ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.3, 1349–1362
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.3.59

THE CONSTRUCTION OF BLOCK CIPHER ENCRYPTION KEY BY USING A LOCAL SUPER ANTIMAGIC TOTAL FACE COLORING

TITA KHALIS MARYATI[1], KHAMIDA SITI NUR ATIQOH, ROSANITA NISVIASARI, IKA HESTI AGUSTIN, DAFIK, AND M.VENKATACHALAM

ABSTRACT. The strength of cryptosystem relays on the management of encryption key. The key should be managed such that it is hard for any intruder to analyze the key. Thus, the main issue is how to make the relation between plaintext, ciphertext and the key is hidden. We aimed to study the use of local super antimagic total face labeling of planar graph on generating encryption keys that can be used to establish a modified of affine and block cipher. We are using graph $G = (V(G), E(G), F(G))$ be a nontrivial, finite, connected graph, and a g bijective function mapping total labeling of graph to natural number start form 1 until the sum of vertices, edges label start from the sum of vertices plus 1 until the sum of vertices and edges, and faces label start form the sum of vertices and edges plus one until the sum of vertices, edges, and faces. If any adjacent two faces f_1 and f_2 have different weights $w(f_1) \neq w(f_2)$ for $f_1, f_2 \in F(G)$, then g is called a local super antimagic total face labeling. The local super antimagic total face coloring is induce a proper faces coloring of G into faces f that assign by the color of face weights on local super antimagic total face labeling. The local antimagic total face chromatic number is the minimum number of colors that super antimagic total face coloring needed. It can be denoted by $\gamma_{latf}(G)$. The resulting of local super antimagic total face coloring can potentially generates encryption keys that can be used to establish a modified of affine and block cipher. As the result, we have two algorithms for establishing modern cryptosystem polyalphabetics of local super antimagic total face coloring of planar graphs.

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 05C78.
Key words and phrases. local super antimagic total face coloring, cryptosystem, encryption.

1. INTRODUCTION

Cryptosystem is a system which converts plain text to cipher text or cipher text to plain text by the application of encryption or decryption algorithm. The strength of cryptosystem relays on the management of encryption key. The key should be managed such that it is hard for any intruder to analyze the key. The main issue is how to develop a secure modern cryptosystem such that the key between plaintext and ciphertext is hidden [11].

There are four purpose of cryptography. First of all, authentication to prove the identity of an entity. Secondly, data confidentiality to guarantee that only those parties can understand the data exchanged. Third, data integrity refers to data that has not been changed, destroyed, or lost. Fourth, non repudiation is to protect against the denial by one of the entities. Encryption process is transforming information so it is unintelligible to anyone but the intended recipient. Decryption process is transforming encrypted information so that it is intelligible again [8], [9].

The local super antimagic total face coloring is induce a proper faces coloring of G into faces f that assign by the color of face weights on local super antimagic total face labeling. The local antimagic total face chromatic number is the minimum number of colors that super antimagic total face coloring needed. It can be denoted by $\gamma_{latf}(G)$.

Arumugam et al. [4] was introduced local antimagic vertex coloring of a graph for the first time. Their results are local antimagic vertex chromatic number of path, cycle, complete, friendship, wheel, bipartite, and complete bipartite graph and also lower and upper bound local antimagic vertex chromatic number of joint graph. Another research that related with local super antimagic total face labeling is super edge-antimagic total labeling of $mK_{n,n}$ [6].

There is another type of local antimagic coloring. It is local antimagic edge coloring. Agustin et al. [2] was studied about local antimagic edge coloring. Their results are the exact value of path, cycle, friendship, ladder, star, wheel, complete, prism, $C_n \odot mK_1$ and $G \odot mK_1$ and also a lower bound of local edge antimagic chromatic number ($\gamma_{lea} \geq \Delta(G)$). The other results of on super local antimagic total edge coloring of some wheel related graphs [1], and local edge antimagic coloring of comb product of graphs [3].

2. A Useful Lemma and Corollary

We present the result of local super antimagic total face coloring in this section to prove the main theorem.

Observation 1. *[7] For any G graph, $\gamma_{latf}(G) \geq \chi_f(G)$, where $\chi_f(G)$ is chromatic number of face coloring in graph G.*

Theorem 2.1. *[7] Let $n \geq 3$ be a natural number and W_n be a wheel graph. Then,*

$$\gamma_{latf}(W_n) = \begin{cases} 2; & \text{for } n \text{ is even} \\ 3; & \text{for } n \text{ is odd.} \end{cases}$$

Theorem 2.2. *[7] Let $n \geq 3$ be a natural number and $J(2,n)$ be a jahangir graph. Then,*

$$\gamma_{lfat}(J(2,n)) = \begin{cases} 2; & \text{for } n \text{ is even} \\ 3; & \text{for } n \text{ is odd.} \end{cases}$$

Some lemma contain partition technique that we use to proof the main theorem.

