

THE ULAM STABILITY OF FUNCTIONAL EQUATION IN MATRIX NORMED SPACES

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ABSTRACT. This paper focuses on the stability results of quattuordecic functional equation in matrix normed spaces with the help of direct method and fixed point method and also we give an example for non-stability.

1. INTRODUCTION

An beguiling talk delivered by Ulam [14] started the study of stability problems of various functional equations in 1940. He raised a question relating to the stability of homomorphism. In 1941, Hyers [5] gave a partial solution to Ulam's question. The Hyers results was generalized by Aoki [1] for additive mappings in 1950. Further Hyers result was generalized by Th. M. Rassias [8] in 1978 for linear mapping. In 1994, Th. M. Rassias result was generalized by P. Gavruta [4] for a general control function. His result is called Generalized Hyers-Ulam-Rassias stability. This type of stability results for several functional equations [2, 9–11, 13, 16] is investigated by a number of authors. In 2016, K. Ravi et al. [12] introduced the following quattuordecic functional equation

$$\begin{aligned} & f(x+7y) - 14f(x+6y) + 1001f(x+3y) - 364f(x+4y) + 91f(x+5y) \\ & + 1001f(x-3y) + f(x-7y) - 2002f(x+2y) - 3432f(x) \\ (1.1) \quad & + 3003f(x+y) + 3003f(x-y) - 364f(x-4y) \\ & - 2002f(x-2y) - 14f(x-6y) + 91f(x-5y) = 14!f(y), \end{aligned}$$

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where $14! = 87178291200$ in quasi- β normed spaces. In the last five years, some authors have established the stability of different types of functional equations in matrix normed spaces ([6], [15]). Being encouraged by the above result, we mainly aim at studying the stability results of quattuordecic functional equation (1.1) in matrix normed spaces by using both methods (direct and fixed point).

Through out this paper, we consider $(X, \|\cdot\|_n)$ as matrix normed spaces and $(Y, \|\cdot\|_n)$ as matrix Banach spaces. Here, n be a fixed non-negative integer.

2. STABILITY OF QUATTUORDECIC FUNCTIONAL EQUATION: FIXED POINT METHOD AND DIRECT METHOD.

We use the following abbreviation for a given mapping $f : X \rightarrow Y$, we define an operator $\mathcal{D}f : X^2 \rightarrow Y$ and $\mathcal{D}f_n : M_n(X)^2 \rightarrow M_n(Y)$ by

$$\begin{aligned} \mathcal{D}f(b, c) &= f(b + 7c) - 14f(b + 6c) + 91f(b + 5c) - 364f(b + 4c) + 1001f(b + 3c) \\ &\quad - 2002f(b + 2c) + 3003f(b + c) - 3432f(b) + 3003f(b - c) - 14f(b - 6c) \\ &\quad - 2002f(b - 2c) + 1001f(b - 3c) - 364f(b - 4c) + 91f(b - 5c) + f(b - 7c) - 14!f(c), \\ \mathcal{D}f_n([x_{rs}], [y_{rs}]) &= f_n([x_{rs} + 7y_{rs}]) - 14f_n([x_{rs} + 6y_{rs}]) + 91f_n([x_{rs} + 5y_{rs}]) \\ &\quad - 364f_n([x_{rs} + 4y_{rs}]) + 1001f_n([x_{rs} + 3y_{rs}]) - 2002f_n([x_{rs} + 2y_{rs}]) \\ &\quad - 3432f_n([x_{rs}]) + 3003f_n([x_{rs} - y_{rs}]) - 2002f_n([x_{rs} - 2y_{rs}]) \\ &\quad + 1001f_n([x_{rs} - 3y_{rs}]) + 3003f_n([x_{rs} + y_{rs}]) - 364f_n([x_{rs} - 4y_{rs}]) \\ &\quad + 91f_n([x_{rs} - 5y_{rs}]) - 14f_n([x_{rs} - 6y_{rs}]) + f_n([x_{rs} - 7y_{rs}]) - 14!f_n([y_{rs}]) \\ \forall b, c \in X \text{ and all } x = [x_{rs}], y = [y_{rs}] \in M_n(X). \end{aligned}$$

