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CARTESIAN PRODUCT ON FUZZY IDEALS OF A TERNARY Γ-SEMIGROUP

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ABSTRACT. In this paper, we introduce and study the concept of cartesian product of fuzzy set of a ternary Γ-semigroup and we characterize fuzzy ternary Γ-semigroup, fuzzy left(resp. right, latral) ideal and fuzzy bi-ideal of a ternary Γ-semigroup in terms of cartesian product of fuzzy ternary Γ-semigroup, fuzzy left(resp. right, latral) ideal and fuzzy bi-ideal respectively.

1. INTRODUCTION

Los, J. [10] showed that any ternary semigroup however may be embedded in an ordinary semigroup in such a way that the operation in ternary semi groups is an (ternary) extension of the (binary) operation of the containing semigroup. Kim, J. [9], Lyapin, E.S. [11] and Sioson, F.M. [15] have also studied the properties of ternary semi groups. Sen, M.K. [14] defined the concepts of Γ-semigroup. It is known that Γ-semigroup is a generalization of semigroup. Many classical notions of semigroups have been extended to Γ-semigroups.

Zadeh, L.A. [17] introduced the study of fuzzy set in 1965. Mathematically a fuzzy set on a set U is a mapping μ into [0,1] of real numbers; for p in U, $\mu(p)$ is called the membership of p belonging to U. Further, Vague sets are introduced which are extension of fuzzy sets. Later, Bhargavi, Y. and Eswarlal, T. [1–7] introduced and studied vague sets on Γ-semirings. After that, Ragamayi, S.

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[12, 13] studied vague sets on Γ-nearrings.Ersoy, B.A., Tepecik, A. and Demir, I. [8] studied cartesian product of fuzzy prime ideals of rings. Sujit Kumar Surdar and Sarbani Goswamy [16] studied cartesian product of fuzzy prime and fuzzy semiprime ideals of semigroups. In this paper, we introduce and study the concept of cartesian product of fuzzy set of a ternary Γ-semigroup and we characterize fuzzy ternary Γ-semigroup, fuzzy left(resp. right, latral) ideal and fuzzy bi-ideal of a ternary Γ-semigroup in terms of cartesian product of fuzzy ternary Γ-semigroup, fuzzy left(resp. right, latral) ideal and fuzzy bi-ideal respectively.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

Notations: Throughout this article, we use the following notations.

1) **TΓSG** stands for Ternary Γ-Semigroup
2) **RI** stands for right ideal.
3) **LI** stands for left ideal.
4) **LAI** stands for latral ideal.
5) **BI** stand for fuzzy bi-ideal.
6) **FS** stands for fuzzy set.
7) **FRI** stands for fuzzy right ideal.
8) **FLI** stands for fuzzy left ideal.
9) **FLAI** stand for fuzzy latral ideal.
10) **FI** stand for fuzzy ideal.
11) **FBI** stands for fuzzy bi-ideal.

Definition 2.1. *A **TΓSG** is an algebraic structure $(E, \Gamma, .)$ such that $E \neq \emptyset$ and $. : E \times \Gamma \times E \times \Gamma \times E \rightarrow E$ is a ternary operation satisfying the following associative law*

$$(a\alpha b\beta c)\gamma d\delta e = a\alpha(b\beta c\gamma d)\delta e = a\alpha b\beta(c\gamma d\delta e),$$

$\forall\ e, f, c, d, e \in E;\ \alpha, \beta, \gamma, \delta \in \Gamma.$

Definition 2.2. *A non-empty subset F of a **TΓSG** E is called*

(i) ***TΓSG** of E if $F\Gamma F\Gamma F \subseteq F$.*

(ii) ***LI(resp. RI, LAI)*** *of* E *if* $E\Gamma E\Gamma F \subseteq F,\ (F\Gamma E\Gamma E \subseteq F, E\Gamma F\Gamma E \subseteq E)$.

(iii) ***BI*** *of* E *if* $F\Gamma E\Gamma F\Gamma E\Gamma F \subseteq F$.

Definition 2.3. *A mapping from a non empty set to an interval* $[0, 1]$ *is called* ***FS***.

Definition 2.4. *Let* $\mu : U \to [0, 1]$ *be any* ***FS***. *Then the set* $\{\mu(p)/p \in U\}$ *is called the image of* μ *and is denoted by* $Im(\mu)$. *For* $t \in [0, 1]$, $\mu_t = \{p \in U/\mu(p) \geq t\}$ *is called a level subset of* μ.

