z, d_z), (E, d)$ *denote Polish spaces and* $(\Omega, \mathcal{F}, \mathbb{P})$ *be a probability space.*

Suppose that $(X^{\varepsilon}, \varepsilon > 0)$ *is a family of random variables with values in* E_x *satisfing a LDP with a rate function* I_x*, and* $(Y^{\varepsilon}, \varepsilon > 0)$ *a random variable with values in* E_y *satisfing a LDP with a rate function* I_y*.*

Suppose that for each $\varepsilon > 0$*,* X^{ε} *is independent of* Y^{ε} *then the family of random variable* $Z = F(X^{\varepsilon}, Y^{\varepsilon})$ *satisfing a LDP with rate function* $I_F(z)$ *defined by*

$$I_F(z) = \inf_{F(x,y)=z} \Big\{ I_x(x) + I_y(y) \Big\}$$

where $F : E_x \times E_y \to E_z$ *is continuous.*

As a reminder for Azencott's method

Proposition 2.1. *let* (E_i,d_i)*,* $i = 1, 2$ *be two Polish spaces and* $X_{\varepsilon}^{i} \to\ E_i$*,* $\varepsilon > 0, i = 1, 2$ *two families of random variables. Assume that* $\{X_1^{\varepsilon}\ , \varepsilon > 0\}$ *satisfies a LDP with rate function* $I_1 : E_1 \to [0, +\infty]$*. Let* $\Phi : \{I_1 < \infty\} \to E_2$ *be a mapping such that its restriction to the compact sets* $\{I_1 \leq a\}$ *is continuous in the topology of* E_1*. For any* $g \in E_2$ *we set* $I(g) = \inf\ \Big\{ I_1(f), \Phi(f) = g \Big\}$*. Suppose that for* $R, \rho, a > 0$ *there exist* α *and* $\varepsilon_0 > 0$ *such that for any* $h \in E_1$ *satisfying* $I_1(h) \leq a$ *and* $\varepsilon \leq \varepsilon_0$ *we have:*

$$P\Big\{ d_2(X_2^{\varepsilon}, \Phi(h)) \geq \rho, d_1(X_1^{\varepsilon}, h) \leq \alpha \Big\} \leq exp(-\frac{R}{\varepsilon^2}) .$$

Then the family $\{X_2^\varepsilon, \varepsilon > 0\}$ *satisfies a LDP with rate function* I.

$$I(g) = \inf\{I_1(h); \Phi(h) = g\}\,.$$

Let us now introduce some function spaces that will be used in the analysis of solutions of the stochastic differential equation (1.1) .

2.2. **Hölderian norm.** For $\alpha \in]0, 1/2[$, we define the $\alpha-$Hölder space $C^\alpha([0,1],\mathbb{R})$ as the space of continuous functions f such that:

$$\| f \|_\alpha = \sup_{0\le u\ne v\le 1} \frac{|f(u)-f(v)|}{|v-u|^\alpha} < \infty\,.$$

Define the Hölderian modulus of continuity of f by

$$D_\alpha(f,\delta) = \sup_{0\le u\ne v\le \delta} \frac{|f(v)-f(u)|}{|v-u|^\alpha}\,.$$

It is well known that $C^\alpha([0,1],\mathbb{R})$ is not separable but its closed subspace, defined by

$$C^{\alpha,0}([0,1],\mathbb{R}) = \{f \in C^\alpha([0,1],\mathbb{R}); \lim_{\delta\to 0} D_\alpha(f,\delta) = 0\}\,,$$

is separable. Both $C^\alpha([0,1],\mathbb{R})$ and $C^{\alpha,0}([0,1],\mathbb{R})$ are Banach spaces for the norm $\| f \|_\alpha$ and $\| f \|_\infty$. It is well known that $P(\| D \|_\alpha < \infty) = 1$ for $0 < \alpha < 1/2$.

It is a remarkable fact that

$$\| f \|_\infty \le \| f \|_\alpha\ . \tag{2.1}$$

2.3. **Fractional Brownian motion.** We consider $\mathcal{W} = \{x \in C([0,1],\mathbb{R}) : x(0) = 0\}$ equipped with the supremum norm. We denote by $\mathcal{W}'$ the strong topological dual of $\mathcal{W}$. For $H \in (0,1)$ we denote by P_H the unique probability measure on $\mathcal{W}$ such that the canonical process $\{W_t^H\}_{t\in[0,1]}$ is a fBm with Hurst parameter H. Recall that the covariance R_H of W is given by

$$R_H(s,t) = \frac{C_H}{2}\Big(s^{2H} + t^{2H} - |t-s|^{2H}\Big),$$

$$C_H = \frac{\Gamma(2-2H)\cos\pi H}{\pi H(1-2H)}\,.$$

In order to represent the fBm in terms of standard Brownian motion (obtained for $H = \frac{1}{2}$) we need hypergeometric functions.

We consider the Gauss hypergeometric function $F(\lambda,\beta,\gamma,z)$ which is the analytic continuation on $C\times C\times C\backslash\{0,-1,-2,\dots\}\times\{z\in C : Arg(1-z)<\pi\}$ of the power series

$$F(\lambda,\beta,\gamma,z)=\sum_{k=0}^{\infty}\frac{(\lambda)_k(\beta)_k}{(\gamma)_k k!}\,z^k,$$

$(a)_k=a(a+1)....(a+k-1)$. Consider the square integrable kernel

$$K_H(t,r)=\frac{(t-r)^{H-\frac{1}{2}}}{\Gamma(H+\frac{1}{2})}\,F\Big(\frac{1}{2}-H,H-\frac{1}{2},H+\frac{1}{2},1-\frac{t}{r}\Big)1_{[0,t)}(r)\,.$$

It is known that, see [15]:

$$R_H(t,s)=\int_0^{t\wedge s}K_H(t,r)\,K_H(s,r)dr\,.$$

Next we shall denote by ∇_H the Gross-Sobolev derivative operator, δ its dual (divergence operator) and we define the stochastic integral with respect the fBm by

$$\int_0^1 u_s\,\delta_H W_s=\delta_H(K_H u)\,,$$

for processes u for which $K_H u$ is in the domain of δ_H.