Theorem 2.1. Let $l \in \{1, -1\}$ be fixed and let δ be a real number with $0 < \delta < 1$. Suppose that the mapping $\zeta : X^2 \rightarrow [0, \infty)$ satisfies the inequality

$$(2.1) \quad \zeta(b, c) \leq 2^{14l} \delta \zeta\left(\frac{b}{2^l}, \frac{c}{2^l}\right),$$

for all $b, c \in X$. Let $f : X \rightarrow Y$ be an even mapping satisfying $f(0) = 0$ and

$$(2.2) \quad \|\mathcal{G}f_n([x_{rs}], [y_{rs}])\| \leq \sum_{r,s=1}^n \zeta(x_{rs}, y_{rs})$$

for all $x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there is a unique quattuordecic function $\mathcal{Q} : X \rightarrow Y$ such that

$$(2.3) \quad \|f_n([x_{rs}]) - \mathcal{Q}_n([x_{rs}])\|_n \leq \sum_{i,j=1}^n \frac{\delta^{\frac{1-l}{2}}}{2^{14}(1-\delta)} \zeta^*(x_{rs})$$

$\forall x = [x_{rs}] \in M_n(X)$, where

$$\begin{aligned}\zeta^*(x_{rs}) = & \frac{2}{14!} \left[\frac{1}{2} \zeta(0, 2x_{rs}) + \zeta(7x_{rs}, x_{rs}) + 14\zeta(6x_{rs}, x_{rs}) \right. \\ & + 91\zeta(5x_{rs}, x_{rs}) + 364\zeta(4x_{rs}, x_{rs}) + 1001\zeta(3x_{rs}, x_{rs}) \cdot \\ & \left. + 2002\zeta(2x_{rs}, x_{rs}) + 3003\zeta(x_{rs}, x_{rs}) + 1716\zeta(0, x_{rs}) \right]\end{aligned}$$

Proof. Putting $n = 1$ in inequality (2.2), we obtain

$$(2.4) \quad \|\mathcal{D}f(b, c)\| \leq \zeta(b, c)$$

$\forall b, c \in X$. By utilizing ([12], Theorem 1), we have

$$\begin{aligned}(2.5) \quad \|-f(2b) + 16384f(b)\| \leq & \frac{2}{14!} \left[\frac{1}{2} \zeta(0, 2b) + \zeta(7b, b) + 14\zeta(6b, b) + 91\zeta(5b, b) \right. \\ & \left. + 364\zeta(4b, b) + 1001\zeta(3b, b) + 2002\zeta(2b, b) + 3003\zeta(b, b) + 1716\zeta(0, b) \right]\end{aligned}$$

for all $b \in X$. For case $l = 1$ and $l = -1$, then we see that

$$(2.6) \quad \left\| f(b) - \frac{1}{2^{14l}} f(2^l b) \right\| \leq \frac{\delta^{(\frac{1-l}{2})}}{2^{14}} \zeta^*(b)$$

$\forall b \in X$. The generalized metric ρ defined on $\mathcal{N}$ by

$$\rho(g, h) = \inf \{ \mu \in \mathcal{R}_+ : \|g(b) - h(b)\| \leq \mu \zeta^*(b), \forall b \in X \},$$

where $\mathcal{N} = \{g : X \rightarrow Y\}$. Then it is simply to verify that $(\mathcal{N}, \rho)$ is a generalized complete metric space (see [7]).

Let us define $\mathcal{T} : \mathcal{N} \rightarrow \mathcal{N}$ by $\mathcal{T}g(b) = \frac{1}{2^{14l}} g(2^l b)$ for all $b \in X$.

Given $g, h \in \mathcal{N}$, let $\mu \in [0, \infty]$ is an arbitrary constant with $\rho(g, h) \leq \mu$. From the definition, we have $\|g(b) - h(b)\| \leq \mu \zeta^*(b) \forall b \in X$. Therefore,

$$\|\mathcal{T}g(b) - \mathcal{T}h(b)\| = \left\| \frac{1}{2^{14l}} g(2^l b) - \frac{1}{2^{14l}} h(2^l b) \right\| \leq \delta \mu \zeta^*(b).$$

Hence, $\rho(\mathcal{T}g, \mathcal{T}h) \leq \delta \mu \leq \delta \rho(g, h)$ for all $g, h \in \mathcal{N}$. Thus, $\mathcal{T}$ is a strictly contractive operator on $\mathcal{N}$ with $L = \delta$. By (2.6), we get

$$\rho(f, \mathcal{T}f) \leq \frac{\delta^{(\frac{1-l}{2})}}{2^{14}}.$$

Applying ([3], Theorem 2.2), we obtain the existence of a fixed point of $\mathcal{T}$, that is, the existence of a function $\mathcal{Q}$ satisfies $\mathcal{Q}(2^l b) = 2^{14l} \mathcal{Q}(b) \forall b \in X$. Moreover, $\rho(\mathcal{T}^k f, \mathcal{Q}) \rightarrow 0$, which implies $\mathcal{Q}(b) = \lim_{k \rightarrow \infty} \mathcal{T}^k f(b) = \lim_{k \rightarrow \infty} \frac{1}{2^{14kl}} f(2^{kl} b)$ for all $b \in X$.