Definition 2.5. *A* ***FS*** μ *of a* ***T***Γ***SG*** E *is said to be* ***FT***Γ***SG*** *if for all* $e, f, g \in E; \alpha, \beta \in \Gamma$, $\mu(e\alpha f\beta g) \geq \min\{\mu(e), \mu(f), \mu(g)\}$.

Definition 2.6. *A* ***FS*** μ *of a* ***T***Γ***SG*** E *is said to be* ***FLI (resp. FRI, FLAI)*** *ideal of* E *if for all* $e, f, g \in E; \alpha, \beta \in \Gamma$, $\mu(e\alpha f\beta g) \geq \mu(g)\ (resp.\mu(e\alpha f\beta g) \geq \mu(e),\ \mu(e\alpha f\beta g) \geq \mu(f))$. *If* μ *is* ***FLI, FRI*** *and* ***FLAI*** *of* E, *then* μ *is called* ***FI*** *of* E.

Definition 2.7. *A* ***FS*** μ *of a* ***T***Γ***SG*** E *is said to be* ***FBI*** *of* E *if forall* $e, f, g, x, y \in E; \alpha, \beta, \gamma, \delta \in \Gamma$, $\mu(e\alpha x\beta f\gamma y\delta g) \geq \min\{\mu(e), \mu(f), \mu(g)\}$.

3. Cartesian Product on Fuzzy Ideals and Fuzzy Bi-ideals of a Ternary Γ-Semigroup

In this section, we introduce and study the concept of cartesian product of **FS** of a **T**Γ**SG**. Throughout this section, E stands for a **T**Γ**SG** unless otherwise mentioned.

Let E_1 and E_2 be two **T**Γ**SG**. Then the cartesian product $E_1 \times E_2$ becomes a **T**Γ**SG** with the ternary composition $(E_1 \times E_2) \times \Gamma \times (E_1 \times E_2)\Gamma \times (E_1 \times E_2) \to E_1 \times E_2$ defined by

$$(e_1, f_1)\alpha(e_2, f_2)\beta(e_3, f_3) = (e_1\alpha e_2\beta e_3, f_1\alpha f_2\beta f_3),$$

$\forall\ (e_1, f_1), (e_2, f_2), (e_3, f_3) \in E_1 \times E_2;\ \alpha, \beta \in \Gamma$.

Definition 3.1. *Let* μ *and* ν *be* ***FS*** *of* E. *Then the cartesian product of* μ *and* ν *is defined by* $(\mu \times \nu)((e, f)) = \min\{\mu(e), \nu(f)\},\ \forall\ (e, f) \in E \times E$.

Lemma 3.1. *If* μ *and* ν *are* ***FS*** *of* E, *then* $(\mu \times \nu)_t = \mu_t \times \nu_t$, *where* $t \in [0, 1]$.

Proof. Suppose $(e, f) \in \mu_t \times \nu_t \Leftrightarrow e \in \mu_t$ and $f \in \nu_t \Leftrightarrow \mu(e) \geq t$ and $\nu(f) \geq t$ $\Leftrightarrow \min\{\mu(e), \nu(f)\} \geq t \Leftrightarrow (e, f) \in (\mu \times \nu)_t$. Thus $(\mu \times \nu)_t = \mu_t \times \nu_t$. □

Theorem 3.1. *If μ and ν are **FT**Γ**SG** of E, then $\mu \times \nu$ is a **FT**Γ**SG** of $E \times E$.*

Proof. Suppose μ and ν are **FT**Γ**SG** of E. Let $(e,f),(g,h),(i,j) \in E \times E;\ \alpha,\beta \in \Gamma$. Now,

$$\begin{aligned}(\mu \times \nu)&((e,f)\alpha(g,h)\beta(i,j))\\ &= (\mu \times \nu)((e\alpha g\beta i, f\alpha h\beta j))\\ &= \min\{\mu(e\alpha g\beta i), \nu(f\alpha h\beta j)\}\\ &\geq \min\{\min\{\mu(e),\mu(g),\mu(i)\}, \min\{\nu(f),\nu(h),\nu(j)\}\}\\ &= \min\{\min\{\mu(e),\nu(f)\}, \min\{\mu(g),\nu(h)\}, \min\{\mu(i),\nu(j)\}\}\\ &= \min\{(\mu \times \nu)((e,f)), (\mu \times \nu)((g,h)), (\mu \times \nu)((i,j))\}.\end{aligned}$$

Thus $\mu \times \nu$ is a **FT**Γ**SG** of $E \times E$. □

Theorem 3.2. *If μ and ν are **FLI** (resp. **FRLI, FLAI**) of E, then $\mu \times \nu$ is a **FLI** (resp. **FRI, FLAI**) of $E \times E$.*