Remark 2.1. *It is known that [15]*

(i) $\Big\{\displaystyle\int_0^1 K_H(t,s)\,\delta_H W_s^H\Big\}_{t\in T}=\{W_t^H\}_{t\in T}$.

(ii) $\Big\{\displaystyle\int_0^1 1_{[0,t]}\,\delta_H W_s^H\Big\}_{t\in T}:=\{W\}_{t\in T}$ *is a standard Brownian motion on* $(\mathcal{W},\mathcal{B}(\mathcal{W}),P_H)$.

(iii) *For every* $u\in L^2(\mathcal{W}\times[0,1])$, u *adapted,*

$$\int_0^t u_s\,\delta_H W_s^H=\int_0^t u_s\,dW_s,\ t\in[0,1],$$

(the last integral is the usual Itô integral).

(iv) *We have the following representation for the fBm*

$$W_t^H=\int_0^t K_H(t,s)\,dW_s,\ t\in[0,1].$$

(v) *When H>1/2, the square integrable kernel can be written*

$$K_H(t,s)=c_H\,s^{\frac{1}{2}-H}\int_s^t\Big(u-s\Big)^{H-\frac{3}{2}}u^{H-\frac{1}{2}}\,du$$

where $c_H = \frac{(t-r)^{H-\frac{1}{2}}}{\Gamma(H+\frac{1}{2})}$.

2.4. **Preliminaries results.** The following LDP proved by Baldi et al. (1992) extends the classical Schilder theorem (see Schilder 1996; Deuschel and Strook 1989)

Theorem 2.1. *Let P^ε be the law of $\sqrt{\varepsilon}W$ on $C([0,1],\mathbb{R})$ equipped with the norm $\|.\|_\infty$ satisfying the LDP with the good rate function $\lambda(.)$ defined by:*

$$\lambda(h) = \begin{cases} \frac{1}{2}\int_0^T |\dot{h}(s)|^2 \, ds & \textit{if } h \in \mathcal{H} \\ +\infty & \textit{otherwise} . \end{cases}$$

One of the basic tools in large deviation theory is the 'contraction principle' (see Deuschel and Strook 1989). It enables the new rate function to be computed after the data have been transformed by a continuous map [25].

Theorem 2.2. *Let Q^ε be a family of probability measure on a Polish space E and satisfies the LDP with a good rate function λ.*
Let $F : E \to E'$ be countinuous. Denote by $Q^\varepsilon = P^\varepsilon \circ F^{-1}$ the family of image measure of P^ε, then $\{Q^\varepsilon\}$ satisfies the LDP with a good rate function $\tilde{\lambda}$ defined by

$$\tilde{\lambda}(y) = \inf_{x:F(x)=y} \lambda(x).$$

Lemma 2.2. *Let $\{\mathcal{D}(t)\}_{t\in T}$ be a bounded nonanticipating real process and $0 < \alpha < H$. Then, for any $a > 0$,*

$$P\left(\| \int_0^\cdot K_H(.,s)\mathcal{D}(s)dW_s \|_\alpha > a\right) \leq K_1 exp\{-(aK_2-1)^2\},$$

where $0 < K_i = K_i\left(\alpha, H, \|\mathcal{D}\|_\infty\right) < \infty$.
In particular

$$P\left(\sup_{0\leq t\leq 1}\left|\int_0^t K_H(t,s)\mathcal{D}(s)dW_s\right| > a\right) \leq K_1 exp\{-(aK_2-1)^2\} .$$

For the proof of this result we refer to [12].

3. LDP FOR PERTURBED DIFFUSION PROCESSES

In this section, we will give a LDP of the perturbed diffusion process solution of (1.3). Let $\mathcal{H}$ be the Cameron Martin space associated to the standard Brownian motion, that is:

$$\mathcal{H} = \left\{ \begin{array}{c} h : [0,1] \to \mathbb{R}, h \text{ is absolutely continuous such that} \\ h(0) = 0 \text{ and } \displaystyle\int_0^1 |\dot{h}_s|^2 \, ds < +\infty \, . \end{array} \right\}$$

Let $\Phi^x(h)(t)$ be the unique solution of the following deterministic perturbed equation:

(3.1)

$$\Phi^x(h)(t) = x_0 + \int_0^t K_H(t,s) b\Big(\Phi^x(h)(s)\Big) ds + \int_0^t K_H(t,s)\sigma\Big(\Phi^x(h)(s)\Big)\dot{h}(s) ds$$
$$+\beta \sup_{0\le s\le t} \Phi^x(h)(s) \, .$$

We have the following main results.

Theorem 3.1. *For $H \in (0,1)$, $\alpha \in]0,1/2[$, $\beta \in]0,1[$, let $\{\eta_\varepsilon, \varepsilon > 0\}$ be the probability measure induced by X^ε on $C^\alpha([0,1],\mathbb{R})$ equipped with the norm $\| \,.\, \|_\alpha$, then η_ε is satisfying the LDP with the good rate function $I(.)$ defined by:*

$$I(g) = \inf_{h\in H; g=\Phi^x(h)} \lambda(h) \, ,$$

where the inf over the empty set is taken to be ∞.