Also, $\rho(f, \mathcal{Q}) \leq \frac{1}{1-L} \rho(f, \mathcal{T}f)$ implies the following inequality

$$(2.7) \quad \|f(b) - \mathcal{Q}(b)\| \leq \frac{\delta^{\frac{1-l}{2}}}{2^{14}(1-\delta)} \zeta^*(b) \quad \forall b \in X.$$

From (2.1) and (2.4), we have $\|\mathcal{D}\mathcal{Q}(b, c)\| = \lim_{k \rightarrow \infty} \frac{1}{2^{14kl}} \|\mathcal{D}f(2^{kl}b, 2^{kl}c)\| = 0$ for all $b, c \in X$. By using ([6], Lemma 2.1) and (2.7), we arrive at (2.3). Hence, $\mathcal{Q}$ is a quattuordecic function satisfying (2.3). $\square$

Theorem 2.2. Let $l = \{1, -1\}$ and let $\zeta : X^2 \rightarrow [0, \infty)$ be a function satisfies

$$\sum_{m=0}^{\infty} \frac{\zeta(2^{ml}b, 2^{ml}c)}{2^{14ml}} < +\infty$$

and $\lim_{m \rightarrow \infty} \frac{\zeta(2^{ml}b, 2^{ml}c)}{2^{14ml}} = 0$ for all $b, c \in X$. Let $f : X \rightarrow Y$ be a mapping satisfies (2.2). Then there is a unique quattuordecic function $\mathcal{Q} : X \rightarrow Y$ such that

$$(2.8) \quad \|f_n([x_{rs}]) - \mathcal{Q}_n([x_{rs}])\|_n \leq \sum_{i,j=1}^n \frac{1}{2^{14}} \left(\sum_{m=\lfloor \frac{1-l}{2} \rfloor}^{\infty} \frac{\zeta^*(2^{ml}x_{rs})}{2^{14ml}} \right)$$

for all $x = [x_{rs}] \in M_n(X)$, where

$$\begin{aligned} \zeta^*(2^{ml}x_{rs}) = & \frac{1}{14!} [\zeta(0, 2 \cdot 2^{ml}x_{rs}) + \zeta(8 \cdot 2^{ml}x_{rs}, 2^{ml}x_{rs}) + 14\zeta(7 \cdot 2^{ml}x_{rs}, 2^{ml}x_{rs}) \\ & + 11428\zeta(0, 2^{ml}x_{rs}) + 470\zeta(5 \cdot 2^{ml}x_{rs}, 2^{ml}x_{rs}) + 1470\zeta(4 \cdot 2^{ml}x_{rs}, 2^{ml}x_{rs}) \\ & + 3458\zeta(3 \cdot 2^{ml}x_{rs}, 2^{ml}x_{rs}) + 6370\zeta(2 \cdot 2^{ml}x_{rs}, 2^{ml}x_{rs}) \\ & + 9428\zeta(2^{ml}x_{rs}, 2^{ml}x_{rs}) + 106\zeta(6 \cdot 2^{ml}x_{rs}, 2^{ml}x_{rs})]. \end{aligned}$$

Proof. From (2.5)

$$(2.9) \quad \left\| f(b) - \frac{1}{2^{14}} f(2b) \right\| \leq \frac{1}{2^{14}} \zeta^*(b)$$

$\forall b \in X$. Considering b and $2b$ in (2.9) and divide it by 2^{14} , we get

$$(2.10) \quad \left\| \frac{f(2^2b)}{2^{28}} - \frac{f(2b)}{2^{14}} \right\| \leq \frac{1}{2^{28}} \zeta^*(2b)$$

for all $b \in X$. Merging (2.9) and (2.10), we obtain

$$\left\| \frac{f(2^2b)}{2^{28}} - f(b) \right\| \leq \frac{1}{2^{14}} [\zeta^*(b) + \frac{1}{2^{14}} \zeta^*(2b)].$$

