

ANALYSIS OF FUZZY GRAPH THEORY AND ITS APPLICATION

K. SREEJIL¹ AND R. BALAKUMAR

ABSTRACT. Interval-valued fuzzy models give more accuracy, adaptability, and similarity to the framework when contrasted with the fuzzy models. The structure of the supplement of a fuzzy cycle is additionally examined. Ideas of graph theory have applications in numerous territories of software engineering, image segmentation, clustering, image catching, networks, and so forth. A stretch esteemed fuzzy set is a speculation of the idea of a fuzzy set. In this paper measure for connectivity of a fuzzy graph and its complement is analyzed.

1. INTRODUCTION

A fuzzy set, as a superset of a fresh set, owes its starting point to crafted by Zadeh [1] in 1965 that has been acquainted with manage the idea of fractional truth between true and false. Zadeh's surprising thought has discovered numerous applications in a few fields, including compound industry, media transmission, dynamic, organizing, software engineering, and discrete arithmetic. Rosenfeld [2] utilized the idea of a fuzzy subset of a set to present the thought of a fuzzy subgroup of a gathering. Rosenfeld's paper led to the advancement of fuzzy unique variable based math. A graph is a numerical portrayal of a system and it depicts the connection between vertices and edges. Graph theory is utilized to speak to genuine wonders, yet now and then graphs can't appropriately speak to numerous marvels since the vulnerability of various properties

¹*corresponding author*

Key words and phrases. Fuzzy relation, Graph theory, connectivity of fuzzy graph.

of the frameworks exists normally. Some true wonders gave the inspiration to characterize the fuzzy graphs.

Fuzzy-graph theory is developing quickly, with various applications in numerous areas, including organizing, correspondence, data mining, clustering, image catching, image segmentation, arranging, and planning. Harinath and Lavanya talked about fuzzy graph structures for wheel, rudder, and star graphs. Nair and Cheng [3] talked about the ideas of faction and fuzzy coteries in fuzzy graphs. Kauffman [4] presented fuzzy graphs utilizing Zadeh's fuzzy connection. Moderson and Yao [5] analyzed diverse connectedness levels in fuzzy graphs.

Bhattacharya [6] gave a few comments on fuzzy graphs, and a few procedure on fuzzy graphs were presented by Mordeson J. Lavanya characterized the all out fuzzy graph and considered complete chromatic number of all out graphs of fuzzy graphs. As a progression fuzzy shading of fuzzy graph was characterized by Eslahchi and Onagh in 2004, and later created by them as Fuzzy vertex shading in 2006 [7]. Somasundaram [8] examined a few procedure on fuzzy graphs, for example, association, join, creation, cartesian item and got their mastery parameters. Somasundaram [9] and A. S. [10]. Dinesh presented the idea of fuzzy-graph structures and portrayed some related ideas. Graph structures are the speculation of graphs and generally valuable in the investigation of certain structures, similar to graphs, marked graphs, semigraphs, edge-hued graphs, and edge-named graphs. Lavanay. Kavitha and S. Yeh and Bang's [11] approach for the investigation of fuzzy graphs were persuaded by its materialness to design arrangement and clustering examination.

Somasundaram S. The auxiliary property of fuzzy limited graphs gave an apparatus that permitted to the arrangement of activities inquire about issues. S. They worked more with the fuzzy lattice of a fuzzy graph, presented ideas like vertex network $\Omega(G)$, edge availability $\lambda(G)$ and built up the fuzzy simple of Whitney's hypothesis. N. Somasundaram [12] presented the ideas of control and all out mastery in fuzzy graphs and decided the mastery number for a few classes of fuzzy graphs and acquired limits for the equivalent. In a similar paper the creators analyzed the properties of different sorts of fuzzy cycles, fuzzy trees, fuzzy extensions, and fuzzy cut hubs in fuzzy graphs.