Theorem 3.2. *For $H \in (0,1)$, $\alpha \in]0,1/2[$, $\beta \in]0,1[$ and $h \in \mathcal{H}$. For any $R, \delta > 0$ there exist $\rho > 0$ such that*

$$\mathbb{P}\Big(\| X^\varepsilon - \Phi^x(h) \|_\alpha > \rho, \| \sqrt{\varepsilon} W - h \|_\infty < \delta \Big) \le \exp(-\frac{R}{\varepsilon}).$$

Proof. By (1.3) and (3.1),

$$\begin{aligned} X_t^\varepsilon - \Phi^x(h)(t) &= \int_0^t \sigma_H(X_s^\varepsilon) \left(\sqrt{\varepsilon}\, dW_s - \dot{h}(s)\right) ds \\ &+ \int_0^t \sigma_H(X_s^\varepsilon) \ - \sigma_H\Big(\Phi^x(h)(s)\Big) \, \dot{h}(s) \, ds \\ &+ \int_0^t b_H(X_s^\varepsilon) \ - b_H\Big(\Phi^x(h)(s)\Big) \, ds \\ &+ \beta\Big(\sup_{0\le s\le t} X_s^\varepsilon - \sup_{0\le s\le t} \Phi^x(h)(s)\Big) \, , \end{aligned}$$

where $\sigma_H(x) = K_H(t,s)\,\sigma(x)$ and $b_H(x) = K_H(t,s)\,b(x)$. Consequently,

$$\begin{aligned}|X_t^\varepsilon - \Phi^x(h)(t)| &\leq \int_0^t |\sigma_H(X_s^\varepsilon)\,(\sqrt{\varepsilon}\,dW_s - \dot{h}(s)\,ds)| \\ &+ L\int_0^t \Big(|X_s^\varepsilon - \Phi^x(h)(s)|\,|K_H(t,s)| \\ &\quad (1+|\dot{h}(s)|)\Big)\,ds \\ &+ \beta \sup_{0\leq s\leq t} |X_s^\varepsilon - \Phi^x(h)(s)|\,,\end{aligned}$$

where $L>0$ is the Lipschitz coefficient, and we also used the fact that

$$\Big|\sup_{0\leq s\leq t} u(s) - \sup_{0\leq s\leq t} v(s)\Big| \leq \sup_{0\leq s\leq t}\big|u(s)-v(s)\big|\,.$$

For any two continuous functions u and v on $\mathbb{R}_+$. Thus, it follows from that, for $t\in[0,1]$

$$\begin{aligned}&\sup_{0\leq u\leq t}|X_u^\varepsilon - \Phi^x(h)(u)| \leq \\ &\tfrac{1}{1-\beta}\sup_{0\leq u\leq t}\int_0^u |\sigma_H(X_s^\varepsilon)\,(\sqrt{\varepsilon}\,dW_s - \dot{h}(s)ds)| \\ &+\tfrac{L}{1-\beta}\sup_{0\leq u\leq t}\int_0^u |X_s^\varepsilon - \Phi^x(h)(s)|\,|K_H(u,s)|\,(1+|\dot{h}(s)|)\,ds\,.\end{aligned}$$

By the Gronwall lemma and Cauchy-Schwarz inequality this yields that

$$\begin{aligned}&\| X_t^\varepsilon - \Phi^x(h)(t)\|_\infty \leq \\ &\tfrac{1}{1-\beta}\sup_{0\leq t\leq 1}\int_0^t |\sigma_H(X_s^\varepsilon)\,(\sqrt{\varepsilon}\,dW_s - \dot{h}(s)|\,ds \\ &\times\exp\Big(\int_0^t \frac{L}{1-\beta}\,|K_H(t,s)|\,(1+\big|\dot{h}(s)\big|\Big)\,ds \\ &\leq C_1(h)\sup_{0\leq t\leq 1}\int_0^t |\sigma_H(X_s^\varepsilon)\,\Big(\sqrt{\varepsilon}\,dW_s - \dot{h}(s)\Big)|\,ds \\ &\leq C_1(h)\ \Big\|\int_0^t \sigma_H(X_s^\varepsilon)\,\Big(\sqrt{\varepsilon}\,dW_s - \dot{h}(s)\Big)\,ds\,\Big\|_\infty\,,\end{aligned}$$

where $C_1(h) = \frac{1}{1-\beta}\exp\Big(C(H)\,L\,(1+\| h\|_{\mathcal{H}})/(1-\beta)\Big)$, with $\| h\|_{\mathcal{H}} = \Big(\int_0^1 |\dot{h}_s|^2\,ds\Big)^{1/2}$ for $h\in\mathcal{H}$. When $H\in(0,1)$, since $t\leq 1$, we have:

$$\Big(\int_0^1 |K_H(t,s)|^2 ds\Big)^{1/2} = \| K_H(t,.)\|_{L^2([0,1])} = C(H)\,.$$

By using the inequality (2.1) and $h = 0$, we deduce that

$$\| X_t^\varepsilon - \Phi^x(0) \|_\infty \le C_1(0) \| \int_0^t \sqrt{\varepsilon}\, K_H(t,s)\, \sigma(X_s^\varepsilon)\ dW_s \|_\alpha\ . \tag{3.2}$$

For any $t \in [0,1]$, for any continuous functions $f : [0,1] \to \mathbb{R}$, denote by

$$\| f \|_{\alpha,t} = \sup_{0 \le u \ne v \le t} \frac{|f(u) - f(v)|}{|v-u|^\alpha} < \infty\,.$$

Next, we set

$$\mathbf{D}_{\Phi^x(0)}^{X^\varepsilon}(u) = X_u^\varepsilon - \Phi^x(0)(u)\,.$$

By the reflexion principle, we have

$$\begin{aligned}
&\frac{\left|\mathbf{D}_{\Phi^x(0)}^{X^\varepsilon}(t) - \mathbf{D}_{\Phi^x(0)}^{X^\varepsilon}(s)\right|}{|t-s|^\alpha} \le \\
&\frac{1}{|t-s|^\alpha}\Bigg(\Big|\frac{1}{1-\beta}\int_s^t \sqrt{\varepsilon}\,\sigma_H(X_v^\varepsilon)\, dW_v \\
&+ \frac{\beta}{1-\beta}\sup_{s\le u\le t}\Big(\sqrt{\varepsilon}\int_s^u \sigma_H(X_v^\varepsilon)\, dW_v\Big) \\
&+ \frac{\beta}{1-\beta}\sup_{s\le u\le t}\int_s^u b(X_v^\varepsilon) - b\Big(\Phi^x(0)(v)\Big)\ dv\Big|\Bigg)\,.
\end{aligned}$$