For case $l = 1$ and $l = -1$, By induction on any positive integer q , we have

$$(2.11) \quad \left\| \frac{f(2^{lq}b)}{2^{14lq}} - f(b) \right\| \leq \frac{1}{2^{14}} \sum_{m=(\frac{1-l}{2})}^{q-1} \frac{\zeta^*(2^{ml}b)}{2^{14ml}}.$$

To show that the sequence $\frac{f(2^{lq}b)}{2^{14lq}}$ is convergence. Putting $b = 2^{ml}b$ in (2.11) and divide it by 2^{14ml} , for any positive integers q and m , we get

$$\begin{aligned} \left\| \frac{f(2^{(q+m)l}b)}{2^{14l(q+m)}} - \frac{f(2^{ml}b)}{2^{14ml}} \right\| &= \frac{1}{2^{14ml}} \left\| \frac{f(2^{lq}2^{ml}b)}{2^{14l(q+m)}} - f(2^{ml}b) \right\| \\ &\leq \frac{1}{2^{14ml}} \frac{1}{2^{14}} \sum_{i=0}^{q-1} \frac{\zeta^*(2^i 2^{ml}b)}{2^{14i}} \rightarrow 0, \end{aligned}$$

as $m \rightarrow \infty$ for all $b \in X$. Therefore, the sequence $\frac{f(2^{lq}b)}{2^{14lq}}$ is Cauchy in Y and so it converges. Therefore, the mapping $\mathcal{Q}$ is defined by $\mathcal{Q}(b) = \lim_{q \rightarrow \infty} \frac{f(2^{lq}b)}{2^{14lq}}$ for all $b \in X$. It follows from the above equation and (2.4) that

$$\|\mathcal{D}\mathcal{Q}(b, c)\| = \lim_{k \rightarrow \infty} \frac{1}{2^{14kl}} \|\mathcal{D}f(2^{kl}b, 2^{kl}c)\| \leq \lim_{k \rightarrow \infty} \frac{1}{2^{14kl}} \zeta(2^{kl}b, 2^{kl}c) = 0$$

for all $b, c \in X$. So, the mapping $\mathcal{Q}$ is quattuordecic. Hence $\mathcal{Q}$ satisfies (1.1). By Lemma 2.1 in [6] and (2.11), we can get (2.8). Hence $\mathcal{Q} : X \rightarrow Y$ is a unique quattuordecic function satisfying (2.8). $\square$

Corollary 2.1. Let $l = \{1, -1\}$ be fixed and let $t, \nu \in \mathcal{R}_+$ with $t \neq 14$. Suppose that $f : X \rightarrow Y$ is an even mapping such that

$$\|\mathcal{D}f_n([x_{rs}], [y_{rs}])\|_n \leq \sum_{r,s=1}^n \nu(\|x_{rs}\|^t + \|y_{rs}\|^t),$$

for all $x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there is a unique quattuordecic function $\mathcal{Q} : X \rightarrow Y$ such that

$$\|f_n([x_{rs}]) - \mathcal{Q}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^n \frac{\nu_0}{|2^{14} - 2^t|} \|x_{rs}\|^t$$

for all $x = [x_{rs}] \in M_n(X)$, where $\nu_0 = \frac{2\nu}{14!} [11195 + 2002.5(2^t) + 1001(3^t) + 364(4^t) + 91(5^t) + 14(6^t) + 7^t]$.

Proof. The proof is identical to that of Theorem 2.1 and Theorem 2.2. $\square$

Corollary 2.2. Let $l = \{1, -1\}$ be fixed and let $t, \nu \in \mathcal{R}_+$ with $t = d + e \neq 14$. Suppose that $f : X \rightarrow Y$ is an even mapping such that

$$\|\mathcal{D}f_n([x_{rs}], [y_{rs}])\|_n \leq \sum_{r,s=1}^n \nu(\|x_{rs}\|^d \cdot \|y_{rs}\|^e)$$

for all $x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there is a unique quattuordecic function $\mathcal{Q} : X \rightarrow Y$ such that

$$\|f_n([x_{rs}]) - \mathcal{Q}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^n \frac{\nu_1}{|2^{14} - 2^t|} \|x_{rs}\|^t$$

for all $x = [x_{rs}] \in M_n(X)$, where $\nu_1 = \frac{2\nu}{14!} [3003 + 2002(2^d) + 1001(3^d) + 364(4^d) + 91(5^d) + 14(6^d) + 7^d]$

