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A STUDY ON CONCEPTS OF BALLS IN A INTUITIONISTIC FUZZY $D-$METRIC SPACES

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ABSTRACT. Dhage [2] introduced the concept of open balls in a D-Metric space in two different ways and discussed at length the properties of the topologies generated by the family of all open balls of each kind. In this paper, a new concept of balls in a Intuitionistic Fuzzy D-Metric spaces are introduced.

1. INTRODUCTION

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets are introduced by Lotti.A. Zadeh [3] (1965) as an extension of the classical notion of sets. The concept of an intuitionistic fuzzy set can be viewed as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In general, the theory of intuitionistic fuzzy sets is the generalization of fuzzy sets.

The idea of an intuitionistic fuzzy set was first published by Krassimir Atanassov [1]. In general, the theory of intuitionistic fuzzy sets is the generalization of fuzzy sets. Several researches have shown interest in the intuitionistic fuzzy set

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theory and successfully applied in many other field. Fuzzy application in almost every direction of mathematics such as arithmetic, topology, graph theory, probability theory, logic etc.,

In 1992, B.C.Dhage [2] proposed the notion of a $D-$metric space in an attempt to obtain analogous results to those for metric spaces, but in a more general setting. This paper is organized as follows. The definition of intuitionistic fuzzy metric space, $D-$metric space and fuzzy $D-$metric space are introduced in section 2. In section 3, we introduce the new concept of intuitionistic fuzzy $D-$metric spaces and also we discuss about the theorems on balls in a intuitionistic fuzzy $D-$metric spaces.

2. Preliminaries

Definition 2.1. *Let A be a non-empty set. A function $\rho : A \times A \times A \to [0, \infty)$ is called a $D-$metric on X if*

(1) $\rho(a, b, c) = 0$ *if and only if* $a = b = c$ *(coincidence);*
(2) $\rho(a, b, c) = \rho(p(a, b, c))$ *for all* $a, b, c \in A$ *and for any permutation* $p(a, b, c)$ *of* $a, b, c(symmetry)$,
(3) $\rho(a, b, c) \leq \rho(a, b, r) + \rho(a, r, c) + \rho(r, b, c)$ *for all* $a, b, c, r \in A$ *(tetrahedral inequality).*

If A is a non-empty set and ρ is a $D-$metric on A, then the ordered pair (A, ρ) is called a $D-$metric space. When the $D-$metric ρ is understood, X itself is called a $D-$metric space.

Definition 2.2. *The 3-tuple $(A, M, *)$ is said to be a fuzzy $D-$ metric space, where A is an arbitrary set, $*$ is continuous t-norm and M is a fuzzy set on $A \times A \times A \to [0, \infty)$ satisfying the following conditions: For all $a, b, c, r \in A, s, t, u > 0$.*

(1) $M(a, b, c, t) = 0$
(2) $M(a, b, c, t) = 1$ *if and only if* $a = b = c$
(3) $M(a, b, c, t) = M(p(a, b, c, t))$ *for all* $a, b, c \in A$ *and for any permutation* $p(a, b, c)$ *of* a, b, c, t
(4) $M(a, b, c, t + s + u) \geq M(a, b, r, t) * M(a, r, c, s) * M(r, b, c, u)$
(5) $M(a, b, c,) : [0, \infty) \to [0, 1]$ *is continuous.*

Definition 2.3. *A 5-tuple $(A, M, N, *, \circ)$ is said to be an intuitionistic fuzzy metric space if A is an arbitrary set, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm and M, N are fuzzy sets on $A^2 \times [0, \infty)$ satisfying the conditions:*

