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SQUARE DIVISOR CORDIAL LABELING OF VARIOUS GRAPHS

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ABSTRACT. We discuss here square divisor cordial labeling of vertex switching of cycle C_n, vertex switching of cycle C_n with one chord, vertex switching of cycle C_n with twin chords and vertex switching of cycle C_n with triangle. And we will also prove that Petersen graph is square divisor cordial.

1. INTRODUCTION

We begin with simple, finite undirected graph $G = (V, E)$. In the present work C_n denote the cycle with n vertices. For all other terminology and notations we follow Harary [1]. Other valuable references are [2-7].

Bosmia and Kanani [2] proved that vertex switching of bistar graph, armed crown, helm and gear graph are square divisor cordial.

Definition 1.1. *Let $G(V(G), E(G))$ be simple graphs and let function $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $f(u)^2|f(v)$ or $f(v)^2|f(u)$ and the label 0 otherwise. f is called a square divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a square divisor cordial labeling is called a square divisor cordial graph.*

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2. MAIN RESULTS

Theorem 2.1. *Switching of vertex of Cycle C_n is square divisor cordial, for all n.*

Proof. Suppose $G = C_n$ and $u_1, u_2, \ldots, u_n$ be successive vertices of C_n. Suppose $(C_n)_{u_1}$ represent the switched vertex of C_n with respect to u_1 of C_n. Consider u_1 as the switched vertex and we start labeling pattern from u_1. Then $|E(G)| =$ 2n-5 and $|V(G)| = n$. We define labeling function $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$, as $f(u_i) = i$, $1 \leq i \leq n$, since, $|e_f(0) - e_f(1)| \leq 1$. Hence, switching of vertex of cycle C_n is square divisor cordial, for all n. □

Example 1. *Switching of vertex of cycle C_7 admitting Square divisor cordial labeling is shown in* FIGURE 1.

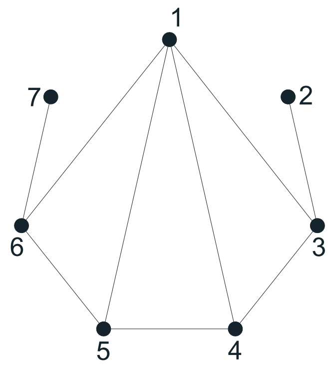

FIGURE 1

Theorem 2.2. *Switching of vertex of cycle $C_n (n \geq 4, n \in \boldsymbol{N})$ having one chord admits square divisor cordial labeling, where chord make a triangle with two edges of cycle C_n.*

Proof. Suppose G be the cycle having one chord. Suppose $u_1, u_2, \ldots, u_n$ be the successive vertices of cycle C_n and $e = u_n u_2$ be the chord of cycle C_n. The edges $e = u_n u_2$, $e_1 = u_2 u_1$, $e_2 = u_n u_1$ form a triangle.

Now the graph get by switching of vertices u_i and u_j of degree 2 are isomorphic to each other for all i and j. Similarly the graph get by switching of vertices u_i and u_j of degree 3 are isomorphic to each other for all i and j.Hence we need to talk about two cases:

(i) switching of an arbitrary vertex of G of degree 3,

(ii) switching of an arbitrary vertex of G of degree 2.
Without detriment of generality suppose the switched vertex be u_1 (of either degree 3 or degree 2) and suppose G_{u_1} denote the switching of vertex of G with respect to vertex u_1.

To define labeling function $f : V(G) \to \{1, 2, \dots, |V(G)|\}$, we consider following cases.

Case 1: Degree of u_1 is 2. Here the number of vertices is n and number of edges is $2n - 4$.

Subcase I: $n \equiv 0(mod4)$

$f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 3$, $f(u_n) = 4$, $f(u_{n-1}) = n - 1$, $f(u_{n-2}) = n$,
$f(u_i) = i + 1$, $4 \leq i \leq n - 3$.

Subcase II: $n \not\equiv 0(mod4)$

$f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 3$, $f(u_n) = 4$,
$f(u_i) = i + 1$, $4 \leq i \leq n - 1$.

Since, $e_f(0) = e_f(1) = n - 2$.

Case 2: Degree of u_1 is 3. Here the number of vertices is n and number of edges is $2n - 6$. $f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 4$, $f(u_4) = 3$,
$f(u_i) = i$, $5 \leq i \leq n$.

Since, $e_f(0) = e_f(1) = n - 3$.

Hence, switching of vertex of cycle C_n having one chord admits square divisor cordial labeling. □

Example 2.

