

NOTE ON AN ADJACENCY MATRIX OF A GRAPH G ROHIT M. PATNE ¹ AND GAJANAN R. AVACHAR

ABSTRACT. Let $G = (V(G), E(G))$ be a finite undirected graph with no loops or multiple edges with a vertex set $V(G)$ and an edge set $E(G)$. In this paper, we have considered the statement of the lemma in [4]. We have given some new results on the largest eigenvalue of an adjacency matrix of a graph G .

1. INTRODUCTION

Let $G = (V(G), E(G))$ be a finite undirected graph with no loops or multiple edges with a vertex set $V(G) = \{v_1, \dots, v_n\}$ and an edge set $E(G)$ such that $|E(G)| = m$. We denote an edge which connects the vertex v_i and v_j by $e_{ij} = v_i v_j = (v_i, v_j)$. Let $A(G) = [a_{ij}]$ denote an adjacency matrix of a graph G . Let $\rho_1(G), \dots, \rho_n(G)$ be the eigenvalues of $A(G)$ such that $\rho_n(G) \leq \dots \leq \rho_1(G)$. The spectral radius, $\rho(G)$ of a graph G corresponding to an adjacency matrix $A(G)$ is the largest eigenvalues of $A(G)$. Here $\rho(G) = \rho_1(G)$.

Let $L(G) = [a_{ij}]$ denote a Laplacian matrix of a graph G . Let $\lambda_1(G), \dots, \lambda_n(G)$ be the eigenvalues of $L(G)$ such that $\lambda_n(G) \leq \dots \leq \lambda_1(G)$. The spectral radius, $\lambda_1(G)$ of a graph G corresponding to a Laplacian matrix $L(G)$ is the largest eigenvalues of $L(G)$.

If we compare the proof of Lemma 2.1 given in [4] with example, then we can observe that the proof of lemma 2.1 given in [4] need modification.

¹corresponding author

2010 Mathematics Subject Classification. 05C62, 68R10.

Key words and phrases. Graph, Adjacency matrix, Spectra of graph, Largest eigenvalue of graph.

In this paper, we have checked the lemma 2.1 given in [4] by using example. We have given some results on an eigenvalues of $A(G)$. For spectral radius, $\rho(G)$ of a graph G (see [1–8].)

2. SOME RESULTS ON LARGEST EIGENVALUE OF AN ADJACENCY MATRIX OF A GRAPH G

In 1988, Hong Yuan shown that the spectral radius

$$(2.1) \quad \rho(G) \leq \sqrt{2m - n + 1}$$

with equality if and only if G is isomorphic to $K_{1,n-1}$ or K_n (see [1]). From this results one can show that if G is a simple planar graph with $n \geq 3$ vertices and m edges then $\rho(G) \leq \sqrt{5n - 11}$. If G is a connected graph with n vertices then $\sum_{i=2}^n \rho_i^2(G) \geq n - 1$.

We can also find the another upperbound of $\rho(G)$ given as follows:

Lemma 2.1. *Let $G = (V(G), E(G))$ be a connected graph with n vertices and m edges. Let vertex set $V(G) = \{v_1, \dots, v_n\}$. Let d_i is the degree of vertex v_i . Let $\delta_1 = \min\{d_i : v_i \in V(G)\}$. Let $\delta_2 = \max\{d_i : v_i \in V(G)\}$. If $X = (x_1, \dots, x_n)^t$ be an eigenvector of $A(G)$ corresponding to an eigenvalue $\rho(G)$ such that $\|x\| = 1$ Then*

$$\rho(G) \leq \sqrt{2m - (n - 1) \sum_{i=1}^n x_{v_i}^2},$$

where $x_{v_i} = \min\{|x_j| : (v_i, v_j) \notin E(G)\}$.

Proof. Let $G = (V(G), E(G))$ be a connected graph with n vertices and m edges. Let vertex set $V(G) = \{v_1, \dots, v_n\}$. Let d_i is the degree of vertex v_i . Let $\delta_1 = \min\{d_i : v_i \in V(G)\}$. Let $A(G)$ be an adjacency matrix of G . Let $A_i(G)$ denote i^{th} row of $A(G)$ which is corresponding to a vertex v_i . Let $S_{v_i}(A(G))$ denotes i^{th} row sum of $A(G)$. Therefore $S_{v_i}(A(G)) = d_i$. Let $X = (x_1, \dots, x_n)^t$ be an eigenvector of $A(G)$ corresponding to an eigenvalue $\rho(G)$ such that $\|x\| = 1$ i.e

$$x_1^2 + \dots + x_n^2 = 1.$$

Therefore

$$(2.2) \quad A(G)X = \rho(G)X$$

Here x_i denote an eigenvector of X correspond to a vertex v_i .