Lemma 2.1. *[5] Let c, b, d and j be positive integers, $\Sigma_{i=1}^{b}\mathcal{P}_{b,d}^{c}(i,j) = \{(i-1)c + j, 1 \leq i \leq b\}$ and $\Sigma_{i=1}^{b}\mathcal{P}_{b,d}^{c}(i,j) = \{(j-1)b + i, 1 \leq i \leq b\}$ form an arithmetic sequence of difference $d \in \{b, b^2\}$, respectively.*

Proof. By simple calculation, for $1 \leq j \leq c$, it gives $\Sigma_{i=1}^{b}\mathcal{P}_{b,b}^{c}(i,j) = \frac{b^2c - cb}{2} + bj \iff \Sigma_{i=1}^{b}\mathcal{P}_{b,b}^{c}(i,j) = \{\frac{b^2c - cb}{2} + b, \frac{b^2c - cb}{2} + 2b, \ldots, \frac{b^2c - cb}{2} + (c-1)b, \ldots, \frac{b^2c - cb}{2} + cb\}$ and
$\Sigma_{i=1}^{b}\mathcal{P}_{b,b^2}^{c}(i,j) = \frac{(b - b^2)}{2} + b^2j \iff \Sigma_{i=1}^{b}\mathcal{P}_{b,b^2}^{c}(i,j) = \{\frac{(b-b^2)}{2} + b^2, \frac{(b-b^2)}{2} + 2b^2, \ldots, \frac{b^2c - cb}{2} + (c-1)b^2, \ldots, \frac{b^2c - cb}{2} + cb^2\}$.

It concludes the proof. $\square$

Lemma 2.2. *[10] Let c and b be positive integers. For $1 \leq j \leq c$, the sum of $\mathcal{P}_{b,d_1}^{c}(i,j) = \{1 + ci - j; 1 \leq i \leq b\}$ and $\mathcal{P}_{b,d_2}^{c}(i,j) = \{bc + i - bj; 1 \leq i \leq b\}$ form an arithmetic sequence of difference $d_1 = -b,\ d_2 = -b^2$.*

Proof. By simple calculation, for $j = 1, 2, \ldots, c$, it gives $\Sigma_{i=1}^{b}\mathcal{P}_{b,d_1}^{c}(i,j) = \mathcal{P}_{b,d_1}^{c}(j) \iff \mathcal{P}_{b,d_1}^{c}(j) = \{\frac{c}{2}(b^2 + b) + b - bj\}\} \iff \mathcal{P}_{b,d_1}^{c}(j) =$

$\{\frac{c}{2}(b^2 + b), \frac{c}{2}(b^2 + b) - b, \frac{c}{2}(b^2 + b) - 2b, \dots, \frac{c}{2}(b^2 + b)b - bc\}\}$ and $\Sigma_{i=1}^{b}\mathcal{P}_{b,d_2}^{c}(j) = \mathcal{P}_{b,d_2}^{c}(j) \iff \mathcal{P}_{b,d_2}^{c}(j) = \{\{\frac{b}{2}(2bc + b + 1) - b^2 j\}\} \iff \mathcal{P}_{b,d_2}^{c}(j) = \{\frac{b}{2}(2bc + b + 1) - b^2, \frac{b}{2}(2bc + b + 1) - 2b^2, \dots, \frac{b}{2}(2bc + b + 1) - b^2 c\}$.

It is easy to see that the differences of those sequence are $d_1 = -b, d_2 = -b^2$. It concludes the proof. □

Lemma 2.3. *Let b and c be positive integers.*

- *For $1 \le i \le b$, $b = 2$ and c is even,* $\mathcal{P}_{b,2w}^{c}(i,j) = \{j; i = 1\} \cup \{bc - j; i = 2, j \text{ } is \text{ } odd\} \cup \{bc - j + 2; i = 2, j \text{ } is \text{ } even\}$.
- *For $1 \le i \le b$, $b = 2$ and c is odd,* $\mathcal{P}_{b,2w}^{c}(i,j) = \{\frac{j+1}{2}; i = 1, j \text{ } is \text{ } odd\} \cup \{\frac{2c-j+2}{2}; i = 1, j \text{ } is \text{ } even\} \cup \{\frac{3c-j+2}{2}; i = 2, j \text{ } is \text{ } odd\} \cup \{\frac{3c+j+1}{2}; i = 2, j \text{ } is \text{ } even\}$.
- *For $1 \le i \le b$, $b = 3$ and c is even,* $\mathcal{P}_{b,2w}^{c}(i,j) = \{\frac{1+j+2c(i-1)}{2}; 1 \le i \le 2, j \text{ } is \text{ } odd\} \cup \{\frac{j+c(2i-1)}{2}; 1 \le i \le 2, j \text{ } is \text{ } even\} \cup \{bc - j + 1; i = 3\}$.
- *For $1 \le i \le b$, $b = 3$ and c is odd,* $\mathcal{P}_{b,2w}^{c}(i,j) = \{\frac{j+1}{2}; i = 1, j \text{ } is \text{ } odd\} \cup \{\frac{c+j+1}{2}; i = 1, j \text{ } is \text{ } even\} \cup \{2c - j + 1; i = 2\} \cup \{\frac{4c+j+1}{2}; i = 3, j \text{ } is \text{ } odd\} \cup \{\frac{5c+j+1}{2}; i = 3, j \text{ } is \text{ } even\}$.
- *For $1 \le j \le c$, the sum of partition $\mathcal{P}_{b,2w}^{c}(i,j)$ form two different weights.*

