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NX2 FLOW SHOP SCHEDULING PROBLEM WITH PARALLEL MACHINES AT EVERY STAGE, PROCESSING TIME ASSOCIATED WITH PROBABILITIES

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ABSTRACT. Scheduling models concern with the determination to find most favourable succession of jobs so as to reduce whole elapsed time or some other appropriate measure of performance. In this study we find out the solution procedure for the problem of reducing the cost of production and makespan in Nx2 flow shop scheduling dilemma with equipotential parallel machines at every phase in which processing time are related with their individual probabilities. Scheduling jobs on equipotential machines is an important activity for Industrial scheduling. The algorithm is evaluated by solving a mathematical illustration.

1. INTRODUCTION

Production scheduling can be characterized as the allotment of existing manufacture resources over time to do a set of tasks. There are different kind of scheduling problems in industries and production concerns like flow shop scheduling. In our study we have considered the case of flow shop scheduling problems in which the order of machines in which jobs are to be processed is fixed that is no passing over the machines is allowed. Over the last decade a considerable amount of research has been done to develop scheduling algorithm for flow shop

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scheduling problems with parallel equipotential machines at first stage. Numerous realistic and investigational situations, which usually occur in production industries to get the most favourable schedule of tasks in group of machines in order to optimize some well defined criterion, diverted the attention of researchers and engineers. The aim of the study is to find the order of the jobs so that after processing on machines in a particular order, it optimizes certain measures. In various problems related to production industries, manufacturing, it is not always possible to work on a single machine of one type, but there are more than one machine of each type to meet the demand of the customer in time and with minimum cost. In our problem we have taken the parallel equipotential machines at each stage in two stage flow shop scheduling problem. Nowadays due to changing customer's demand and rising competition in global market it is very necessary for the production industries to satisfy the customer's demand and minimize the production cost. Hence bicriteria flow shop scheduling problems are very important in production scheduling. In practical situations processing time are not always deterministic due to weather conditions, quality of raw material, quality of finished product required, designing etc, so we have associated probabilities with processing times of all the jobs.

Practical situation:

In universal supply chains, consumer fulfilment is most significant to carry on the trade, for example in readymade garment industry it is not always possible to work on a single machine of cutting and sewing type. There is more than one machine of each type with different operating cost. The need of the industry is to allocate different machines to different jobs in order to find the most favourable schedule to diminish the total time to execute all the jobs. The objective is to decrease the price of commodity and fulfil the demand of the customers of superior quality of product at least possible cost with in time.

Flow shop scheduling problems are classical scheduling problem in manufacturing industries. Johnson et. al. (1954) [1] studied problems related to production scheduling involving two or three machines of different types to find the optimal schedule. Work was extended by various researchers by considering different parameters like, transportation time, setup time, job block criterion, weight age of jobs etc. nx2 and nx3 stage flow shop scheduling problems were solved by Lomnieki(1965) [2] by applying Branch and Bound Procedure to find the most favourable schedule in problems related to production involving two or three machines of different types. Ignall et. al. (1965) [3] and Brown et. al. (1966) [4] gave algorithm to solve scheduling problems involving different measures using Branch and Bound technique. McMahon (1967) [5] studied some flow shop scheduling problems. Scheduling problems under different hiring circumstances were explored by a number of investigators like Narain (2002) [8], Narain (2005) [9], Singh, et.al (2005) [10], Sharma et.al (2011) [12] etc. Gupta (2011) [13], Gupta et. al. (2012) [14], Gupta et.al. (2013) [15] employ the technique provided by Lominiki in 1965 [2] to find out the solution of problems related to production involving a range of parameters. Prakash(1995) [6], Shen et. al. (2013) [7] worked on some classical parallel machines problems.

Traditional equivalent machine scheduling comprises of scheduling of n tasks over m similar parallel machines. Each and every job is measured with a fixed processing time to optimize different measures. A lot of work has been done on flow shop scheduling problems with parallel machines having fixed processing time or under fuzzy environment for optimizing different measures such as total completion time, numeral of delayed jobs, weighted flow time, elapsed time etc. Many investigators like Nailwal et. al. (2015) [16] use the concept of equivalent parallel machines to optimize different measures such as elapsed time, maximum lateness, weighted flow time. Sharma et. al. (2016) [17] extended the work on scheduling problems with equipotential machines by considering multiple objectives under fuzzy environment. In real world production on large scale is not possible with a single machine of one type optimizing different criterion, so in most of production cases, industries use multiple parallel machines of each type with same efficiency or different efficiency depending upon time, cost etc. Singh et. al. (2008) [11] investigated three stage scheduling problems by taking into account multiple equipotential machines for the first machine and sole machine for second and third type machine, Carrying time is also taken into consideration. Gupta et. al. (2018) [18] extended the work done by Singh et. al. (2008) [11] by associating the probabilities with the processing time of jobs. Gupta et. al. (2019) [18] studied two stage flow shop scheduling problems with equipotential machines at every stage. In this research, we widen the work done by Gupta et. al. (2019) [18] by associating the probabilities with the processing time of the jobs.