(1) $M(a, b, t) + N(a, b, t) \leq 1$ *for all* $a, b \in A$ *and* $t > 0$*;*
(2) $M(a, b, 0) = 0$ *for all* $a, b \in A$*;*
(3) $M(a, b, t) = 1$ *for all* $a, b \in A$ *and* $t > 0$ *if and only if* $a = b$*;*
(4) $M(a, b, t) = M(b, a, t)$ *for all* $a, b \in A$ *and* $t > 0$*;*
(5) $M(a, b, t) * M(b, c, s) \leq M(a, c, t + s)$*, for all* $a, b, c \in A$ *and* $s, t0$*;*
(6) $M(a, b, .) : [0, \infty) \to [0, \infty]$ *is left continuous, for all* $a, b \in A$*;*
(7) $\lim_{t\to\infty} M(a, b, t) = 1$ *for all* $a, b \in A$ *and* $t > 0$*;*
(8) $N(a, b, 0) = 1$ *for all* $a, b \in A$*;*
(9) $N(a, b, t) = 0$*, for all* $a, b \in A$ *and* $t > 0$ *if and only if* $a = b$*;*
(10) $N(a, b, t) = N(b, a, t)$ *for all* $a, b \in A$ *and* $t > 0$*;*
(11) $N(a, b, t) \circ N(b, c, s) \geq N(a, c, t + s)$ *for all* $a, b, c \in A$ *and* $s, t > 0$*;*
(12) $N(a, b, .) : [0, \infty) \to [0, 1]$ *is right continuous, for all* $a, b \in A$*;*
(13) $\lim_{t\to\infty} N(a, b, t) = 0$ *for all* $a, b \in A$*.*

The functions $M(a, b, t)$ and $N(a, b, t)$ denote the degree of nearness and the degree of non-nearness between a and b w.r.t. t respectively.

3. Balls in an intuitionistic fuzzy $D-$metric spaces

Definition 3.1. *A 5-tuple $(A, M, N, *, \circ)$ is said to be an intuitionistic fuzzy $D-$metric space if A is an arbitrary set, $*$ is a continuous t-norm, $\circ$ is a continuous $t-$ conorm and M, N are fuzzy sets on $A^3 \times [0, \infty)$ satisfying the conditions:*

(1) $M(a, b, c, t) + N(a, b, c, t) \leq 1$ *for all* $a, b, c \in A$ *and* $t > 0$*;*
(2) $M(a, b, c, 0) = 0$ *for all* $a, b, c \in A$*,*
(3) $M(a, b, c, t) = 1$ *for all* $a, b, c \in A$ *and* $t > 0$ *if and only if* $a = b = c$*;*
(4) $M(a, b, c, t) = M(p(a, b, c, t))$ *for all* $a, b, c \in A$ *and for any permutation* $p(a, b, c)$ *of* $a, b, c, t > 0$*;*
(5) $(M(a, b, c, t + s + u) \geq M(a, b, r, t) * M(a, r, c, t) * M(r, b, c, u)$*, for all* $a, b, c, r \in A$ *and* $s, t, u > 0$*;*
(6) $M(a, b, c, .) : [0, \infty) \to [0, \infty]$ *is left continuous for all* $a, b, c \in A$*;*
(7) $\lim_{t\to\infty} M(a, b, c, t) = 1$ *for all* $a, b, c \in A$ *and* $t > 0$*;*
(8) $N(a, b, c, 0) = 1$ *for all* $a, b, c \in A$*;*

(9) $N(a,b,c,t) = 0$ *for all* $a,b,c \in A$ *and* $t > 0$ *if and only if* $a = b = c$*;*

(10) $N(a,b,c,t) = N(p(a,b,c,t))$ *for all* $a,b,c \in A$ *and for any permutation* $p(a,b,c)$ *of* $a,b,c,t > 0$*;*

(11) $N(a,b,r,t) \circ N(a,r,c,s) \circ N(r,b,c,s) \geq N(a,b,c,t+s+u)$ *for all* $a,b,c,r \in A$ *and* $s,t,u > 0$*;*

(12) $N(a,b,c,.) : [0,\infty) \to [0,1]$ *is right continuous, for all* $a,b,c \in A$*;*

(13) $\lim_{t\to\infty} N(a,b,c,t) = 0$ *for all* $a,b,c \in A$*. The functions* $M(a,b,c,t)$ *and* $N(a,b,c,t)$ *denote the degree of nearness and the degree of non-nearness between* a,b *and* c *w.r.t.* t *respectively.*

Definition 3.2. *Let* $(A,M,N,*,\circ)$ *be an intuitionistic fuzzy* $D-$*metric space. Then*

(1) *A sequence* $\{a_n\}$ *in* A *is said to be Cauchy sequence if for all and* $t > 0$ *and* $p,q > 0, \lim_{n\to\infty} M(a_{n+p+q}, a_{n+p}, a_n, t) = 1$ *and* $\lim_{n\to\infty} N(a_{n+p+q}, a_{n+p}, a_n, t) = 1$.