Proof. By simple calculation, for $b = 2$ and c is odd, it gives $\Sigma_{i=1}^{b}\mathcal{P}_{b,2w}^{c}(i,j) = \mathcal{P}_{b,2w}^{c}(j) \iff \mathcal{P}_{b,2w}^{c}(j) = \{\frac{3c+3}{2}, \frac{5c+3}{2}, \frac{3c+3}{2}, \frac{5c+3}{2}, \dots\}$.
For $b = 2$ and c is even, it gives $\Sigma_{i=1}^{b}\mathcal{P}_{b,2w}^{c}(i,j) = \mathcal{P}_{b,2w}^{c}(j) \iff \mathcal{P}_{b,2w}^{c}(j) = \{bc, bc + 2, bc, bc + 2, \dots\}$.
For $b = 3$ and c is odd, it gives $\Sigma_{i=1}^{b}\mathcal{P}_{b,2w}^{c}(i,j) = \mathcal{P}_{b,2w}^{c}(j) \iff \mathcal{P}_{b,2w}^{c}(j) = \{4c + 2, 5c + 2, 4c + 2, 5c + 2, \dots\}$.
For $b = 3$ and c is even, it gives $\Sigma_{i=1}^{b}\mathcal{P}_{b,2w}^{c}(i,j) = \mathcal{P}_{b,2w}^{c}(j) \iff \mathcal{P}_{b,2w}^{c}(j) = \{bc + c + 2, bc + 2c + 1, bc + c + 2, bc + 2c + 1, \dots\}$.
It is easy to see that the sum of this partition have two different. It concludes the proof. □

Lemma 2.4. *Let b and c be positive integers.*

- *For* $1 \leq i \leq b, b = 2$*, and* c *is odd,* $\mathcal{P}^c_{b,3w}(i,j) = \{j; i = 1, 1 \leq j \leq c\} \cup \{2c - j + 2; i = 2, 1 \leq j \leq c, j\ is\ odd\} \cup \{\frac{3c-2j+7}{2}; i = 2, 1 \leq j \leq c, j\ is\ even\}$.
- *For* $1 \leq i \leq b, b = 3$*, and* c *is odd,* $\mathcal{P}^c_{b,3w}(i,j) = \{\frac{2c(i-1)+j+1}{2}; 1 \leq i \leq 2, 1 \leq j \leq c, j\ is\ odd, j \neq m\} \cup \{\frac{c(2i-1)+j-1}{2}; 1 \leq i \leq 2, 1 \leq j \leq c, j\ is\ even\} \cup \{ic; 1 \leq i \leq 2, j = m\} \cup \{3c - j + 1; i = 3, 1 \leq j \leq c\}$.
- *For* $1 \leq j \leq c$*, the sum of partition* $\mathcal{P}^c_{b,3w}(i,j)$ *form three different weights.*

Proof. By simple calculation, for $b = 2$ and c is odd, it gives $\Sigma^b_{i=1}\mathcal{P}^c_{b,3w}(i,j) = \mathcal{P}^c_{b,3w}(j) \Longleftrightarrow \mathcal{P}^c_{b,3w}(j) = \{2c{+}1, \frac{3c+7}{2}, 2c{+}2, \frac{3c+7}{2}, 2c{+}2, \frac{3c+7}{2}, \dots, \}$. For $b = 3$ and c is odd, it gives $\Sigma^b_{i=1}\mathcal{P}^c_{b,3w}(i,j) = \mathcal{P}^c_{b,3w}(j) \Longleftrightarrow \mathcal{P}^c_{b,3w}(j) = \{4c + 2, 5c, 4c + 2, 5c, \dots, 4c + 2, 5c, 4c + 2, 5c, 5c + 1\}$.

It is easy to see that the sum of this partition have three different. It concludes the proof. □

3. The Results

The main theorem that show the existence of local super antimagic total face coloring is follows.