Notations

- x_i, y_i = Time of processing on machine X and Y for i^{th} job respectively.
- X_j (j = 1, 2,. . . ,r),Y_j(j = 1, 2,. . . , s)= r,s equivalent machines of type X and Y respectively
- p_i,q_i = probability related with time of processing of i^{th} job on machine of type X and Y respectively
- t_{1j},t_{2j} = time for which equipotential machine of type X and Y are available for processing
- β_{ij},γ_{ij} = unit functional charge of i^{th} job on X_j and Y_j equivalent machine respectively

2. Problem formulation

Suppose there are n tasks which are to be executed on two machines of sort X and Y. Assume that X_1, X_2 , X_3, . . . , X_r are r equipotential machines of sort X and Y_1 , Y_2 , Y_3 ,. . . ,Y_s are s equipotential machines of sort Y respectively. Let x_i, y_i for i = 1, 2, . . . , n be the time to process the i^{th} job on first and second machine respectively. Suppose p_i and q_i be the probabilities related to the time of processing of i^{th} job on first and second machine respectively. Consider machine X and machine Y and n tasks are processed on these machines in a fixed order that is X, Y. We consider that each job on like/equipotential machines of sort X is executed in different parts. After finishing on machine X the job as a whole goes on machine Y and executed on machine Y in different parts. Equipotential machines X_1, X_2 , X_3 ,. . . , X_r of sort X and Y_1, Y_2 , Y_3 ,. . . , Y_s of sort Y are available for fixed period of time given by t_{11}, t_{12},. . . , t_{1r} and t_{21}, t_{22},. . . ,t_{2s} respectively as explained in table1.

We consider the condition $\sum_{j=1}^{r} t_{1j}=\sum_{i=1}^{n} x_i^{'}$, $\sum_{j=1}^{s} t_{2j}=\sum_{i=1}^{n} y_i^{'}$ is satisfied, where $x_i^{'}$ and $y_i^{'}$ are the expected time to process all the jobs on machines X and Y respectively. Let β_{ij}(i = 1, 2,. . . , n, j= 1, 2,. . . , r) is the unit functional cost of $i^t h$ job on machine X_1, X_2, X_3,. . . , X^r and suppose γ_{ij}(i = 1,2,. . . ,n, j= 1, 2,. . . , s) is unit functional cost of $i^t h$ job on machine $Y_1, Y_2, Y_3, \ldots, Y_s$. The objective of the study is to allocate the processing time of jobs of machine X and machine Y to like/ equipotential machines $X_1, X_2, X_3, \ldots, X_r$ and $Y_1, Y_2, Y_3, \ldots, Y_s$ in order to minimize the production cost and to discover an optimal schedule so that time of doing all the jobs is minimized.

Assumptions

1. No machine can perform more than one task at a time,the jobs are obtainable initially at time zero.
2. The jobs do not depend on each other,every machine of type X and type Y has distinctive working cost.

TABLE 1. Mathematical model

Job	Machine X	Processing time of X		Machine Y	Processing time of X	
i	$X_1\ X_2 \ldots X_r$	x_i	p_i	$Y_1\ Y_2 \ldots Y_s$	y_i	q_i
(1)	$\beta_{11}\ \beta_{12} \ldots \beta_{1r}$	x_1	p_1	$\gamma_{11}\ \gamma_{12} \cdots \gamma_{1s}$	y_1	q_1
(2)	$\beta_{21}\ \beta_{22} \ldots \beta_{2r}$	x_2	p_2	$\gamma_{21}\ \gamma_{22} \cdots \gamma_{2s}$	y_2	q_2
(3)	$\beta_{31}\ \beta_{32} \ldots \beta_{3r}$	x_3	p_3	$\gamma_{31}\ \gamma_{32} \cdots \gamma_{3s}$	y_3	q_3
(n)	$\beta_{n1}\ \beta_{n2} \ldots \beta_{nr}$	x_n	p_n	$\gamma_{n1}\ \gamma_{n2} \cdots \gamma_{ns}$	y_n	q_n
	$t_{11}\ t_{12} \ldots t_{1r}$			$t_{21}\ t_{22} \ldots t_{2s}$		

Algorithm / Methodology:

Step 1: Locate out the expected processing time of i^{th} job on machine X and Y by using the formula, $x_i{'} = x_i p_i$, $y_i{'} = y_i q_i$.

Step 2: Check the condition $\sum_{j=1}^{r} t_{1j} = \sum_{i=1}^{n} x_i{'}$, $\sum_{j=1}^{s} t_{2j} = \sum_{i=1}^{n} y_i{'}$.

If the condition is satisfied, use Modified distribution method(U-V method) to calculate the best possible allotment of processing time of every job on machine X and machine Y to the equipotential machines X_j and Y_j correspondingly specified by X_{ij} and Y_{ij}.