Proof. The proof is identical to that of Theorem 2.1 and Theorem 2.2. $\square$

Corollary 2.3. Let $l = \{1, -1\}$ be fixed and let $t, \nu \in \mathcal{R}_+$ with $t = d + e \neq 14$. Suppose that $f : X \rightarrow Y$ is an even mapping such that

$$\|\mathcal{D}f_n([x_{rs}], [y_{rs}])\|_n \leq \sum_{r,s=1}^n \nu(\|x_{rs}\|^d \cdot \|y_{rs}\|^e + \|x_{rs}\|^t + \|y_{rs}\|^t)$$

for all $x = [x_{rs}], y = [y_{rs}] \in M_n(X)$. Then there is a unique quattuordecic function $\mathcal{Q} : X \rightarrow Y$ such that

$$\|f_n([x_{rs}]) - \mathcal{Q}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^n \frac{\nu_2}{|2^{14} - 2^t|} \|x_{rs}\|^t$$

for all $x = [x_{rs}] \in M_n(X)$, where

$$\begin{aligned} \nu_2 = \frac{2\nu}{14!} [14198 + 2002.5(2^t) + 2002(2^d) + 1001(3^t + 3^d) + 364(4^t + 4^d) \\ + 91(5^t + 5^d) + 14(6^t + 6^d) + 7^t + 7^d]. \end{aligned}$$

Proof. The proof is identical to that of Theorem 2.1 and Theorem 2.2. $\square$

3. COUNTER-EXAMPLE

Next we will check that the functional equation (1.1) is not stable for $t = 14$ by taking $n = 1$ in Corollary 2.1.

Example 1. Let $f : \mathcal{R} \rightarrow \mathcal{R}$ be a function and let $\zeta : \mathcal{R} \rightarrow \mathcal{R}$ be a function for some constant $\nu > 0$ defined by $\zeta(b) = \nu b^{14}$, if $|b| < 1$, $\zeta(b) = \nu$, if $|b| \geq 1$ and $f(b) = \sum_{n=0}^{\infty} \frac{\zeta(2^n x)}{2^{14n}}$ for all $b \in \mathcal{R}$. Then f satisfies the following inequality

$$(3.1) \quad |\mathcal{G}f(b, c)| \leq \frac{(87178307580)}{16383} (16384)^2 \nu (|b|^{14} + |c|^{14}) \quad \text{for all } b, c \in \mathcal{R}.$$

Proof. Obviously, f is bounded by $\frac{16384\nu}{16383}$ on $\mathcal{R}$. Next we have to verify that the function f satisfies (3.1). If $|b|^{14} + |c|^{14} = 0$ or $|b|^{14} + |c|^{14} \geq \frac{1}{2^{14}}$, then

$$|\mathcal{G}f(b, c)| \leq \frac{(87178307580)}{16383} (16384).$$

Suppose that $0 < |b|^{14} + |c|^{14} < \frac{1}{2^{14}}$, then there is a non-negative integer k , $\frac{1}{2^{14(k+1)}} \leq |b|^{14} + |c|^{14} < \frac{1}{2^{14k}}$, so that $2^{14(k-1)}b^{14} < \frac{1}{2^{14}}$, $2^{14(k-1)}c^{14} < \frac{1}{2^{14}}$ and $2^n(b), 2^n(c), 2^n(b \pm 7c), 2^n(b \pm 6c), 2^n(b \pm 5c), 2^n(b \pm 4c), 2^n(b \pm 3c), 2^n(b \pm 2c), 2^n(b \pm c) \in (-1, 1)$. Hence, $\mathcal{G}\psi(2^n b, 2^n c) = 0$ for $n = 0, 1, 2, \dots, k-1$. Since $\frac{1}{2^{14(k+1)}} \leq |b|^{14} + |c|^{14} < \frac{1}{2^{14k}}$, thus

$$|\mathcal{G}f(b, c)| \leq \sum_{n=0}^{\infty} \frac{1}{2^{14n}} |\mathcal{G}\psi(2^n b, 2^n c)| \leq \frac{(87178307580)}{16383} (16384)^2 \nu (|b|^{14} + |c|^{14}).$$

Therefore, f satisfies (3.1) for all $b, c \in \mathcal{R}$. □

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