Proof. Suppose μ and ν are **FRI** of E. Let $(e,f),(g,h),(i,j) \in E \times E;\ \alpha,\beta \in \Gamma$. Now,

$$\begin{aligned}(\mu \times \nu)&((e,f)\alpha(g,h)\beta(i,j))\\ &= (\mu \times \nu)((e\alpha g\beta i, f\alpha h\beta j))\\ &= \min\{\mu(e\alpha g\beta i), \nu(f\alpha h\beta j)\}\\ &\geq \min\{\mu(i),\nu(j)\} = (\mu \times \nu)((i,j)).\end{aligned}$$

Thus $\mu \times \nu$ is a **FLI** of $E \times E$. Similarly, we prove for **FRI** and **FLAI**. □

Theorem 3.3. *If μ and ν are **FBI** of E, then $\mu \times \nu$ is a **FBI** of $E \times E$.*

Proof. Suppose μ and ν are **FBI** of E. Let$(e,f),(g,h),(i,j),(k,l),(m,n) \in E \times E;\ \alpha,\beta,\gamma,\delta \in \Gamma$. Now,

$$\begin{aligned}(\mu \times \nu)&((e,f)\alpha(g,h)\beta(i,j)\gamma(k,l)\delta(m,n))\\ &= (\mu \times \nu)((e\alpha g\beta i\gamma k\delta m, f\alpha h\beta j\gamma l\delta n))\\ &= \min\{\mu(e\alpha g\beta i\gamma k\delta m), \nu(f\alpha h\beta j\gamma l\delta n)\}\\ &\geq \min\{\min\{\mu(e),\mu(i),\mu(m)\}, \min\{\nu(f),\nu(j),\nu(n)\}\}\\ &= \min\{\min\{\mu(e),\nu(f)\}, \min\{\mu(i),\nu(j)\}, \min\{\mu(m),\nu(n)\}\}\\ &= \min\{(\mu \times \nu)((e,f)), (\mu \times \nu)((i,j)), (\mu \times \nu)((m,n))\}.\end{aligned}$$

Thus $\mu \times \nu$ is a **FBI** of $E \times E$. □

Theorem 3.4. *Let μ and ν are **FT**Γ**SG** of E, then $\mu \times \nu$ is a **FT**Γ**SG** of $E \times E$ if and only if the level set $(\mu \times \nu)_t$, where $t \in [0,1]$ is a ternary sub Γ-semigroup of $E \times E$.*

Proof. Suppose $\mu\times\nu$ is a **FTΓSG** of $E\times E$. Let $(e,f),(g,h),(i,j)\in(\mu\times\nu)_t;\ \alpha,\beta\in\Gamma$. Then

$$(\mu\times\nu)((e,f))\geq t,\ (\mu\times\nu)((g,h))\geq t$$

and

$$(\mu\times\nu)((i,j))\geq t.$$

Now,

$$(\mu\times\nu)((e,f)\alpha(g,h)\beta(i,j))$$
$$\geq\min\{(\mu\times\nu)(e,f)),(\mu\times\nu)(g,h)),(\mu\times\nu)((i,j))\}\geq t.$$

That implies $(e,f)\alpha(g,h)\beta(i,j)\in(\mu\times\nu)_t$. Thus $(\mu\times\nu)_t$ is a ternary sub Γ-semigroup of $E\times E$.

Conversely, suppose that $(\mu\times\nu)_t$ is a ternary sub Γ-semigroup of $E\times E$.

Let $(e,f),(g,h),(i,j)\in E\times E;\ \alpha,\beta\in\Gamma$. Let $(\mu\times\nu)((e,f))=t_1$, $(\mu\times\nu)((g,h))=t_2$ and $(\mu\times\nu)((i,j))=t_3$, where $t_1,t_2,t_3\in[0,1]$ with $t_3\leq t_2\leq t_1$. Next put $t=\min\{t_1,t_2,t_3\}$. Then $(e,f),(g,h),(i,j)\in(\mu\times\nu)_t$. So,

$$(e,f)\alpha(g,h)\beta(i,j)\in(\mu\times\nu)_t.$$

That implies $(\mu\times\nu)((e,f)\alpha(g,h)\beta(i,j))\geq t=\min\{(\mu\times\nu)(e,f)),(\mu\times\nu)(g,h)),(\mu\times\nu)((i,j))\}$. Thus $\mu\times\nu$ is a **FTΓSG** of $E\times E$. □