Consequently, we obtain :

$$\begin{aligned}
\| (X^\varepsilon - \Phi^x(0)) \|_{\alpha,t} \ &\le \tfrac{1}{1-\beta} \quad \| \sqrt{\varepsilon}\int_0^t \sigma_H(X_v^\varepsilon)\, dW_v \|_{\alpha,t} \\
&+ \tfrac{\beta\ L}{1-\beta} \quad \| X^\varepsilon - \Phi^x(0) \|_\infty \\
&+ \tfrac{\beta\ L}{1-\beta} \quad \int_0^t \| X^\varepsilon - \Phi^x(0) \|_{\alpha,t}\ dv\,.
\end{aligned}$$

By using (3.2), we get

$$\| (X^\varepsilon - \Phi^x(0)) \|_\alpha \le \left(\tfrac{1}{1-\beta} + \tfrac{\beta\ C_1(0)\ L}{1-\beta}\right) \| \sqrt{\varepsilon}\int_0^t K_H(t,s)\,\sigma(X_v^\varepsilon)\, dW_v \|_\alpha$$

$$+ \tfrac{\beta\ L}{1-\beta}\int_0^t \| X^\varepsilon - \Phi^x(0) \|_\alpha\ dv\,.$$

By the Gronwall lemma we have:

$$\| (X^{\varepsilon} - \Phi^{x}(0)) \|_{\alpha} \leq \left(\tfrac{1}{1-\beta} + \tfrac{\beta\ C_1(0)\ L}{1-\beta} \right) \| \times \sqrt{\varepsilon} \int_0^t K_H(t,s)\ \sigma(X_v^{\varepsilon})\ dW_v \|_{\alpha}\ \Theta(0)\,,$$

where $\Theta(0) = \exp\left(\frac{\beta\ L}{1-\beta} \right)$. Similar with the proof of (3.2), we obtain:

$$\| X_t^{\varepsilon} - \Phi^{x}(h)(t) \|_{\infty} \leq C_1(h) \| \int_0^t \sigma_H(X_s^{\varepsilon}) \Big(\sqrt{\varepsilon} dW_s - \dot{h}(s) \Big) ds \|_{\alpha} \tag{3.3}$$

Next we have:

$$\frac{\left| \mathbf{D}^{X^{\varepsilon}}_{\Phi^x(h)}(t) - \mathbf{D}^{X^{\varepsilon}}_{\Phi^x(h)}(s) \right|}{|t-s|^{\alpha}} \leq \tfrac{1}{|t-s|^{\alpha}} \Bigg(\Bigg|$$

$$\tfrac{1}{1-\beta} \int_s^t \sigma_H(X_v^{\varepsilon}) \left(\sqrt{\varepsilon}\ dW_v - \dot{h}(v) \right) dv$$

$$+ \tfrac{\beta}{1-\beta} \sup_{s \leq u \leq t} \left(\int_s^u \sqrt{\varepsilon}\ \sigma_H(X_v^{\varepsilon})\ dW_v \right)$$

$$+ \tfrac{\beta}{1-\beta} \sup_{s \leq u \leq t} \left(\int_s^u b_H(X_v^{\varepsilon}) - b_H\Big(\Phi^x(h)(v)\Big)\ dv \right.$$

$$+ \tfrac{\beta}{1-\beta} \sup_{s \leq u \leq t} \int_s^u \left[\sigma_H(X_v^{\varepsilon})(v) - \sigma_H\Big(\Phi^x(h)\Big)(v) \right] \dot{h}(v)\ dv \Bigg| \Bigg)$$

An application of formula (3.3), we obtain

$$\| (X^{\varepsilon} - \Phi^x(h)) \|_{\alpha,t} \leq$$

$$\tfrac{1}{1-\beta} \| \int_0^{\cdot} \sigma_H(X_s^{\varepsilon}) \left[\sqrt{\varepsilon}\ dW_s - \dot{h}(s) \right] ds \|_{\alpha,t}$$

$$+ \tfrac{\beta\ L}{1-\beta} C_1(h) \| \int_0^t \sigma_H(X_s^{\varepsilon}) \left[\sqrt{\varepsilon}\ dW_s - \dot{h}(s) \right] ds \|_{\alpha}$$

$$+ \tfrac{\beta\ L}{1-\beta} \int_0^{\cdot} (1 + |\dot{h}_s|) \| X^{\varepsilon} - \Phi^x(h) \|_{\alpha,t}\ ds$$

By the Gronwall lemma

(3.4)

$$\| (X^\varepsilon - \Phi^x(h)) \|_\alpha \leq \left(\tfrac{1}{1-\beta} + \tfrac{\beta\ L\ C_1(h)}{(1-\beta)}\right) \times \| \int_0^t K_H(t,s)\ \sigma(X_s^\varepsilon)\ (\sqrt{\varepsilon}\ dW_s -\ \dot{h}_s\ ds) \|_\alpha\ \ \Theta(h)$$

where $\Theta(h) = \exp\left(\frac{\beta L(1+\|h\|_{\mathcal{H}})}{1-\beta}\right)$ $\square$

Theorem 3.3. *For* $\alpha \in]0,1/2[,\ \ \beta \in]0,1[$. *For any* $R, \delta, \tilde{a} > 0$ *there exist* $\rho > 0$ *and* $\varepsilon_0 > 0$ *such that, for any* $h \in C^\alpha([0,1],\mathbb{R})$ *satisfying* $\lambda(h) \leq \tilde{a}$ *and* $\varepsilon \leq \varepsilon_0$

$$P\bigg(\ \| K_H(t,s)\ \sigma(X_s^\varepsilon)\ \left(\sqrt{\varepsilon}\ dW_s - \dot{h}(s)\ ds\right) \|_\alpha > \rho, \| \sqrt{\varepsilon}\ W_s - h \|_\infty < \delta\bigg) \leq \exp\left(-\tfrac{R}{\varepsilon}\right).$$