Theorem 3.1. *Let* G *be a related wheel graph with* n, m, r *is natural number and* $n \geq 3$*, then*

$$\gamma_{lfat}(G) = \begin{cases} 2; & \text{for } n \text{ is even} \\ 3; & \text{for } n \text{ is odd.} \end{cases}$$

Proof. Graph G is related wheel graph has n spokes, m vertices on spoke, and r vertices between two spokes. Related wheel graph has vertex set as $V(G) = \{x\} \cup \{x^j_i; 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{x_k; 1 \leq k \leq nr\}$ and edge set as $E(G) = \{xx^j_1; 1 \leq j \leq n\} \cup \{x^j_1x^j_{i+1}; 1 \leq i \leq m-1, 1 \leq j \leq n\} \cup \{x^j_mx_1; 1 \leq j \leq n\} \cup \{x_kx_{k+1}; 1 \leq k \leq n(r-1)\} \cup \{x_kx^j_m; 1 \leq j \leq n\}$. To prove this theorem, we start proving the lower bound of $\gamma_{latf}(G)$. Based on Observation 1, we have $\gamma_{latf}(G) \geq \gamma_f(G)$. We know that faces with an edge in common has different colors, so the chromatic number of face coloring in graph is at least 2, especially planar graphs. The chromatic number of face coloring in related wheel graph is at least 2, because related wheel graph has at least two adjacent faces.

Hence, $\gamma_f(G) = 2$. For $n = 3$, $c(f_1) \neq c(f_2)$, $c(f_2) \neq c(f_3)$, and $c(f_1) \neq c(f_3)$. We can see that every faces of related wheel n is odd must have 3 different colors. Hence, $\gamma_f(G) = 3$ for n is odd. We get the lower bound of $\gamma_{latf}(G) \geq 2$ for n is even and $\gamma_{latf}(G) \geq 3$ for n is odd. To prove the upper bound of $\gamma_{latf}(G)$, we define the function of all the elements of G with using lemma 2.1-2.4. The functions are as follows:

$$f(x) = 1$$

$$f(x_i^j) = \begin{cases} n(i-1)+j+1; & \text{for } i \text{ is odd} \\ ni-j+2; & \text{for } i \text{ is even} \end{cases}$$

$$\begin{aligned} f(V \backslash \{x_i^j\}) &= \{\mathcal{P}^c_{b_1,b_1} \oplus [nm+1]\} \cup \{\mathcal{P}^c_{b_2,-b_2} \oplus [(nm+1)+cb_1]\} \cup \{\mathcal{P}^c_{b_3,b_3^2} \oplus \\ &\quad [(nm+1)+cb_1+cb_2]\} \cup \{\mathcal{P}^c_{b_4,-b_4^2} \oplus [(nm+1)+cb_1+cb_2+ \\ &\quad cb_3]\} \end{aligned}$$

$$f(xx_1^j) = |V| + n - j + 1$$

$$f(x_i^j x_{i+1}^j) = \begin{cases} |V|+ni+j; & \text{for } i \text{ is odd} \\ |V|+n(i+1)-j+1; & \text{for } i \text{ is even} \end{cases}$$

for n is even,

$$\begin{aligned} f((E \backslash \{xx_i\}) \cup F) &= \{\mathcal{P}^c_{b_5,b_5} \oplus [|V|+nm]\} \cup \{\mathcal{P}^c_{b_6,-b_6} \oplus [|V|+nm+cb_5]\} \cup \\ &\quad \{\mathcal{P}^c_{b_7,b_7^2} \oplus [|V|+nm+cb_5+cb_6]\} \cup \{\mathcal{P}^c_{b_8,-b_8^2} \oplus [|V|+ \\ &\quad nm+cb_5+cb_6+cb_7]\} \cup \{\mathcal{P}^c_{b_9,2w} \oplus [|V|+nm+cb_5+cb_6 \\ &\quad +cb_7+cb_8]\} \end{aligned}$$

for n is odd,

$$\begin{aligned} f((E \backslash \{xx_i\}) \cup F) &= \{\mathcal{P}^c_{b_5,b_5} \oplus [|V|+nm]\} \cup \{\mathcal{P}^c_{b_6,-b_6} \oplus [|V|+nm+cb_5]\} \cup \\ &\quad \{\mathcal{P}^c_{b_7,b_7^2} \oplus [|V|+nm+cb_5+cb_6]\} \cup \{\mathcal{P}^c_{b_8,-b_8^2} \oplus [|V|+ \\ &\quad nm+cb_5+cb_6+cb_7]\} \cup \{\mathcal{P}^c_{b_9,3w} \oplus [|V|+nm+cb_5+ \\ &\quad cb_6+cb_7+cb_8]\} \end{aligned}$$