(2) *A sequence* $\{a_n\}$ *in* X *is said to be convergent to a point* $a \in A$ *if for all* $t > 0$ *and* $p > 0$*,* $\lim_{n\to\infty} M(a_{n+p}, a_n, a, t) = 1$ *and* $\lim_{n\to\infty} N(a_{n+p}, a_n, a, t) = 0$.

Remark 3.1. *Let* $(A,M,N,*,\circ)$ *be an intuitionistic fuzzy* $D-$*metric space,* $a \in A$ *and* $\alpha \in (0,\infty)$.
Let

$$\begin{aligned}
\tilde{B}(a,\alpha,t) &= \{b \in A : M(a,b,b,t) > 1-\alpha, N(a,b,c,t) < \alpha\}, \\
B(a,\alpha,t) &= \{b \in \tilde{B}(a,\alpha,t) : M(a,b,c,t) > 1-\alpha, N(a,b,c,t) < \alpha, \forall c \in \tilde{B}(a,\alpha,t)\} \\
\hat{B}(a,\alpha,t) &= \{\{a\} \cup b \in A : \sup_{c\in A} M(a,b,c,t) > 1-\alpha, \sup_{c\in A} N(a,b,c,t) < \alpha\}.
\end{aligned}$$

Remark 3.2. (i) *It is clear that* $B(a,\alpha,t) \subset \tilde{B}(a,\alpha,t)$,

(ii) *If* $0 < \alpha_1 < \alpha_2$ *then* $\tilde{B}(a,\alpha_1,t) \subset \tilde{B}(a,\alpha_2,t), B(a,\alpha_1,t) \subseteq \tilde{B}(a,\alpha_2,t)$.

By $\overline{B}(a,\alpha,t)$ *we mean a set in* A *is given by* $\overline{B}(a,\alpha,t) = \{b \in \tilde{B}(a,\alpha,t) :$ *if* $b,c \in \tilde{B}(a,\alpha,t)\}$ *. Then*

$$\begin{aligned}
M(a,b,c,t) &> 1-\alpha, N(a,b,c,t) < \alpha\} \\
&= \{b,c \in A : M(a,b,c,t) > 1-\alpha, N(a,b,c,t) < \alpha\}
\end{aligned}$$

It is clear that $B(a,\alpha,t) \subseteq \overline{B}(a,\alpha,t)$.

Theorem 3.1. *Let* $(A, M, N, *, \circ)$ *be an intuitionistic fuzzy* $D-$*metric space. Then for a fixed* $a \in A$*, the balls* $\tilde{B}(a, \alpha, t)$ *and* $B(a, \alpha, t)$ *are the sets in* A *given by,* $\tilde{B}(a, \alpha, t) = (\frac{a - \alpha t}{1 - \alpha}, \frac{a + \alpha t}{1 - \alpha})$*,* $B(a, \alpha, t) = (\frac{a - \alpha t}{2(1 - \alpha)}, \frac{a + \alpha t}{2(1 - \alpha)})$.