For $\varepsilon > 0$, define a probability measure P^ε on Ω by

$$\mathrm{dP}^\varepsilon = M_\varepsilon\ \mathrm{dP} = \exp\left(-\frac{2}{\sqrt{\varepsilon}}\int_0^1 \dot{h}_s\ dW_s + \frac{1}{\varepsilon}\int_0^1 |\dot{h}_s|^2\ ds\right)\mathrm{dP} \tag{3.5}$$

Then, Girsanov's theorem implies that

$\left\{W_t^\varepsilon = W_t - \frac{1}{\sqrt{\varepsilon}}\ \dot{h}_t,\ t \in [0,1]\right\}$ is a Wiener process with respect to the probability P^ε. Let $\{U_t^\varepsilon, 0 \leq t \leq 1\}$ be the solution of SDE

(3.6)

$$U^\varepsilon(t) = x_0 + \int_0^t K_H(t,s)\ b(U^\varepsilon(s))\ ds + \int_0^t K_H(t,s)\ \sigma(U^\varepsilon(s))\ \dot{h}(s)\ ds + \beta \sup_{0\leq s\leq t} U^\varepsilon(s)$$

To simplify the notation, set for any ρ ,α, $\varepsilon > 0$

$$A^\varepsilon = \bigg\{\ \| K_H(t,s)\ \sigma(X_s^\varepsilon)\ \left(\sqrt{\varepsilon}\ dW_s - \dot{h}(s)\ ds\right) \|_\alpha > \rho, \| \sqrt{\varepsilon}\ W_s - h \|_\infty < \delta\bigg\}$$

Then by the Cauchy-Schwarz inequality,

$$P(A^\varepsilon) = \int_\Omega M_\varepsilon^{-1} \chi_{\{A^\varepsilon(w)\}} \mathrm{P}^\varepsilon(dW) \leq \left(\int_\Omega M_\varepsilon^{-2}(w)\ \mathrm{P}^\varepsilon(dW)\right)^{1/2} \left(\mathrm{P}^\varepsilon(A^\varepsilon)\right)^{1/2}$$

An application of formula (3.5), we obtain

$$\int_\Omega M_\varepsilon^{-2}(w) \mathrm{P}^\varepsilon(dW) = \mathrm{E}_{\mathrm{P}^\varepsilon}\left[\exp\left(-\frac{2}{\sqrt{\varepsilon}}\int_0^1 \dot{h}_s\ dW_s + \frac{1}{\varepsilon}\int_0^1 |\dot{h}_s|^2\ ds\right)\right]$$

$$= \mathrm{E}_{\mathrm{P}^\varepsilon}\left[\exp\left(-\tfrac{2}{\sqrt{\varepsilon}}\int_0^1 \dot{h}_s\ dW_s - \frac{2}{\varepsilon}\int_0^1 |\dot{h}_s|^2\ ds\right)\right] \times \exp\left(\frac{1}{\varepsilon}\int_0^1 |\dot{h}_s|^2\ ds\right)$$

Finally, we obtain

$$\int_\Omega M_\varepsilon^{-2}(w)\ \mathrm{P}^\varepsilon(dW) = \exp\Big(\frac{1}{\varepsilon} \parallel h \parallel_H^2\Big)$$

Therefore, if $\lambda(h) \leq a$ then

$$P(A^\varepsilon) \leq \exp\Big(\tfrac{\tilde{a}}{\varepsilon}\Big)\Big(\mathrm{P}^\varepsilon(A^\varepsilon)\Big)^{1/2}. \tag{3.7}$$

Therefore,

$$\mathrm{P}^\varepsilon(A^\varepsilon) = \mathrm{P}^\varepsilon\Big(\parallel K_H(t,s)\ \sigma(X_s^\varepsilon)\ \Big(\sqrt{\varepsilon}\ dW_s - \dot{h}(s)ds\Big) \parallel_\alpha > \rho, \parallel \sqrt{\varepsilon}\ W_s - h \parallel_\infty < \delta\Big)$$

$$= \mathrm{P}^\varepsilon\Big(\parallel K_H(t,s)\ \sigma(X_s^\varepsilon)\sqrt{\varepsilon}\ dW_s^\varepsilon \parallel_\alpha > \rho, \parallel \sqrt{\varepsilon}\ W_s^\varepsilon - h \parallel_\infty < \delta\Big)$$

$$= \mathrm{P}\Big(\parallel K_H(t,s)\ \sigma(X_s^\varepsilon)\sqrt{\varepsilon}\ dW_s \parallel_\alpha > \rho, \parallel \sqrt{\varepsilon}\ W_s - h \parallel_\infty < \delta\Big).$$

Theorem 3.4. *For $\alpha \in]0,1/2[,\ \beta \in]0,1[$. For any $R, \delta, \tilde{a} > 0$ there exist $\rho > 0$ and $\varepsilon_0 > 0$ such that, for any $h \in C^\alpha([0,1],\mathbb{R})$ satisfying $\lambda(h) \leq \tilde{a}$ and $\varepsilon \leq \varepsilon_0$*

$$P\Big(\parallel \int_0^t K_H(t,s)\ \sqrt{\varepsilon}\ \sigma(U_s^\varepsilon)\ \ dW_s \parallel_\alpha > \rho \parallel \sqrt{\varepsilon}\ W_t - h \parallel_\infty < \delta\Big) \leq \exp\Big(-\frac{R}{\varepsilon}\Big).$$

For any $n \in \mathbb{N}^*$ we consider the approximation sequence of the process U^ε defined by

$$U_t^{\varepsilon,n} = U_{\frac{j}{n}}^\varepsilon, if\ s \in \Big[\frac{j}{n}, \frac{j+1}{n}\Big[\ \ for\ \ all\ j = 0,1,2,...,n-1$$