Let for the vertex v_i , $x_{v_i} = \min\{|x_j| : (v_i, v_j) \notin E(G)\}$.

Let $Y(v_i)$ denote the vector obtained from X by replacing x_j with 0 if $(v_i, v_j) \notin E(G)$. From i^{th} equation of (2.2), we have

$$A_i(G)Y(v_i) = A_i(G)X = \rho(G)X = \rho(G)x_i.$$

Hence by the Cauchy-Schwartz inequality

$$(2.3) \quad \rho^2(G)x_i^2 = |A_i(G)Y(v_i)|^2 \leq |A_i(G)|^2 |Y(v_i)|^2.$$

We know that $|A_i(G)|^2 = \sum_{j=1}^n a_{ij}^2 = d_i$.

We know that $x_1^2 + \cdots + x_n^2 = 1$. Therefore

$$\begin{aligned} \sum_{(v_i v_j) \in E(G)} x_j^2 + \sum_{(v_i v_j) \notin E(G)} x_j^2 &= 1 \\ \sum_{(v_i v_j) \in E(G)} x_j^2 &= 1 - \sum_{(v_i v_j) \notin E(G)} x_j^2 \\ |Y(v_i)|^2 &= \sum_{(v_i v_j) \in E(G)} x_j^2 = 1 - \sum_{(v_i v_j) \notin E(G)} x_j^2. \end{aligned}$$

From equation (2.3), we have

$$\rho^2(G)x_i^2 \leq d_i \left(1 - \sum_{(v_i v_j) \notin E(G)} x_j^2\right).$$

Taking sum on both side over all vertices of a graph G , we have

$$\begin{aligned} \sum_{i=1}^n \rho^2(G)x_i^2 &\leq \sum_{i=1}^n \left(d_i \left(1 - \sum_{(v_i v_j) \notin E(G)} x_j^2\right)\right). \\ \rho^2(G) \left(\sum_{i=1}^n x_i^2\right) &\leq 2m - \left(\sum_{i=1}^n d_i \sum_{(v_i v_j) \notin E(G)} x_j^2\right). \end{aligned}$$

$$(2.4) \quad \rho^2(G) \leq 2m - \left(\sum_{i=1}^n d_i \sum_{(v_i v_j) \notin E(G)} x_j^2\right).$$

Let

$$\begin{aligned}
 \sum_{i=1}^n d_i \left(\sum_{(v_i v_j) \notin E(G)} x_j^2 \right) &= \sum_{i=1}^n d_i (x_i^2 + \sum_{(v_i v_j) \notin E(G), i \neq j} x_j^2) \\
 &= \sum_{i=1}^n d_i x_i^2 + \sum_{i=1}^n (d_i \sum_{(v_i v_j) \notin E(G), i \neq j} x_j^2) \\
 &\geq \sum_{i=1}^n d_i x_i^2 + \sum_{i=1}^n \left(\sum_{(v_i v_j) \notin E(G), i \neq j} x_j^2 \right) \\
 &\geq \sum_{i=1}^n d_i x_i^2 + \sum_{i=1}^n \left(\sum_{(v_i v_j) \notin E(G), i \neq j} x_{v_i}^2 \right) \\
 &\geq \sum_{i=1}^n d_i x_i^2 + \sum_{i=1}^n ((n - (d_i + 1)) x_{v_i}^2) \\
 &\geq \sum_{i=1}^n d_i x_{v_i}^2 + \sum_{i=1}^n ((n - (d_i + 1)) x_{v_i}^2) \\
 &\geq \sum_{i=1}^n [d_i + (n - (d_i + 1))] x_{v_i}^2 \\
 &\geq (n - 1) \sum_{i=1}^n x_{v_i}^2.
 \end{aligned}$$

Therefore

$$\begin{aligned}
 \sum_{i=1}^n d_i \left(\sum_{(v_i v_j) \notin E(G)} x_j^2 \right) &\geq (n - 1) \sum_{i=1}^n x_{v_i}^2, \\
 (2.5) \quad - \sum_{i=1}^n d_i \left(\sum_{(v_i v_j) \notin E(G)} x_j^2 \right) &\leq -(n - 1) \sum_{i=1}^n x_{v_i}^2.
 \end{aligned}$$

Put (2.5) in (2.4), we get

$$\begin{aligned}
 \rho^2(G) &\leq 2m - (n - 1) \sum_{i=1}^n x_{v_i}^2, \\
 \rho^2(G) &\leq 2m - (n - 1) \sum_{i=1}^n x_{v_i}^2,
 \end{aligned}$$

where $x_{v_i} = \min\{|x_j| : (v_i, v_j) \notin E(G)\}$.