- *Square divisor cordial labeling of the graph obtained by switching of a vertex of degree 2 in cycle C_9 with one chord is shown in Fig. 2(a).*
- *Square divisor cordial labeling of the graph obtained by switching of a vertex of degree 3 in cycle C_9 with one chord is shown in Fig. 2(b).*

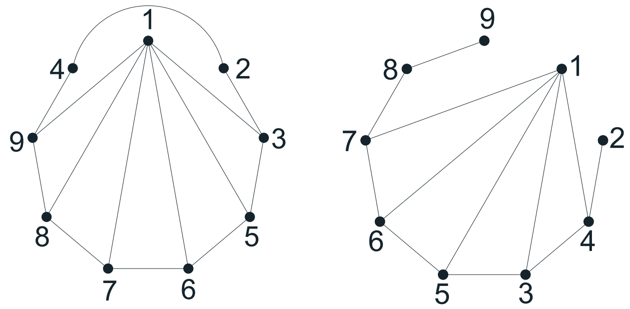

Fig. 2(a) Fig. 2(b)

Theorem 2.3. *Switching of vertex of cycle C_n having twin chords $C_{n,3}$ admits square divisor cordial labeling, where chords form two triangles and one cycle C_{n-2}.*

Proof. Suppose G be the cycle having twin chords. Suppose $u_1, u_2, \dots, u_n$ be the successive vertices of cycle C_n and $e_1 = u_n u_2$ and $e_2 = u_n u_3$ be the chords of cycle C_n which form two triangles and one cycle C_{n-2}.

Now the graph get by switching of vertices u_i and u_j of degree 2 are isomorphic to each other for all i and j. Similarly the graph get by switching of vertices u_i and u_j of degree 3 are isomorphic to each other and the graph get by switching of vertices u_i and u_j of degree 4 are isomorphic to each other for all i and j. Hence we need to talk about three cases:

(i) switching of an arbitrary vertex of G of degree 2,

(ii) switching of an arbitrary vertex of G of degree 3,

(iii) switching of an arbitrary vertex of G of degree 4.

With out detriment of generality suppose the switched vertex be u_1 and suppose G_{u_1} denote the switching of vertex of G with respect to vertex u_1.

To define labeling function $f : V(G) \to \{1, 2, \dots, |V(G)|\}$, we consider following cases.

Case 1: Degree of u_1 is 2. Here the number of vertices is n and number of edges is 2n - 3.

$f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 3$, $f(u_n) = 4$,
$f(u_i) = i + 1$, $4 \leq i \leq n - 1$.

Case 2: Degree of u_1 is 3 and Degree of u_1 is 4.
Here the number of vertices is n and number of edges is 2n - 5.
Here the number of vertices is n and number of edges is 2n - 7.

$f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 4$, $f(u_4) = 3$,
$f(u_i) = i$, $4 \leq i \leq n$.

Thus, in each case $|e_f(0) - e_f(1)| \leq 1$. Hence, Switching of vertex of cycle C_n having twin chords $C_{n,3}$ admits square divisor cordial labeling. □

Example 3.

- *Square divisor cordial labeling of the graph obtained by switching of a vertex of degree* 2 *in cycle* C_6 *with twin chords is shown in Fig. 3(a).*
- *Square divisor cordial labeling of the graph obtained by switching of a vertex of degree* 3 *in cycle* C_6 *with twin chords is shown in Fig. 3(b).*
- *Square divisor cordial labeling of the graph obtained by switching of a vertex of degree* 4 *in cycle* C_6 *with twin chords is shown in Fig. 3(c).*

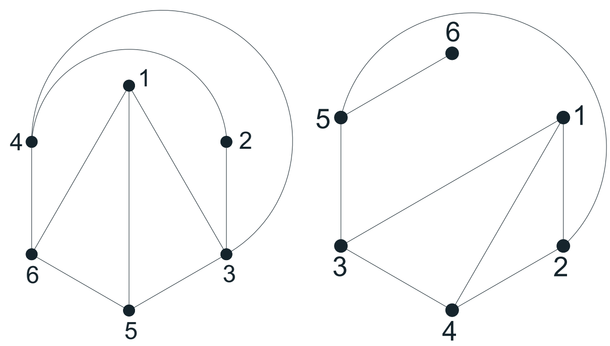

Fig. 3(a) Fig. 3(b) Fig. 3(c)

Theorem 2.4. *Switching of vertex of cycle having triangle* $C_n(1, 1, n - 5)$ *admits square divisor cordial labeling.*

Proof. Suppose G be the cycle having triangle $C_n(1, 1, n-5)$. Suppose $u_1, u_2, \ldots, u_n$ be the successive vertices of G.Suppose $e_1 = v_1v_{n-1}$, $e_2 = v_1v_3$, $e_3 = v_{n-1}v_3$ be the chords of G.