It is easy to see that f is a local super antimagic total face labeling with the face weights are as follows:

$$\begin{aligned}
w(f_j) &= \bigcup_{k=1}^{3} w_k(f_j)\\
w_1(f_j) &= f(x) + f(x_i^j) + f(x_i^{j+1}) + f(xx_1^j) + f(xx_1^{j+1}) + f(x_i^j x_{i+1}^j) + \\
&\quad f(x_i^{j+1} x_{i+1}^{j+1})\\
w_2(f_j) &= \Sigma_{i=1}^{b_1}\{\mathcal{P}_{b_1,b_1}^c \oplus [nm+1]\} \cup \Sigma_{i=1}^{b_2}\{\mathcal{P}_{b_2,-b_2}^c \oplus [(nm+1)+cb_1]\} \cup \\
&\quad \Sigma_{i=1}^{b_3}\{\mathcal{P}_{b_3,b_3^2}^c \oplus [(nm+1)+cb_1+cb_2]\} \cup \Sigma_{i=1}^{b_4}\{\mathcal{P}_{b_4,-b_4^2}^c \oplus [(nm+1)+ \\
&\quad cb_1+cb_2+cb_3]\} \cup \Sigma_{i=1}^{b_5}\{\mathcal{P}_{b_5,b_5}^c \oplus [|V|+nm]\} \cup \Sigma_{i=1}^{b_6}\{\mathcal{P}_{b_6,-b_6}^c \oplus [|V|+ \\
&\quad nm+cb_5]\} \cup \Sigma_{i=1}^{b_7}\{\mathcal{P}_{b_7,b_7^2}^c \oplus [|V|+nm+cb_5+cb_6]\} \cup \Sigma_{i=1}^{b_9}\{\mathcal{P}_{b_8,-b_8^2}^c \oplus \\
&\quad [|V|+nm+cb_5+cb_6+cb_7]\}
\end{aligned}$$

$$w_3(f_j) = \begin{cases} \Sigma_{i=1}^{b_9}\{\mathcal{P}_{b_9,3w}^c \oplus [|V|+nm+cb_5+cb_6+cb_7+cb_8]\}; & \text{for } n \text{ is odd}\\ \Sigma_{i=1}^{b_9}\{\mathcal{P}_{b_9,2w}^c \oplus [|V|+nm+cb_5+cb_6+cb_7+cb_8]\}; & \text{for } n \text{ is even} \end{cases}$$

$$\begin{aligned}
w_1(f_j) &= 2nm^2 + 4m + 2m|V| + 1\\
w_2(f_j) &= \frac{n}{2}(b_1^2 + b_2^2 + b_5^2 + b_6^2 - b_1 + b_2 - b_5 + b_6) + nm(b_1 + b_2 + b_3 + b_4 + b_5 + \\
&\quad b_6 + b_7 + b_8) + n(b_4^2 + b_8^2 + b_1b_2 + b_1b_3 + b_2b_3 + b_1b_4 + b_2b_4 + b_3b_4 + \\
&\quad b_5b_6 + b_5b_7 + b_6b_7 + b_5b_8 + b_6b_8 + b_7b_8) + \frac{1}{2}(-b_3^2 + 3b_3 + b_4^2 + 3b_4 - \\
&\quad b_7^2 + b_7 + b_8^2 + b_8) + b_1 + 2b_2 + b_6 + |V|(b_5 + b_6 + b_7 + b_8) + j(b_1 - \\
&\quad b_2 + b_3^2 - b_4^2 + b_5 - b_6 + b_7^2 - b_8^2)
\end{aligned}$$

$$w_3(f_j) = \begin{cases} b_9(|V| + n(1 + m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 2,\; n \text{ is even}, \\ & \phantom{\text{for}} \;\; j \text{ is odd} \\ 2 + b_9(|V| + n(1 + m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 2,\; n \text{ is even}, \\ & \phantom{\text{for}} \;\; j \text{ is even} \\ n + 2 + b_9(|V| + n(1 + m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 3,\; n \text{ is even}, \\ & \phantom{\text{for}} \;\; j \text{ is odd} \\ 2n + 1 + b_9(|V| + n(1 + m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 3,\; n \text{ is even}, \\ & \phantom{\text{for}} \;\; j \text{ is even} \\ 2n + 1 + b_9(|V| + n(m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 2,\; n \text{ is odd}, \\ & \phantom{\text{for}} \;\; j = 1 \\ \frac{3n+7}{2} + b_9(|V| + n(m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 2,\; n \text{ is odd}, \\ & \phantom{\text{for}} \;\; j \text{ is even} \\ 2n + 2 + b_9(|V| + n(m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 2,\; n \text{ is odd}, \\ & \phantom{\text{for}} \;\; j \text{ is odd}, j \neq 1 \\ 4n + 2 + b_9(|V| + n(m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 3,\; n \text{ is odd}, \\ & \phantom{\text{for}} \;\; j \text{ is odd}, j \neq n \\ 5n + b_9(|V| + n(m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 3,\; n \text{ is odd} \\ & \phantom{\text{for}} \;\; j \text{ is even} \\ 5n + 1 + b_9(|V| + n(m + b_5 + b_6 + b_7 + b_8)); & \text{for} \;\; b_9 = 3,\; n \text{ is odd}, \\ & \phantom{\text{for}} \;\; j = n \end{cases}$$

From the above face weight $w(f_j) = \bigcup_{k=1}^{3} w_k(f_j)$, the set of face weight contains only two elements for n is even and three elements for n is odd. It concludes that we get $\gamma_{latf}(G) \geq 2$ for n is even and $\gamma_{latf}(G) \geq 3$ for n is odd. $\square$

4. Establishing Modern Cryptosystem Polyalphabetic

Modern cryptosystem polyalphabetic is an improved variant of block cipher and affine cipher. At the block cipher, the plaintext is normally divided into several blocks with the same length. At the affine cipher, the whole process relies on working $mod m$ (the length of alphabet used). Our developed cryptosystem can be described as follows.