Therefore,

$$\rho(G) \leq \sqrt{2m - (n-1) \sum_{i=1}^n x_{v_i}^2}.$$

□

Remark 2.1. If $\sum_{i=1}^n x_{v_i}^2 = 1$, we get the equation (2.1), one can observe that

$$\rho(G) \leq \sqrt{2m - n + 1} \leq \sqrt{2m - (n-1) \sum_{i=1}^n x_{v_i}^2}.$$

Lemma 2.2. Let $G = (V(G), E(G))$ be a connected graph with n vertices and m edges. Let vertex set $V(G) = \{v_1, \dots, v_n\}$. Let d_i is the degree of vertex v_i . Let $\delta_1 = \min\{d_i : v_i \in V(G)\}$. Let $\delta_2 = \max\{d_i : v_i \in V(G)\}$. Then

$$\rho(G) \leq \sqrt{\delta_2 \sum_{i=1}^n \left(\sum_{(v_i v_j) \in E(G)} x_j^2 \right)}.$$

Proof. Let $G = (V(G), E(G))$ be a connected simple graph with n vertices and m edges. Let vertex set $V(G) = \{v_1, \dots, v_n\}$. Let d_i is the degree of vertex v_i . Let $\delta_1 = \min\{d_i : v_i \in V(G)\}$. Let $A(G)$ be an adjacency matrix of G . Let $A_i(G)$ denote i^{th} row of $A(G)$ which is corresponding to a vertex v_i . Let $S_{v_i}(A(G))$ denotes i^{th} rowsum of $A(G)$. Therefore $S_{v_i}(A(G)) = d_i$. Let $X = (x_1, \dots, x_n)^t$ be an eigenvector of $A(G)$ corresponding to an eigenvalue $\rho(G)$ such that $\|x\| = 1$ i.e

$$x_1^2 + \dots + x_n^2 = 1.$$

Therefore

$$(2.6) \quad A(G)X = \rho(G)X.$$

Here x_i denote an eigencomponent of X correspond to a vertex v_i . Let $Y(v_i)$ denote the vector obtained from X by replacing x_j with 0 if $(v_i, v_j) \notin E(G)$. From i^{th} equation of (2.6), we have

$$A_i(G)Y(v_i) = A_i(G)X = \rho(G)X = \rho(G)x_i.$$

Hence by the Cauchy-Schwartz inequality

$$(2.7) \quad \rho^2(G)x_i^2 = |A_i(G)Y(v_i)|^2 \leq |A_i(G)|^2 |Y(v_i)|^2.$$

We know that $|A_i(G)|^2 = \sum_{j=1}^n a_{ij}^2 = d_i$. We know that $x_1^2 + \cdots + x_n^2 = 1$. Therefore

$$\begin{aligned} \sum_{(v_i v_j) \in E(G)} x_j^2 + \sum_{(v_i v_j) \notin E(G)} x_j^2 &= 1. \\ \sum_{(v_i v_j) \in E(G)} x_j^2 &= 1 - \sum_{(v_i v_j) \notin E(G)} x_j^2. \\ |Y(v_i)|^2 &= \sum_{(v_i v_j) \in E(G)} x_j^2 = 1 - \sum_{(v_i v_j) \notin E(G)} x_j^2. \end{aligned}$$

From equation (2.7), we have

$$\rho^2(G)x_i^2 \leq d_i(1 - \sum_{(v_i v_j) \notin E(G)} x_j^2).$$

Taking sum on both side over all vertices of a graph G , we have

$$\begin{aligned} \sum_{i=1}^n \rho^2(G)x_i^2 &\leq \sum_{i=1}^n (d_i(1 - \sum_{(v_i v_j) \notin E(G)} x_j^2)). \\ \rho^2(G)(\sum_{i=1}^n x_i^2) &\leq 2m - (\sum_{i=1}^n d_i \sum_{(v_i v_j) \notin E(G)} x_j^2). \\ \rho^2(G) &\leq 2m - (\sum_{i=1}^n d_i \sum_{(v_i v_j) \notin E(G)} x_j^2). \end{aligned}$$