Now the graph get by switching of vertices u_i and u_j of degree 2 are isomorphic to each other for all i and j. Similarly the graph get by switching of vertices u_i and u_j of degree 4 are isomorphic to each other. Hence we need to talk about two cases: (i) switching of an arbitrary vertex of G of degree 2, (ii) switching of an arbitrary vertex of G of degree 4. With out detriment of generality suppose the switched vertex be u_1 and suppose G_{u_1} denote the switching of vertex of G with respect to vertex u_1.

To define labeling function $f : V(G) \to \{1, 2, \dots, |V(G)|\}$, we consider following cases.

Case 1: Degree of u_1 is 2. Here the number of vertices is n and number of edges is 2n - 2.

Subcase I: When n is even.
$f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 4$,
$f(u_4) = 3$, $f(u_5) = 6$, $f(u_6) = 5$,
$f(u_n) = n - 1$, $f(u_{n-1}) = n$, $f(u_{n-2}) = n - 3$, $f(u_{n-3}) = n - 2$,
$f(u_i) = i$, $7 \leq i \leq n - 4$.

Subcase II: When n is odd. $f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 4$,
$f(u_4) = 3$, $f(u_n) = n - 1$, $f(u_{n-1}) = n$,
$f(u_i) = i$, $5 \leq i \leq n - 2$.

Case 2: Degree of u_1 is 4. Here the number of vertices is n and number of edges is 2n - 6.

Subcase I: When n is even.
$f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 4$,
$f(u_4) = 3$, $f(u_5) = 6$, $f(u_6) = 5$,
$f(u_i) = i$, $7 \leq i \leq n$.

Subcase II: When n is odd.
$f(u_1) = 1$, $f(u_2) = 2$, $f(u_3) = 4$,
$f(u_4) = 3$, $f(u_5) = 6$, $f(u_6) = 5$,
$f(u_n) = n - 1$, $f(u_{n-1}) = n$,
$f(u_i) = i$, $7 \leq i \leq n - 2$.

Thus, in each case $|e_f(0) - e_f(1)| \leq 1$. Hence, switching of vertex of cycle having triangle $C_n(1, 1, n - 5)$ admits square divisor cordial labeling. □

Example 4.

- *Square divisor cordial labeling of the graph obtained by switching of a vertex of degree* 2 *in cycle* C_6 *with triangle is shown in Fig. 4(a).*
- *Square divisor cordial labeling of the graph obtained by switching of a vertex of degree* 4 *in cycle* C_6 *with triangle is shown in Fig. 4(b).*

Theorem 2.5. *Petersen graph admits square divisor cordial labeling.*

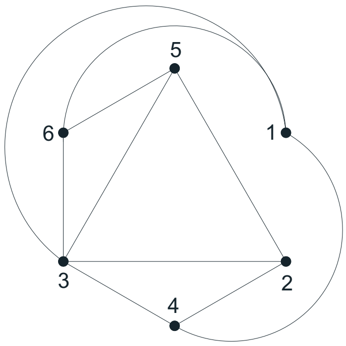

Fig. 4(a)

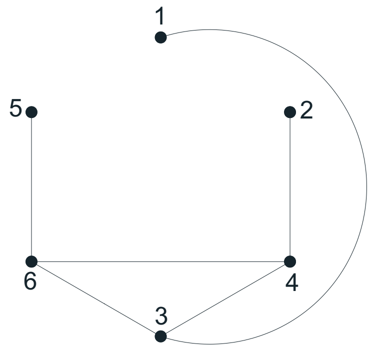

Fig. 4(b)

Proof. Suppose u_1, u_2, u_3, u_4, u_5 as the internal vertices and v_1, v_2, v_3, v_4, v_5 as the external vertices of Petersen graph such that each u_i is adjacent to v_i, $1 \leq i \leq 5$. We define labeling function $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ as follows:

$f(u_i) = 2n - 1$, $1 \leq i \leq 5$.

$f(v_i) = 2n$, $1 \leq i \leq 5$.

Thus, $|e_f(0) - e_f(1)| \leq 1$. Hence, Petersen graph admits square divisor cordial labeling. □

Example 5. *Petersen graph admitting Square divisor cordial labeling is shown in* FIGURE 5.

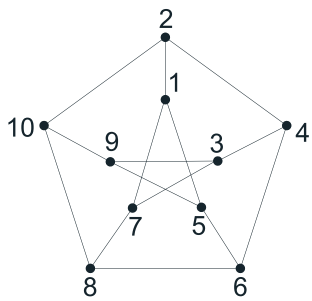

Fig. 5 Petersen Graph

3. CONCLUSION

It is very interesting to investigate graph or graph families which are square divisor cordial as all the graphs do not admit square divisor cordial labeling. Here, it has been proved that vertex switching of cycle, cycle with one chord, cycle with twin chords and cycle with triangle are square divisor cordial graphs. Further, we proved that Petersen graph is square divisor cordial graph.

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