Step 3: Compute $g_1 = \max\left\{\sum_{i=1}^{n} X_{ij}\right\}_{j=1,2,\ldots,r} + min_{i\in j_t{'}}\left\{\max_{j=1,2,\ldots,s} Y_{ij}\right\}$
$g_2 = \left\{\max_{i\in j_t X_{ij}}\right\}_{j=1,2,\ldots,r} + \max\left\{\sum_{i=1}^{n} Y_{ij}\right\}_{j=1,2,\ldots,s}$.

Step 4: Calculate g = $\max\ (g_1, g_2)$ and find out g for the n-component of permutations, ranging from 1, 2, 3,..., n respectively, having classified the appropriate vertices of the scheduling hierarchy by these values.

Step 5: Examine the peak for the smallest value. Now for (n-1) sub sequences compute g, beginning from the above selected apex. Again find out the minimum labelled apex and calculate the value of g for (n-2) sub modules. Continuing in such a manner until we arrive at the closing stages of the hierarchy symbolized by two solitary permutations for which we assess the intact task length. We find out the most advantageous sequence of executing the jobs on two machines X and Y by moving according to above steps.

Step 6: Formulate in/out table for the most advantageous series which we get in the step 6 to find out the minimum time of executing all the jobs.

MATHEMATICAL PROBLEM:

To diminish the entire elapsed time and operating cost of executing the jobs on two machines, find out the most advantageous sequence of jobs, given four jobs that are to be performed on two machines X and Y given by Table 2.

TABLE 2. Numerical Problem

Jobs/Machines	**X**				**Y**			
i	X_1	X_2	x_i	p_i	Y_1	Y_2	y_i	q_i
1	6	2	40	0.2	11	8	30	0.3
2	5	7	20	0.3	9	7	40	0.2
3	10	3	10	0.4	6	5	10	0.4
4	8	9	50	0.1	10	12	60	0.1
Available Time	13	10			13	14		

We have taken two machines X_1 and X_2 of sort X and Y_1 and Y_2 of sort Y which are available for fixed given time. All the equipotential machines of type X and type Y have different operating cost which is given as in Table 3.

SOLUTION:

Step 1 Calculate expected processing time of all the jobs (i = 1, 2, 3, 4) to machine X and Y as per step 1 shown in Table 3.

Step 2: By the use of modified distribution method allocate the best possible time of processing for (i = 1, 2, 3, 4) of job i to equivalent machineX_j (j = 1, 2), Yj (j = 1, 2) as revealed in Table 4.

Step-3: Find out the lower bound for one scheduled job as

LB (1) = Maximum (13+4, 6+14) = Maximum (17, 20) = 20

LB (2) = 20,LB (3)=18, LB (4) = 19.

Corresponding to the job 3 we get the least value of lower bound (18). Hence in the optimal sequence we fix the job 3 at first place and start working to find the second job in the optimal sequence.

TABLE 3. Cost matrix of machines of type X and Y with expected processing time of jobs

Jobs/Machines	**X**			**Y**		
i	X_1	X_2	x_i	Y_1	Y_2	y_i
1	6	2	8	11	8	9
2	5	7	6	9	7	8
3	10	3	4	6	5	4
4	8	9	5	10	12	6
Available Time	13	10		13	14	

TABLE 4. Condensed problem

Jobs/Machines	X_1	X_2	Y_1	Y_2
1	2	6	0	9
2	6	0	3	5
3	0	4	4	0
4	5	0	6	0

Step-4: (LB) (3, 1)= 24,(LB) (3, 2)= 20, LB (3, 4)= 19
Least value of LB is 19 which are corresponding to the subsequence <3, 4>

Step-5: Lower Bound (3, 4, 1)= 24, Lower Bound (3, 4, 2) = 25. Hence we get the best possible sequence as 3, 4, 1, and 2.

Step-6: Formulate in-out table and find out the minimum elapsed time as revealed in Table 5.

TABLE 5. In-Out Table

Jobs/Machines	X_1	X_2	Y_1	Y_2
3	-	0-4	4-8	-
4	0-5	-	8-14	-
1	5-7	4-10	-	10-19
2	7-13	-	14-17	19-24

Total elapsed time = 24 hours.

3. Results analysis

This paper provides a mathematical model to reduce the cost of production and total elapsed time in case of two stage flow shop scheduling problem with equipotential parallel machines at each stage.

4. Conclusion and future scope

We provide a solution algorithm based on Branch and Bound technique which is an exact method to find the best possible schedule. The mathematical model proposed here assist the business manager or a decision-maker to select the best possible production schedule with the purpose of minimizing the cost of production and save energy consumption. The algorithm presented in this paper gave significant saving in terms of time, cost and meet the demand of the customer in time. Study is similar to that of Gupta et. al. (2018) [18], if the processing time of all the jobs is fixed and we take single machine at second stage. We can further supplement the work by taking three stage flow shop scheduling problem with equipotential machines at each stage.

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