Proof. Let $a, b, c \in A$ be arbitrary. Let $\alpha > 0$ be fixed. Then

$$\begin{aligned}
\tilde{B}(a, \alpha, t) &= \{b \in A : (M(a, b, b, t) > 1 - \alpha, N(a, b, b, t) < \alpha)\} \\
&= \{b \in A : (M(a, b, b, t) > 1 - \alpha)\} \\
&= \{b \in A : |a - b| < \frac{\alpha t}{1 - \alpha}\} \\
\tilde{B}(a, \alpha, t) &= (a - \frac{\alpha t}{1 - \alpha}, \frac{a + \alpha t}{1 - \alpha}) .
\end{aligned}$$

Again,

$$\begin{aligned}
B(a, \alpha, t) &= \{b \in \tilde{B}(a, \alpha, t) : M(a, b, c, t) > 1 - \alpha, N(a, b, c, t) < \alpha, \forall c \in \tilde{B}(a, \alpha, t)\} \\
&= \{b \in \tilde{B}(a, \alpha, t) : M(a, b, c, t) > 1\alpha, \forall c \in \tilde{B}(a, \alpha, t)\} \qquad (3.1) \\
B(a, \alpha, t) &= \{b \in \tilde{B}(a, \alpha, t) : Max\{|a - b|, |b - a|, |c - a| < \frac{\alpha t}{1 - \alpha}\}\} .
\end{aligned}$$

This relation (3.1) implies that the set $B(a, \alpha, t)$ contain all the points $b, c \in A$ for which one has

$$|a - b| < \frac{\alpha t}{1 - \alpha}, |a - c| < \frac{\alpha t}{(1 - \alpha)} \ \ with \ \ |b - c| < \frac{\alpha t}{1 - \alpha} . \qquad (3.2)$$

In order to hold inequalities in (3.2) we must have,

$$|a - b| + |a - c| < \frac{\alpha t}{1 - \alpha} ,$$

since

$$|b - c| \leq |b - a| + |a - c| .$$

Therefore if we take

$$|a - b| < \frac{\alpha t}{2(1 - \alpha)} ,$$

and

$$|a - c| < \frac{\alpha t}{2(1 - \alpha)} ,$$

then the inequalities in (3.2) are satisfied . Thus we have

$$\begin{aligned} B(a,\alpha,t) &= \{b \in \tilde{B}(a,\alpha,t) : |a-b| < \frac{\alpha t}{2(1-\alpha)}\} \\ B(a,\alpha,t) &= (a - \frac{\alpha t}{2(1-\alpha)}, a + \frac{\alpha t}{2(1-\alpha)})\,. \end{aligned}$$

□

Theorem 3.2. *Every ball $B(a,\alpha,t), a \in A, \alpha > 0$, is an open set in A. (ie.) it contains a ball of each of its points*

Proof. Let $a \in A$ be arbitrary and $\alpha > 0$. Consider the ball $B(a,\alpha,t)$ in A and supposed that $x \in B(a,\alpha,t)$. We will show that there is an $\tilde{\alpha} > 0, \tilde{\alpha} > \alpha$ such that $B(x,\tilde{\alpha},t) \subseteq B(a,\alpha,t)$. Since $x \in B(a,\alpha,t)$ there is number $\alpha_1 > 0$ such that $M(a,x,x,t) > 1-\alpha$ and $N(a,x,x,t) < \alpha_1$ and $\alpha_1 < \alpha$. We may choose an arbitrary $\epsilon > 0$ such that $\tilde{B}(a,\alpha_1 + ,t) \subseteq B(a,\alpha,t)$ which is possible in view of $\alpha_1 < \alpha$. Since $\tilde{B}(a,\alpha_1 + ,t)$ is open ,there is an open ball $\tilde{B}(x,\tilde{\alpha},t), \tilde{\alpha} < 0$ such that $\tilde{B}(x,\tilde{\alpha},t) \subseteq \tilde{B}(a,\alpha_1+\epsilon,t) \subseteq B(a,\alpha,t)$. Again $B(x,\tilde{\alpha},t) \subseteq \tilde{B}(x,\tilde{\alpha},t)$. Hence $B(x,\tilde{\alpha},t) \subseteq B(x,\tilde{\alpha},t)$.

Thus proves that $B(a,\alpha,t)$ is an open in A. □

4. CONCLUSION

Last three decades were very productive for the fuzzy mathematics and the recent literature has observed the fuzzy application in almost every direction of mathematics. In this paper a general analysis has been done open balls in intuitionistic fuzzy $D-$metric space. In future from this concept can be to extend in various spaces.

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