For $\gamma > 0$ and for every $n \in \mathbb{N}$, we have

$$A^\varepsilon = \Big\{ \parallel \sqrt{\varepsilon} \int_0^\cdot K_H(t,s)\sigma(Y_s^\varepsilon)\ dW_s^\varepsilon \parallel_\alpha \geq \rho, \parallel \sqrt{\varepsilon} W^\varepsilon \parallel_\infty \leq \delta\Big\} \subset A_1^\varepsilon \cup A_2^\varepsilon \cup A_3^\varepsilon$$

where

$$\begin{cases} A_1^\varepsilon = \Big\{ \parallel \sqrt{\varepsilon} \int_0^\cdot K_H(t,s)\Big(\sigma(U_s^\varepsilon) - \sigma(U_s^{\varepsilon,n})\Big)dW_s^\varepsilon \parallel_\alpha \geq \frac{\rho}{2}, \parallel U^\varepsilon - U^{\varepsilon,n} \parallel_\infty \leq \gamma\Big\} \\ A_2^\varepsilon = \Big\{ \parallel U^\varepsilon - U^{\varepsilon,n} \parallel_\infty \geq \gamma\Big\} \\ A_3^\varepsilon = \Big\{ \parallel \sqrt{\varepsilon} \int_0^\cdot K_H(t,s)\ \sigma(U_s^{\varepsilon,n})\ dW_s^\varepsilon \parallel_\alpha \geq \frac{\rho}{2}, \parallel \sqrt{\varepsilon} W^\varepsilon \parallel_\infty \leq \delta\Big\} \end{cases}$$

On the set $\{\| U^\varepsilon - U^{\varepsilon,n} \|_\infty \leq \gamma\}$, we have the following estimates $\| \sqrt{\varepsilon}[\sigma(U_s^\varepsilon) - \sigma(U_s^{\varepsilon,n})] \|_\alpha \leq \sqrt{\varepsilon} L\gamma$ and by Lemma (2.2), it follows that

$$P(A_1^\varepsilon) \leq K_1 \exp\bigg\{ - \Big(\frac{\rho}{2\sqrt{\varepsilon}L\gamma} K_2 - 1 \Big)^2 \bigg\}$$

To treat $P\Big(A_3^\varepsilon\Big)$. On the set $\{\| \sqrt{\varepsilon} W^\varepsilon \|_\infty \leq \delta\}$, if σ is bounded by M, we get

$$\| \sqrt{\varepsilon} \int_0^t K_H(t,s)\sigma(U_s^{\varepsilon,n})\, dW_s^\varepsilon \|_\alpha =$$

$$\sqrt{\varepsilon} \| \sum_{j=0}^{n-1} \; K_H(t,s)\; \sigma(U_{t_j}^{\varepsilon,n})\Big(W^\varepsilon(t_{j+1} \wedge .) - W^\varepsilon(t_j \wedge .)\Big) \|_\alpha$$

$$\leq M \sum_{j=0}^{n-1} \; K_H(t,s)\; \sqrt{\varepsilon} \; \| \; \Big(W^\varepsilon(t_{j+1}) - W^\varepsilon(t_j)\Big) \|_\infty$$

$$\leq n\; M\; \delta \; \| \; K(t,.) \; \|_\infty$$

where $M > 0$ is a common bound of b and σ. Therefore, if $\delta \leq \frac{\rho}{2\; n\; M\; \|K(t,.)\|_\infty}$ then $P(A_3^\varepsilon) = 0$. By using the formula (2.17) in Bo and Zhang [7], we have

$$P(A_2^\varepsilon) \leq n \exp\bigg\{ - \frac{n\; \gamma^2(1-\beta)^2}{8L^2\varepsilon} \bigg\}$$

Proposition 3.1. *Let* $\alpha \in]0, \frac{1}{2}[$ *and* $\beta \in]0,1[$ *be. For any* $\tilde{a} \leq 0$*, the map* $F : C^{\alpha,0}([0,1],\mathbb{R}) \cap \bigg(\Big\{h \in \mathcal{H} : \| h \|_{\mathcal{H}}^2 \leq \tilde{a}\Big\}\bigg) \longrightarrow (C^{\alpha,0}([0,1],\mathbb{R}), \| \;.\; \|_\alpha)$ *is continuous.*

4. LDP FOR PERTURBED REFLECTED DIFFUSION PROCESSES

In this section, we will prove the LDP for the solution of the perturbed reflected diffusion equation (1.4)

For $y \geq 0$ and $f \in C_y([0,1],\mathbb{R})$ as the space of continuous functions in $[0,1]$ to $\mathbb{R}$ starting from y.

Define two operators $\Gamma : C_y([0,1],\mathbb{R}) \longrightarrow C_y([0,1],\mathbb{R}_+)$ and $K : C_y([0,1],\mathbb{R}) \longrightarrow C_y([0,1],\mathbb{R}_+)$

by $\Gamma f = f + \tilde{f}$ and $Kf = \tilde{f}$, where $\tilde{f} = -\inf_{s\leq t}(f(s) \wedge 0)$, $t \in [0,1]$.