- The number of block on plaintext is taken from chromatic number of LATF of planar graphs

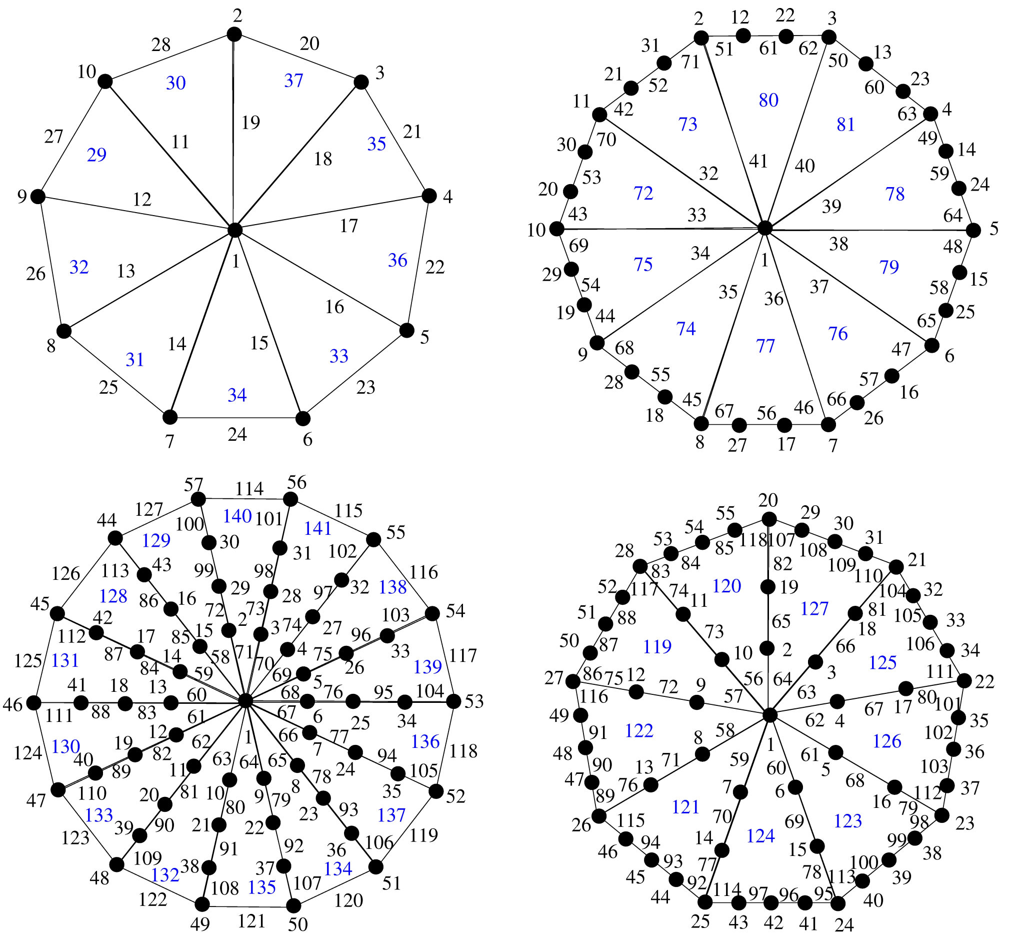

FIGURE 1. Some example of local super antimagic total face coloring on related wheel graph.

- The source of key system is taken from LATF of planar graphs
- The key has a length of all element on planar graphs such as vertex, edges and faces

4.1. **Generating Key Modern Cryptosystem Polyalphabetic.** We modify the LATF algorithm for constructing encryption key to generate a key modern cryptosystem polyalphabetic. This algorithm yields a sequence of labels taken from LATF which has same weight of faces. The sequence is then utilized as an encryption key. The mechanism is encrypt each block of plaintext based on same weight of faces. Let we are working on 26 English alphabets. The key stream construction is taken through the following algorithm.

Algorithm 1. **Generating key system**

1. Define f for labeling the graph elements
2. If f is bjection, do 3, otherwise back to 1
3. Take a certain b for chromatic number of LATF
4. Take z_ij is the sequence for label of vertex, edges, and face at the same weight face in a face which i = the number of face with the same weight and $1 \leq j \leq b$
5. Put z_{ij} and sort the sequence based on the smaller label of vertex, edges, and face.
6. Take k = element of sequence z_{ij}

Outputs of algorithm 1 are the key k. This labeling is illustrated in Figure 2. It shows that the vertex, edge, and face labels start from 1 to 7, 8 to 19, and 20 to 25, respectively.