We know that

$$\begin{aligned} \sum_{i=1}^n (d_i \sum_{j=1}^n x_j^2) &= \sum_{i=1}^n (d_i \sum_{(v_i v_j) \in E(G)} x_j^2) + \sum_{i=1}^n (d_i \sum_{(v_i v_j) \notin E(G)} x_j^2). \\ \sum_{i=1}^n (d_i \sum_{(v_i v_j) \notin E(G)} x_j^2) &= \sum_{i=1}^n (d_i \sum_{j=1}^n x_j^2) - \sum_{i=1}^n (d_i \sum_{(v_i v_j) \in E(G)} x_j^2). \\ &\geq \sum_{i=1}^n d_i - \sum_{i=1}^n (\delta_2 \sum_{(v_i v_j) \in E(G)} x_j^2) \\ &\geq 2m - \sum_{i=1}^n (\delta_2 \sum_{(v_i v_j) \in E(G)} x_j^2) \\ (2.8) \quad & - \sum_{i=1}^n (d_i \sum_{(v_i v_j) \notin E(G)} x_j^2) \leq -(2m - \sum_{i=1}^n (\delta_2 \sum_{(v_i v_j) \in E(G)} x_j^2)) \end{aligned}$$

From (2.4) and (2.8), we have

$$\begin{aligned}\rho^2(G) &\leq 2m - (2m - \sum_{i=1}^n (\delta_2 \sum_{(v_i v_j) \in E(G)} x_j^2)) \\ &\leq \delta_2 \sum_{i=1}^n (\sum_{(v_i v_j) \in E(G)} x_j^2).\end{aligned}$$

Hence

$$\rho(G) \leq \sqrt{\delta_2 \left(\sum_{i=1}^n \left(\sum_{(v_i v_j) \in E(G)} x_j^2 \right) \right)}.$$

□

In 1997, Hong Yuan shown that if G be a simple graph with n vertices and $X = (x_1, \dots, x_n)^t$ be an eigenvector corresponding to an eigenvalue $\rho(G)$ such that $\|x\| = 1$ then $\rho(G) \leq \sum_{i=1}^n d_i x_i^2$, see [2].

3. NOTE ON AN ADJACENCY MATRIX

In this section, we have considered the statement of the lemma 2.1 and its proof which is given in [4] for completion of this article.

Lemma 3.1. [4] *Let G be a connected graph with vertex set $V(G) = \{v_1, \dots, v_n\}$ and d_i be the degree of vertex v_i , $i = 1, \dots, n$. Then*

$$(3.1) \quad \rho(G) \leq \max_{(v_i, v_j) \in E(G)} \sqrt{d_i d_j},$$

where $E(G)$ is the edge set of G . Moreover, the equality in (3.1) holds if and only if G is a regular or bipartite semiregular graph.

Remark 3.1. In [4], it is given in the proof that X be a Perron vector of G , where x_i corresponds to the vertex v_i . Let $x_s = \max_{v_i \in V(G)} x_i$ and $x_t = \max_{(v_i, v_s) \in E(G)} x_i$. From $A(G)X = \rho(G)X$, we have

$$(3.2) \quad \rho(G)x_s = \sum_{v_i \in N_G(v_s)} x_i \leq \sum_{v_i \in N_G(v_s)} x_t = d_s x_t,$$

$$(3.3) \quad \rho(G)x_t = \sum_{v_i \in N_G(v_t)} x_i \leq \sum_{v_i \in N_G(v_t)} x_s = d_t x_s,$$

where $N_G(S)$ denotes the neighbors in G of S . Hence

$$\begin{aligned}\rho(G)^2 x_s x_t &\leq d_s d_t x_s x_t \\ \rho(G) &\leq \sqrt{d_s d_t}.\end{aligned}$$

3.1. Verification proof by an example. In this subsection, we claim that equation (3.2) and (3.3) not correct step in general. The proof of this statement is given as follows:

Proof. Suppose if possible equation (3.2) is correct.