By the reflection principle, the solution T^ε of (1.4) is given by

$$T_t^\varepsilon = (\Gamma Z^\varepsilon)(t) \text{ and } L_t^\varepsilon = (KZ^\varepsilon)(t),\; t \in [0,1] \tag{4.1}$$

Where Z^ε is a solution of the following stochastic equation

(4.2) $$Z^\varepsilon = y + \sqrt{\varepsilon}\int_0^t \varsigma(\Gamma Z^\varepsilon)(t))\, dW_s^H + \beta \sup_{0\le s\le t}(\Gamma Z_s^\varepsilon)\ t \in [0,1]$$

For $h \in \mathcal{H}$, let $\tilde\Phi^y(h)$ the unique solution of the following equation:

(4.3)
$$\tilde\Phi^y(h)(t) = y + \int_0^t K_H(t,s)\,\varsigma(\Gamma\tilde\Phi^y(h))(s))\,\dot h_s\, ds + \beta \sup_{0\le s\le t}(\Gamma\tilde\Phi^y(h)(s)) + \eta_t\ t \in [0,1]$$

where $\tilde\Phi^y(h)$ is continuous, non-negative and η is an increasing continuous function satisfying $\eta_t = \int_0^t \chi_{\{\tilde\Phi^y(h)=0\}}\, d\eta_s$. The existence and uniqueness of the solution to (4.4) might be obtained by Theorem (4.1) and Theorems 4.2 and 4.3 in [7]

Similar as (4.1), $\tilde\Phi^y(h)$ can also be written as

(4.4) $$\tilde\Phi^y(h)(t) = (\Gamma V(h))(s) \text{ and } \eta_t = (KV(h))(s),\ t \in [0,1]$$

Where $V(h)$ is a solution of the following stochastic equation

(4.5)
$$V(h)(t) = y + \sqrt{\varepsilon}\int_0^t K_H(t,s)\,\varsigma(\Gamma V(h)(s))\,\dot h_s\, ds + \beta \sup_{0\le s\le t}(\Gamma V(h)(s))\ t \in [0,1]$$

Let ν_ε^1 be the law of Z^ε on $C_y([0,1],\mathbb{R}_+)$ equipped with the Hölderian norm $\|\,.\,\|_\alpha$ We have the following main result.

Theorem 4.1. *For $\alpha \in]0,1/2[,\ \beta \in]0,1/2[$. Let $\{\nu_\varepsilon^1, \varepsilon > 0\}$ be the probability measure induced by Z^ε on $C_y([0,1],\mathbb{R}_+)$ equipped with the norm $\|.\|_\alpha$, then ν_ε^1 satisfying the LDP with the good rate function $\tilde I_y(.)$ defined by:*

(4.6) $$\tilde I_y(g) = \inf_{h\in\mathcal{H};g=\tilde\Phi^x(h)} \lambda(h)$$

where the $\inf$ over the empty set is taken to be ∞.

Theorem 4.2. *For $\alpha \in]0,1/2[,\ \beta \in]0,1/2[$. Let $\{\nu_\varepsilon^2, \varepsilon > 0\}$ be the probability measure induced by T^ε on $C_y([0,1],\mathbb{R}_+)$ equipped with the norm $\|.\|_\alpha$, then ν_ε^2 satisfying the LDP with the good rate function $\bar I_y(.)$ defined by:*

(4.7) $$\bar I_y(g) = \inf_{\bar g=\Gamma g} \tilde I_y(g)$$

Where the $\inf$ over the empty set is taken to be ∞

Proof. By contraction principle, by using the formula $\Gamma\psi - \psi = (K\psi, ..., 0)$ see Anderson et Orey [1], it suffices to prove that Γ is continuous α-Hölderian. Recall that $\| \Gamma\psi_1 - \Gamma\psi_2 \|_\infty \leq \| \psi_1 - \psi_2 \|_\infty$

$$\begin{aligned} \tfrac{\|\Gamma\psi_1 - \Gamma\psi_2\|_\alpha}{\|\psi_1-\psi_2\|_\alpha} &\leq \tfrac{|(\Gamma\psi_1(t)-\Gamma\psi_2(t))-(\Gamma\psi_1(s)-\Gamma\psi_2(s))|}{|\psi_1-\psi_2|} \\ &+ \tfrac{2\|\psi_1-\psi_2\|_\infty + 2\|\psi_1-\psi_2\|_\infty}{\|\psi_1-\psi_2\|_\infty} = 4 \end{aligned}$$

It follows $\| \Gamma\psi_1 - \Gamma\psi_2 \|_\alpha \leq 4 \| \psi_1 - \psi_2 \|_\alpha$ □

Theorem (4.2) is the consequence of the following two propositions.

Theorem 4.3. *For* $\alpha \in]0, 1/2[$, $\beta \in]0, 1/2[$ *and* $h \in \mathcal{H}$. *For any* $R, \rho > 0$ *there exist* $\delta > 0$ *such that*

$$\mathbb{P}\bigg(\| Z^{\varepsilon,y} - V^y(h) \|_\alpha > \rho, \| \sqrt{\varepsilon}w - h \|_\infty < \delta \bigg) \leq \exp\bigg(- \frac{R}{\varepsilon} \bigg).$$

Theorem 4.4. *For* $\alpha \in]0, 1/2[$, $\beta \in]0, 1[$. *For any* $R, \rho > 0$ *there exist* $\delta > 0$ *and* $\varepsilon_0 > 0$ *such that, for any* ε *small enough*

$$\begin{aligned} &\boldsymbol{P}\bigg(\| \int_0^t K_H(t,s) \sqrt{\varepsilon}\, \varsigma(\Gamma Z_s^{\varepsilon,y}(s)) \;\; dW_s \|_\alpha > \rho, \\ &\| \sqrt{\varepsilon}\, w \|_\infty < \delta \bigg) \leq \exp\bigg(- \tfrac{R}{\varepsilon} \bigg). \end{aligned}$$

For any $n \in \mathbb{N}^*$ we consider the approximation sequence of the process Z^ε defined by

$$Z_t^{\varepsilon,n} = Z_{\frac{j}{n}}^{\varepsilon}, if \; s \in \Big[\frac{j}{n}, \frac{j+1}{n}\Big[\;\; for \;\; all \; j = 0, 1, 2, ..., n-1$$