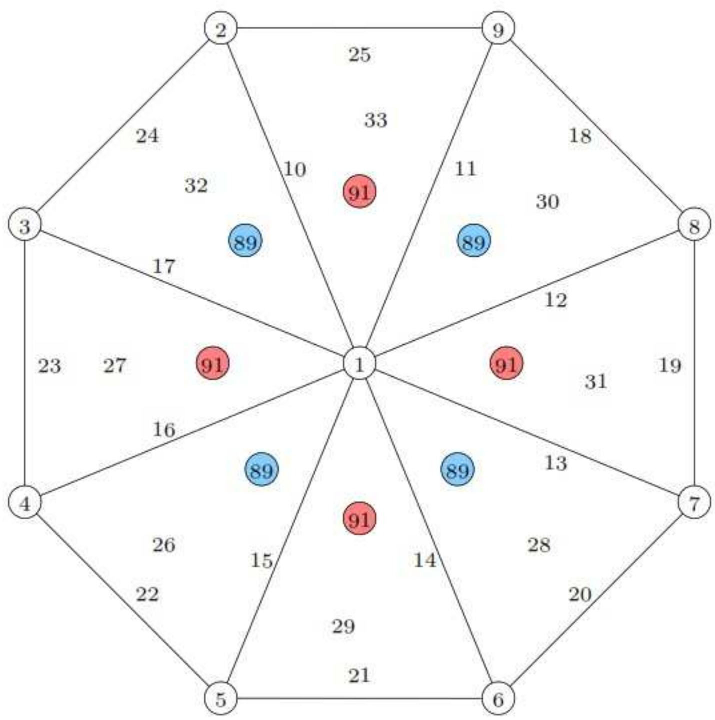

FIGURE 2. Local super antimagic total face labeling of W_8.

Based on illustrated in Figure 2, we have key for two blocks of plaintext, such as the keys for first block are the labels of elements on face with same weight (faces with blue color) and the keys for second block are the labels of elements on face with same weight (faces with red color).

5. Encryption Algorithm

The key modern cryptosystem polyalphabetic produced by the algorithm 1 is implemented to establish encryption in the mode of affine cipher and block cipher. The encryption process is undertaken using algorithm 2.

Algorithm 2. **Encryption in Affine and Block Cipher**

1. Let the plaintext $P = (p_i), 1 \leq i \leq h$
2. Devide P into blocks of the chromatic number of LATF d
3. For $i = 1$ to $\lceil \frac{h}{b} \rceil$, compute the ciphertext blocks using equation (5.1) and compute the plaintext blocks using equation (5.2).

$$C_n = (P_n + K_n) \; mod \; 26 \tag{5.1}$$

$$P_n = (C_n - K_n) \; mod \; 26 \tag{5.2}$$

where P_n, K_n, and C_n are the n-th block of plaintext, key sequence, and ciphertext, respectively. For $n = 1$, C_{n-1} is a null vector.

Table 1 and table 2 exhibits how the key stream obtained from the algorithm 1 is utilozed to encrypt the plaintext "UINJAKARTAHEBAT" and yields the ciphertext "VKWTLJHSUCKMSYZ". The decryption process can be done in the reverse direction.

6. Concluding Remarks

In this paper, we have given the result on the local super antimagic total face labeling of related wheel graphs. Furthermore, we apply this type of labeling to develop a modern algorithm of cryptosystem polyalphabetics, namely building an encryption key by using Affin Block cipher. By means this labeling, we ensure that the key will be completly hard to reveal by any introdur since the decryption process will pass the two layers, i.e. the graph labelings and the Affin Block cipher formula. However, to build a GUI system of this algorithm is

TABLE 1. Encryption Process

Block 1	Plaintext	U	I	N	J	A	K	A	R
	P_i	20	8	13	9	0	10	0	17
	K_i	1	2	9	10	11	25	33	1
	$P_i + K_i$	21	10	22	19	11	35	33	18
	C_i	21	10	22	19	11	9	7	18
	Ciphertext	V	K	W	T	L	J	H	S
Block 2	Plaintext	T	A	H	E	B	A	T	
	P_i	19	0	7	4	1	0	19	
	K_i	1	2	3	10	17	24	32	
	$P_i + K_i$	20	2	10	14	18	24	51	
	C_i	20	2	10	14	18	24	25	
	Ciphertex	U	C	K	M	S	Y	Z	

TABLE 2. Decryption Process

Block 1	Ciphertext	V	K	W	T	L	J	H	S
	C_i	21	10	22	19	11	9	7	18
	K_i	1	2	9	10	11	25	33	1
	$C_i - K_i$	20	8	13	9	0	-16	-26	17
	P_i	20	8	13	9	0	10	0	17
	Plaintext	U	I	N	J	A	K	A	R
Block 2	Ciphertex	U	C	K	M	S	Y	Z	
	C_i	20	2	10	14	18	24	25	
	K_i	1	2	3	10	17	24	32	
	$C_i - K_i$	19	0	7	4	1	0	-7	
	P_i	19	0	7	4	1	0	19	
	Plaintext	T	A	H	E	B	A	T	

not easy job. We need to do a further research, thus we propose the following open problems

Open Problem 6.1. *Develop a GUI software derived from the above algorithm.*

Open Problem 6.2. *Determine the local super antimagic total face labelings of other family of graph and develop the encryption key algorithm.*

Acknowledgement

We are gratefully acknowledge for a wonderful support from PUSLITPEN Universitas Islam Negeri Syarif Hidayatullah Jakarta and CGANT - University of Jember in 2019.