Let $G = (V(G), E(G))$ be a graph with $V(G) = \{v_1, v_2, \dots, v_{10}\}$ and $E(G) = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_1, v_5), (v_5, v_6), (v_7, v_6), (v_7, v_8), (v_7, v_9), (v_7, v_{10})\}$. The adjacency matrix $A(G)$ of G is given by

$$A(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The largest eigenvalue of the matrix $A(G)$ is $\rho(G) = 2.1987$. An eigenvector X corresponding to an eigenvalue $\rho(G)$ is given by

$$X = \begin{bmatrix} -0.0000 & -0.2988 & -0.2988 & -0.2988 & -0.5481 & -0.0000 & -0.6570 & 0 & 0 & 0 \end{bmatrix}.$$

Let $x_s = \max_{v_i \in V(G)} x_i = 0$ (which is corresponding to vertices v_8, v_9, v_{10}). In G , we have $(v_7, v_8), (v_7, v_9), (v_7, v_{10}) \in E(G)$. Hence $x_t = \max_{(v_i, v_s) \in E(G)} x_i = -0.6570$ (which is corresponding to a vertex v_7). From equation (3.2), we have

$$(3.4) \quad \rho(G)x_s = \sum_{v_i \in N_G(v_s)} x_i \leq \sum_{v_i \in N_G(v_s)} x_t = d_s x_t.$$

Put the value of $x_s, x_t, \rho(G), d_s$ in equation (3.4), we get

$$(2.1987)0 = \sum_{v_i \in N_G(v_s)} x_i \leq \sum_{v_i \in N_G(v_s)} x_t = x_7 = d_s x_t = (1)(-0.6570).$$

Therefore $0 \leq -0.6570$, which is not possible. Hence the equation (3.2), given in proof of lemma 2.1 in [4] do not hold in general. $\square$

3.2. New result.

Lemma 3.2. [3] Let G be a connected graph with vertex set $V(G) = \{v_1, \dots, v_n\}$ such that $|V(G)| = n$. Let $A(G)$ be an adjacency matrix of G . Let P be any polynomial and $S_{v_i}(P(A(G)))$ be a rowsums of $P(A(G))$ corresponding to the vertex v_i . Then

$$\min S_{v_i}(P(A(G))) \leq P(\rho(G)) \leq \max S_{v_i}(P(A(G))).$$

Moreover, equality holds if and only if the rowsums of $P(A(G))$ are all equal.

Lemma 3.3. Let G be a connected graph with vertex set $V(G) = \{v_1, \dots, v_n\}$ and d_i be the degree of vertex v_i , $i = 1, \dots, n$. Let $\rho(G)$ be a largest eigenvalue of an adjacency matrix $A(G)$. Let $x = (x_1, \dots, x_n)^t$ be an eigenvector corresponding to an eigenvalue $\rho(G)$. Then

$$\rho(G) \leq \max_{(v_i, v_j) \in E(G)} \sqrt{d_i d_j},$$

where $E(G)$ is the edge set of G .

Proof. Let G be a connected graph with vertex set $V(G) = \{v_1, \dots, v_n\}$ and d_i be the degree of vertex v_i , $i = 1, \dots, n$. Let $\rho(G)$ be a largest eigenvalue of an adjacency matrix $A(G)$. Let $x = (x_1, \dots, x_n)^t$ be an eigenvector corresponding to an eigenvalue $\rho(G)$. Let $N_i = \{v_\alpha : (v_i, v_\alpha) \in E(G)\}$.

Let $S_{v_i}(A^k(G))$ is the number of walks of length k in G which begin at v_i . Therefore $S_{v_i}(A(G))$ is d_i .

$$\begin{aligned} S_{v_i}(A^2(G)) &= d_i + \sum |N_i \cap N_j| \\ S_{v_i}(A^2(G)) &= \sum_{(v_i, v_j) \in E(G)} d_j. \end{aligned}$$

Let $d_k = \max\{d_j : (v_i, v_j) \in E(G)\}$. Therefore we have

$$S_{v_i}(A^2(G)) = \sum_{(v_i, v_j) \in E(G)} d_j \leq \sum_{(v_i, v_j) \in E(G)} d_k = d_i d_k.$$

By using lemma 3.2, we have

$$\begin{aligned}\rho^2(G) &\leq \max_{v_i \in V(G)} S_{v_i}(A^2(G)) \\ \rho^2(G) &\leq \max\{d_i d_j : (v_i, v_j) \in E(G)\} \\ \rho(G) &\leq \max_{(v_i, v_j) \in E(G)} \sqrt{d_i d_j}.\end{aligned}$$