Nagoor Gani and Basheer Ahmed analyzed the properties of different kinds of degree, request and size of fuzzy graphs and thought about the connection

between degree, request and size of fuzzy graphs. Methods of fuzzy clustering examination can likewise be found in [13]. Fuzzy-graph structures are more valuable than graph structures since they manage the vulnerability and vagueness of some true wonders. A. Graph structures are extremely helpful in the investigation of various spaces of software engineering and computational knowledge. Different properties of fuzzy inner circles and portrayal of fuzzy factions were likewise introduced. They likewise demonstrated that for any three genuine numbers a, b, c to such an extent that $0 < a \leq b \leq c$, there exists a fuzzy graph G with $\Omega(G) = a$, $\lambda(G) = b$ and $\delta(G) = c$. what's more, Peng C. Ramakrishnan and Dinesh [4] took a shot at summed up fuzzy-graph structures. S and Sattanathan expanded the idea of fuzzy vertex shading in to a group of fuzzy sets [14].

2. TYPES OF ARCS IN A FUZZY GRAPH

Depending upon the $CONNG - (x, y)$ of arc (x, y) in a fuzzy graph, G Sunil Mathew and Sunitha [15] have characterized the accompanying three distinct sorts of curves. Note that $CONNG - (x, y)(x, y)$ is the quality of connectedness among x and y in the fuzzy graph got from G by erasing the circular segment (x, y) .

Definition 2.1. An arc (x, y) in G is called $\alpha - strong$ if $\mu(x, y) > CONNG - (x, y)(x, y)$.

Definition 2.2. An arc (x, y) in G is called $\beta - strong$ if $\mu(x, y) = CONNG - (x, y)(x, y)$.

Definition 2.3. An arc (x, y) in G is called a $\delta - arc$ if $\mu(x, y) < CONNG - (x, y)(x, y)$.

Definition 2.4. A strong arc is either $\alpha - strong$ or $\beta - strong$ by definition 2.1 and definition 2.2 respectively.

Definition 2.5. A $\delta - arc$ (x, y) is called a $\delta^* - arc$ if $\mu(x, y) > \mu(u, v)$ where (u, v) is a weakest arc of G .

Definition 2.6. A path in an f -graph $G : (\sigma, \mu)$ is called an $\alpha - strong$ path if all its arcs are $\alpha - strong$ and is called a $\beta - strong$ path if all its arcs are $\beta - strong$.

3. PRELIMINARIES

The accompanying fundamental definitions are taken from [10]. A fuzzy graph is a couple $G : (\sigma, \mu)$, where σ is a fuzzy subset of a set V and μ is a fuzzy connection on σ , i.e. $\mu(u, v) \leq \sigma(u) \wedge \sigma(v), \forall u, v \in V$. We accept that V is limited and non-void, μ is reflexive and symmetric. In all the models σ is picked appropriately. Likewise we mean the basic fuzzy graph by $G^* : (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V : \mu(u, v) > 0\}$. $H = (\tau, \nu)$ is known as a halfway fuzzy sub graph of G if $\tau \leq \sigma$ and $\nu \leq \mu$. We call $H = (\tau, \nu)$ a crossing fuzzy sub graph of $G = (\sigma, \mu)$ if $\tau = \sigma$. A way P of length n is an arrangement of unmistakable hubs $u_0, u_1, \dots, u_n$ with the end goal that $\mu(u_{i-1}, u_i) > 0$ and level of participation of a most vulnerable arc is characterized as its quality. In the event that $u_0 = u_n$ and $n \geq 3$, at that point P is known as a cycle and it is fuzzy cycle if there is more than one powerless arc. The quality of connectedness between two hubs u, v is characterized as the limit of qualities of all ways among u and v and is signified by $CONN_G(u, v)$. An arc (u, v) is known as a scaffold in G if the evacuation of (u, v) lessens the quality of connectedness between some pair of hubs in G . An associated fuzzy graph is known as a fuzzy tree on the off chance that it contains a traversing sub graph F which is a tree with the end goal that, for all edges (u, v) not in F , $\mu(u, v) < CONNF(u, v)$. A hub u in a fuzzy graph G is known as a fuzzy cut hub if its expulsion from G lessens the quality of connectedness between some other pair of hubs not including u . A fuzzy graph G is known as a fuzzy block on the off chance that it doesn't have fuzzy cut hubs.