References

[1] I. H. Agustin, R. Alfarisi, Dafik, A. I. Kristiana, R. M. Prihandini, E. Y. Kurniawati: *On Super Local Antimagic Total Edge Coloring of Some Wheel Related Graphs*, AIP Conference Proceedings, **2018**, 2018.

[2] I. H. Agustin, Dafik, M. Hasan, R. Alfarisi, R. M. Prihandini: *Local Edge Antimagic Coloring of Graphs*, Far East Journal of Mathematical Sciences, **102**(9) (2017), 1925–1941.

[3] I. H. Agustin, M. Hasan, Dafik, R. Alfarisi, A. I. Kristiana, R. M. Prihandini: *Local Edge Antimagic Coloring of Comb Product of Graphs*, Journal of Physics: Conference Series, **1008** (2018), 1–10.

[4] S. Arumugam, K. Premalatha, M. Bača, A. Semaničová-Feňovčíková: *Local Antimagic Vertex Coloring of a Graph*, Graphs and Combinatorics, **33**(2) (2017), 275–285.

[5] Dafik, M. Hasan, Y. N. Azizah, I. H. Agustin: *A generalized shackle of any graph H admits a super H-antimagic total labeling*, The Asian Mathematical Conference 2016, **893** (2016), 1–9.

[6] Dafik, M. Miller, J. Ryan, M. Bača: *Super Edge-Antimagic Total Labelings of $mK_{n,n}$*, Ars Combinatoria -Waterloo then Winnipeg, **101**(97) (2011), 97–107.

[7] R. Nisviasari, Dafik, T. K. Maryati, I. H. Agustin, E. Y. Kurniawati: *Local Super Antimagic Total Face Coloring of Planar Graphs*, Earth and Environmental Science, **243** (2019), 1–11.

[8] A. C. Prihandoko, Dafik, I. H. Agustin, D. Susanto, A. I. Kristiana, Slamin: *The Construction of Encryption Key by Using a Super H-antimagic Total Graph*, Program and Abstract the Asian Mathematical Conference AMC, **408** (2016) ISBN 978-602-74668-0-7.

[9] A. C. Prihandoko, Dafik, I. H. Agustin: *Implementation of Super H-antimagic Total Graph on Establishing Stream Cipher*, Indonesian Journal of Combinatorics, **3**(1) (2019), 14–23.

[10] R. M. Prihandini, I. H. Agustin, Dafik, R. Alfarisi: *Super complete-antimagicness of amalgamation of any graph*, International Journal of Advanced Engeneering Research and Science, **4**(5) (2017), 202–206.

[11] R. M. PRIHANDINI, I. H. AGUSTIN, DAFIK: *Chipertext Stream Construction by using Super Total Labeling (a,d)-$P_2 \triangleright H$-Antimagic of Comb Product Graph*, Journal of Physics: Conference Series, **1211**, 2019.

MATHEMATICS EDUCATION DEPARTMENT
UNIVERSITAS ISLAM NEGERI SYARIF HIDAYATULLAH JAKARTA
INDONESIA
E-mail address: tita.khalis@uinjkt.ac.id

MATHEMATICS EDUCATION DEPARTMENT
UNIVERSITAS ISLAM NEGERI SYARIF HIDAYATULLAH JAKARTA
INDONESIA
E-mail address: khamida.siti@uinjkt.ac.id

CGANT-UNIVERSITY OF JEMBER AND DEPARTMENT OF MATHEMATICS
UNIVERSITY OF JEMBER
KALIMANTAN NO 37 JEMBER, EAST JAVA, INDONESIA
E-mail address: nisviasari@gmail.com

CGANT-UNIVERSITY OF JEMBER AND DEPARTMENT OF MATHEMATICS
UNIVERSITY OF JEMBER
KALIMANTAN NO 37 JEMBER, EAST JAVA, INDONESIA
E-mail address: ikahesti.fmipa@unej.ac.id

CGANT-UNIVERSITY OF JEMBER AND DEPARTMENT OF MATHEMATICS EDUCATION
UNIVERSITY OF JEMBER
KALIMANTAN NO 37 JEMBER, EAST JAVA, INDONESIA
E-mail address: d.dafik@unej.ac.id

DEPARTMENT OF MATHEMATICS
KONGUNADU ARTS AND SCIENCE COLLEGE
TAMIL NADU, INDIA
E-mail address: venkatmaths@gmail.com