□

Lemma 3.4. Let G be a connected graph with vertex set $V(G) = \{v_1, \dots, v_n\}$ and d_i be the degree of vertex v_i , $i = 1, \dots, n$. Let $\rho(G)$ be a largest eigenvalue of an adjacency matrix $A(G)$. Let $x = (x_1, \dots, x_n)^t$ be an eigenvector corresponding to an eigenvalue $\rho(G)$. Let for a vertex $v_i \in V(G)$, $d_k = \max\{d_j : (v_i, v_j) \in E(G)\}$ and $d_\alpha = \max\{d_j : (v_i, v_j) \in E(G)\}$ be second maximum degree among all degree of vertices adjacent to v_i , hence $d_\alpha \leq d_k$

Then

$$\rho(G) \leq \max_{(v_i, v_\alpha) \in E(G), (v_i, v_k) \in E(G)} \sqrt{(d_i - 1)d_\alpha + d_k},$$

where $E(G)$ is the edge set of G .

Proof. Let G be a connected graph with vertex set $V(G) = \{v_1, \dots, v_n\}$ and d_i be the degree of vertex v_i , $i = 1, \dots, n$. Let $\rho(G)$ be a largest eigenvalue of an adjacency matrix $A(G)$. Let $x = (x_1, \dots, x_n)^t$ be an eigenvector corresponding to an eigenvalue $\rho(G)$. Let $N_i = \{v_\alpha : (v_i, v_\alpha) \in E(G)\}$.

Let $S_{v_i}(A^k(G))$ is the number of walks of length k in G which begin at v_i . Therefore $S_{v_i}(A(G))$ is d_i .

$$\begin{aligned}S_{v_i}(A^2(G)) &= d_i + \sum |N_i \cap N_j|. \\ S_{v_i}(A^2(G)) &= \sum_{(v_i, v_j) \in E(G)} d_j.\end{aligned}$$

Let $d_k = \max\{d_j : (v_i, v_j) \in E(G)\}$. Therefore we have

$$\begin{aligned}S_{v_i}(A^2(G)) &= \sum_{(v_i, v_j) \in E(G)} d_j = \left(\sum_{(v_i, v_j) \in E(G)} d_j \right) + d_k \\ &\leq \left(\sum_{(v_i, v_j) \in E(G)} d_\alpha \right) + d_k = (d_i - 1)d_\alpha + d_k.\end{aligned}$$

By using lemma 3.2, we have

$$\begin{aligned}\rho^2(G) &\leq \max_{v_i \in V(G)} S_{v_i}(A^2(G)) \\ \rho^2(G) &\leq \max\{(d_i - 1)d_\alpha + d_k : (v_i, v_j) \in E(G)\} \\ \rho(G) &\leq \max_{(v_i, v_\alpha) \in E(G), (v_i, v_k) \in E(G)} \sqrt{(d_i - 1)d_\alpha + d_k}.\end{aligned}$$

□

REFERENCES

- [1] H. YUAN: *A bound on the spectral radius of graphs*, Linear Algebra and its applications, **108** (1988), 135–139.
- [2] H. YUAN: *Upper bounds of the spectral radius of graphs in terms of genus*, Journal of combinatorial theory, series B, **74** (1988), 153–159.
- [3] H. YUAN, S. JIN-LONG, K. FANG: *A sharp upper bound of the spectral radius of graphs*, Journal of combinatorial theory, series B, **81** (2001), 177–183.
- [4] A. BERMAN, Z. XIAO-DONG: *On the spectral radius of graphs with cut vertices*, Journal of combinatorial Theory, series B, **83** (2001), 233–240.
- [5] D. CVETKOVIC, M. DOOB, H. SACHS: *Spectra of Graphs-Theory and Applications*, 2nd ed, Academic Press, New York, 1982.
- [6] D. CVETKOVIC, P. ROWLINSON: *The largest eigenvalue of a graph: A survey*, Linear Multilinear Algebra, **28** (1990), 3–33.
- [7] F. HARARY: *The Determinant of the Adjacency Matrix of a Graph*, SIAM Review, **4** (1962), 202–210.
- [8] F. HARARY: *Graph theory*, Addison-Wesley Reading, Massachusetts, 1996.

DEPARTMENT OF MATHEMATICS
SSES AMRAVATI'S SCIENCE COLLEGE,
CONGRESS NAGAR, NAGPUR, INDIA-440012
E-mail address: iitg2010rohit@gmail.com

DEPARTMENT OF MATHEMATICS
SSES AMRAVATI'S SCIENCE COLLEGE
CONGRESS NAGAR, NAGPUR, INDIA-440012
E-mail address: gravachar@gmail.com