The supplement of G [4] is signified as $G^c : (\sigma^c; \mu^c)$, where $\sigma^c = \sigma$ and $\mu^c(x; y) = \wedge[\sigma(x); \alpha(y)] - \mu(x; y)$.

4. CONNECTIVITY IN G^c

There might be situations when a fuzzy graph will be a fuzzy tree or fuzzy cycle and this auxiliary property may not be fulfilled by its supplement. The class of fuzzy graphs are so wide and needs incredible exertion to comprehend and investigate the basic properties of fuzzy graphs. It was seen that there are fuzzy graphs which are associated however their supplements become detached.

Theorem 4.1. *Let $G = (\sigma, \mu)$ be connected fuzzy graph with no m -strong arcs, then G^c is connected.*

Proof. The fuzzy graph G is associated and contain no m -solid arcs. Assume u, v be two subjective nodes of G^c . At that point they are likewise nodes of G . Since G is associated there exist a way among u and v in G . Leave this way alone P . At that point $P = (u_0, u_1)(u_1, u_2) \dots (u_{n-1}, u_n)$ where $\mu(\mu_{i-1}, \mu_i) > 0 \forall i$.

Hence P will be a (u, v) path in G^c also. Therefore G^c is connected. $\square$

Theorem 4.2. *Let $G = (\sigma, \mu)$ be a fuzzy graph. G and G^c are associated if and just if G contains at any rate one associated spreading over fuzzy subgraph with no m -solid arcs.*

Proof. Assume that G contains a crossing subgraph H that is associated, having no m -solid arcs. Since H contain no m -solid arcs and is associated utilizing $G = (\sigma, \mu)$ at that point G, H^c will be an associated spreading over fuzzy subgraph of G^c and in this way G^c is likewise associated. On the other hand, accept that G and G^c are associated. Need to locate an associated traversing subgraph of G that contains no m -solid arcs.

Let H alone a discretionary associated crossing subgraph of G . In the event that H contains no m -solid arcs, at that point H is the required subgraph. Assume H contains one m -solid arc say (u, v) . At that point arc (u, v) won't be available in G^c . Since G^c is associated there will exist a u - v way in G^c . Leave this way alone P_1 . Let $P_1 = (u_1, u_2)(u_2, u_3) \dots (u_{n-1}, u_n)$, where $u_i = u$ and $u_n = v$.

In the event that all the arcs of P_1 are available in G , at that point $H \cup (u, v)$ along with P_1 will be the required crossing subgraph. If not, there exist at any rate one arc say (u_1, v_1) in P_1 which isn't in G . Since G is associated we can replace (u_1, v_1) by another $u_1 - v_1$ way in G . Leave this way alone P_2 . In the event that P_2 contain no m -solid arcs, at that point $H \cup (u, v) \cup (u_1, v_1)$ along with P_1 and P_2 will be the required traversing subgraph. In the event that P_2 contain m -solid arc, at that point this arc won't be available in G^c . At that point supplant this arc by a way interfacing the comparing vertices in G^c and continue as above and since G contain just limited number of arcs at long last we will get a spreading over subgraph of that contain no m -solid arcs. $\square$

5. COMPLEMENT OF FUZZY CYCLES

Theorem 5.1. *Let $G = (\sigma, \mu)$ be a fuzzy graph to such an extent that G^* is a cycle with in excess of 5 vertices. At that point $(G^*)^c$ can't be a cycle.*

Proof. Given G^* is a cycle having n nodes where $n \geq 6$. Then G^* will have exactly n arcs. Since all the nodes of G are also present in G^c number of nodes of G^c is n . Let the nodes of G and G^c be $v_1, v_2, \dots, v_n$. Then G^c must contain at least the following edges.

$$\begin{aligned} &(v_1, v_3), (v_1, v_4)(v_1, v_n); \\ &(v_2, v_4), (v_2, v_5)(v_2, v_n); \\ &(v_3, v_5), (v_3, v_6)(v_3, v_n). \end{aligned}$$

Since $n \geq 6$ the all-out number of edges in G^c will be more noteworthy than n . Therefore G^c won't be a cycle. $\square$