Theorem 3.5. *Let μ and ν are **FLI**(resp. **FRI, FLAI**) ideals of E, then $\mu\times\nu$ is a **FLI**(resp. **FRI, FLAI**) ideal of $E\times E$ if and only if the level set $(\mu\times\nu)_t$, where $t\in[0,1]$ is a **LI**(resp. **RI, LAI**) ideal of $E\times E$.*

Proof. Suppose $\mu\times\nu$ is a **FLI** of $E\times E$. Let $(e,f),(g,h)\in E\times E;\ (i,j)\in(\mu\times\nu)_t;\ \alpha,\beta\in\Gamma$. Then, $(\mu\times\nu)((i,j))\geq t$. Now, $(\mu\times\nu)((e,f)\alpha(g,h)\beta(i,j))\geq(\mu\times\nu)((i,j))\geq t$. That implies $(e,f)\alpha(g,h)\beta(i,j)\in(\mu\times\nu)_t$. Thus $(\mu\times\nu)_t$ is a **LI** of $E\times E$.

Conversely, suppose that $(\mu\times\nu)_t$ is a **LI** of $E\times E$.

Let $(e,f),(g,h),(i,j)\in E\times E;\ \alpha,\beta\in\Gamma$ and let $(\mu\times\nu)((e,f))=t_1$, $(\mu\times\nu)((g,h))=t_2$ and $(\mu\times\nu)((i,j))=t_3$, where $t_1,t_2,t_3\in[0,1]$ with $t_3\leq t_2\leq t_1$. Next, put $t=\min\{t_1,t_2,t_3\}$. Then $(e,f),(g,h),(i,j)\in(\mu\times\nu)_t$. So,

$$(e,f)\alpha(g,h)\beta(i,j)\in(\mu\times\nu)_t.$$

That implies $(\mu\times\nu)((e,f)\alpha(g,h)\beta(i,j))\geq t=(\mu\times\nu)((i,j))$. Thus $\mu\times\nu$ is a **FLI** of $E\times E$.

Similarly, we can prove for **FRI** and **FLAI**. □

Theorem 3.6. *Let μ and ν are **FBI** of E, then $\mu \times \nu$ is a **FBI** of $E \times E$ if and only if the level set $(\mu \times \nu)_t$, where $t \in [0,1]$ is a **BI** of $E \times E$.*

Proof. Suppose $\mu \times \nu$ is a **FBI** of $E \times E$. Let $(e,f),(g,h),(i,j) \in (\mu \times \nu)_t$; (k,l), $(m,n) \in E \times E$; $\alpha,\beta,\gamma,\delta \in \Gamma$. Then, $(\mu \times \nu)((e,f)) \geq t$, $(\mu \times \nu)((g,h)) \geq t$ and $(\mu \times \nu)((i,j)) \geq t$. Now,

$$\begin{aligned}&(\mu \times \nu)((e,f)\alpha(k,l)\beta(g,h)\gamma(m,n)\delta(i,j))\\ &\qquad\geq \min\{(\mu \times \nu)((e,f)),(\mu \times \nu)((g,h)),(\mu \times \nu)((i,j))\} \geq t.\end{aligned}$$

That implies $(e,f)\alpha(k,l)\beta(g,h)\gamma(m,n)\delta(i,j) \in (\mu \times \nu)_t$. Thus $(\mu \times \nu)_t$ is a **BI** of $E \times E$.

Conversely suppose that $(\mu \times \nu)_t$ is a **BI** of $E \times E$.

Let $(e,f),(g,h),(i,j)(k,l),(m,n) \in E \times E$; $\alpha,\beta,\gamma,\delta \in \Gamma$ and let $(\mu \times \nu)((e,f)) = t_1$, $(\mu \times \nu)((i,j)) = t_2$ and $(\mu \times \nu)((m,n)) = t_3$, where $t_1,t_2,t_3 \in [0,1]$ with $t_3 \leq t_2 \leq t_1$. Next, put $t = \min\{t_1,t_2,t_3\}$. Then $(e,f),(i,j),(m,n) \in (\mu \times \nu)_t$. So, $(e,f)\alpha(g,h)\beta(i,j)\gamma(k,l)\delta(m,n) \in (\mu \times \nu)_t$. That implies

$$\begin{aligned}&(\mu \times \nu)((e,f)\alpha(g,h)\beta(i,j)\gamma(k,l)\delta(m,n))\\ &\qquad\geq t = \min\{(\mu \times \nu)((e,f)),(\mu \times \nu)((i,j)),(\mu \times \nu)((m,n))\}.\end{aligned}$$

Thus $\mu \times \nu$ is a **FBI** of $E \times E$. □

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