For $\alpha > 0$ and for every $n \in \mathbb{N}$, we have

$$\tilde{A}^\varepsilon = \bigg\{ \| \sqrt{\varepsilon} \int_0^{\cdot} \; K_H(t,s)\, \varsigma(Y_s^\varepsilon)\, dW_s^\varepsilon \|_\alpha \geq \rho, \| \sqrt{\varepsilon} W^\varepsilon \|_\infty \leq \delta \bigg\} \subset \tilde{A}_1^\varepsilon \cup \tilde{A}_2^\varepsilon \cup \tilde{A}_3$$

where

$$\begin{cases} \tilde{A}_1^{\varepsilon} = \Big\{ \parallel \sqrt{\varepsilon} \int_0^{\cdot} K_H(t,s) \Big(\varsigma(\Gamma Z_s^{\varepsilon}) - \varsigma(\Gamma Z_s^{\varepsilon,n,y}) \Big) \, dW_s^{\varepsilon} \parallel_{\alpha} \geq \frac{\rho}{2}, \parallel Z^{\varepsilon} - Z^{\varepsilon,n,y} \parallel_{\infty} \leq \gamma \Big\} \\ \tilde{A}_2^{\varepsilon} = \Big\{ \parallel Z^{\varepsilon,y} - Z^{\varepsilon,n,y} \parallel_{\infty} \geq \gamma \Big\} \\ \tilde{A}_3^{\varepsilon} = \Big\{ \parallel \sqrt{\varepsilon} \int_0^{\cdot} K_H(t,s) \, \varsigma(\Gamma Z_s^{\varepsilon,n,y}) \, dW_s^{\varepsilon} \parallel_{\alpha} \geq \frac{\rho}{2}, \parallel \sqrt{\varepsilon} W^{\varepsilon} \parallel_{\infty} \leq \delta \Big\} \end{cases}$$

By the result obtained in Bo and Zhang [7], we have

$$P(\tilde{A}_2^{\varepsilon}) \leq n \exp\Big(- \frac{n\gamma(1-2\beta)^2}{8N^2\varepsilon} \Big)$$

For any $R, \gamma > 0$ there exist $\tilde{\varepsilon}_0 > 0$ and $\tilde{n}_0 > 0$ such that if $\varepsilon \leq \tilde{\varepsilon}_0$ and $n \geq \tilde{n}_0$

$$P(\tilde{A}_2^{\varepsilon}) \leq \exp\Big(- \frac{R}{\varepsilon} \Big)$$

By Lemma (2.2) and Theorem (4.3),

$$P(\tilde{A}_1^{\varepsilon}) \leq C \exp\Big(- \frac{\rho^2}{8L\gamma^2\varepsilon} \Big)$$

Lemma 4.1. *(Existence and uniqueness of solution)*
Assume that σ, b are bounded and Lipschitz, the equation (3.1) *has a unique solution*

Proof. If $\Phi^{(1)}(h)$ and $\Phi^{(2)}(h)$ are continuous solutions of (3.1) then it follows

Put $\mathbf{D}\Phi(h)(t) = \Phi^{(1)}(h)(t) - \Phi^{(2)}(h)(t)$

Then $\parallel \Phi^{(1)}(h) - \Phi^{(2)}(h) \parallel_{\alpha}$ can be rewritten as $\parallel \mathbf{D}\Phi(h) \parallel_{\alpha}$

$$|\mathbf{D}\Phi(h)(t)| \leq L \int_0^t |K_H(t,s)| \, \mathbf{D}\Phi(h)(s)(1+\dot{h}) \, ds + \beta \sup_{0 \leq s \leq t} |\mathbf{D}\Phi(h)(s)| \tag{4.8}$$

Thus,

$$\parallel \mathbf{D}\Phi(h) \parallel_{\alpha} \leq L \frac{1}{1-\beta} \int_0^t |K_H(t,v)| \parallel \mathbf{D}\Phi(h) \parallel_{\alpha} (1+|\dot{h}_v|) \, dv \tag{4.9}$$

Set,

$$\phi_t = |K_H(t,v)| \, (1+|\dot{h}_v|) \in L^2([0,1]) \tag{4.10}$$

By Cauchy-Schwarz inequality we obtain

$$\int_0^1 \phi_s \, ds \leq \left(\int_0^1 |K_H(t,s)|^2 ds \right)^{1/2} \left(\int_0^1 (1 + |\dot{h}_s|)^2 ds \right)^{1/2}$$
$$= C(H)(1 + \parallel h \parallel_{\mathcal{H}}) < \infty$$

We can be deduced from Gronwall's lemma $\parallel \Phi^{(1)}(h) - \Phi^{(2)}(h) \parallel_\alpha = 0$, thus $\Phi^{(1)}(h) = \Phi^{(2)}(h)$

For the existence we use the successive approximations. Define

$$\Phi_t^n = x_0 + \int_0^t K_H(t,s) \, b(\Phi_t^{n-1}) \, ds + \int_0^t K_H(t,s) \, \sigma(\Phi_t^{n-1}) \, \dot{h}(s) \, ds + \beta \sup_{0 \leq s \leq t} \Phi_s^{n-1}$$

Denote by $\Phi_n(t) = \parallel \Phi_t^{n+1} - \Phi_t^n \parallel_\alpha$ then we have

$$\Phi_0(t) \leq \frac{L}{1-\beta} \int_0^t |K_H(t,s)| \, (1 + \dot{h}_s) \, ds < \infty$$

$$\Phi_n(t) \leq \frac{L}{1-\beta} \int_0^t |K_H(t,s)| \, \Phi_{n-1}(s) \, (1 + \dot{h}_s) \, ds < \infty$$

And by iteration

$$\Phi_n(t) \leq \left(\frac{L}{1-\beta} \right)^n D \, K_{n-1}^{(1)}(t)$$

Then we deduce that $\Phi_n(t) \longrightarrow \Phi(t)$ uniformly in t and Φ is the solution of (3.1) $\square$

5. Conclusions

In the present paper, we have etablished a large deviation principle (LDP) for perturbed reflected diffusion processes driven by a fractional brownian motion for any Hurst parameter $H \in (0, 1)$ using the method of Azencott in Hölderian norm. This extends the LDP proved by Lijun Bo in [7].

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