Theorem 5.2. *Let G be fuzzy cycle with 6 or more nodes. Then G^c will not be fuzzy cycle.*

Proof. Given G be fuzzy cycle with at least 6 nodes. All the nodes of G is additionally hub of G^c . At that point G^c must contain in any event the accompanying edges.

$$\begin{aligned} &(v_1, v_3), (v_1, v_4)(v_1, v_n); \\ &(v_2, v_4), (v_2, v_5)(v_2, v_n); \\ &(v_3, v_5), (v_3, v_6)(v_3, v_n). \end{aligned}$$

Since $n \geq 6$ the total number of edges in G^c will be greater than n . Thus G^c will not be a cycle. $\square$

6. CONCLUSION

The study of fuzzy graphs made in this report is far from being complete. We sincerely hope that the wide ranging applications of graph theory and the interdisciplinary nature of fuzzy set theory, if properly blended together could pave a way for a substantial growth of fuzzy graph theory.

REFERENCES

- [1] K. R. BHUTANI, A. ROSENFELD: *Fuzzy end nodes in fuzzy graphs*, Information Sciences, 117 (2003), 319–322.

- [2] J. N. MORDESON, P.S. NAIR: *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica Verlag, 2000.
- [3] M. S. SUNITHA, A. VIJAYAKUMAR: *A characterisation of fuzzy trees*, Information Sciences, **113**(1999), 293–300.
- [4] M. S. SUNITHA, A. VIJAYAKUMAR: *Complement of a fuzzy graph*, Indian Journal of Pure and Applied Mathematics, **9**(33) (2002), 1451–1464.
- [5] M. S. SUNITHA, A. VIJAYAKUMAR: *Blocks in fuzzy graphs*, Indian Journal of Fuzzy Mathematics, **13**(1) (2005), 13–23.
- [6] K. VENGATESAN, E. S. KUMAR, S. YUVARAJ, P. S. TANESH, A. KUMAR: *An Approach for Remove Missing Values in Numerical and Categorical Values Using Two Way Table Marginal Joint Probability*, International Journal of Advanced Science and Technology, **29**(5) (2020), 2745 – 2756.
- [7] K. VENGATESAN, R. R. NIKAM, S. YUVARAJ, A. M. SHANKAR, P. S. TANESH, A. KUMAR: *A Random Forest-based Classification Method for Prediction of Car Price*, International Journal of Psychosocial Rehabilitation, **24**(3) (2020), 2745 – 2756.
- [8] ROSENFELD: *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, Academic Press, 1975.
- [9] K. R. SANDEEP NARAYAN, M. S. SUNITHA: *Connectivity in a Fuzzy Graph and its Complement*, ICRS, **9** (2012), 38–43.
- [10] S. MATHEW, M. S. SUNITHA, N. ANJALI: *Some Connectivity Concepts in Bipolar Fuzzy Graphs*, Annals of Pure and Applied Mathematics, **7** (2014), 98–108.
- [11] S. SAMANTA, M. PAL: *Irregular Bipolar Fuzzy Graphs*, International Journal of Fuzzy Sets, **2** (2012), 91–102.
- [12] A. K. SHARMA, B. V. PADAMWAR, C. L. DEWANGAN: *Trends in Fuzzy Graphs*, IJRSET **2** (2010), 91–102.
- [13] J. N. MORDESON, P.S. NAIR: *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica Verlag, 2000.
- [14] N. JICY, S. MATHEW: *Connectivity Analysis of Cyclically Balanced Fuzzy Graphs*, Elsevier, **7** (2015), 245–255.
- [15] L. ZADEH L: *Fuzzy sets*, Information and control, **8** (1965), 338–353.

DEPARTMENT OF MATHEMATICS
 PRIST DEEMED TO BE UNIVERSITY
 THANJAVUR
E-mail address: sreejilkottayi@gmail.com

DEPARTMENT OF MATHEMATICS
 PRIST DEEMED TO BE UNIVERSITY
 THANJAVUR
E-mail address: balaphdmaths@gmail.com