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SSP STRUCTURE OF COPIES OF CYCLES

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ABSTRACT. A graph G is Super Strongly Perfect (SSP) if every induced sub graph H of G possesses a minimal dominating set that meets all the maximal cliques of H. The structure of super strongly perfect graphs have been investigated by some classes of graphs like complete graphs, complete bipartite graphs, trees etc., In this paper, web graphs are analysed by SSP. Also, it is found the cardinality of minimal dominating set, number of maximal cliques, colouring number and regularity of web graphs.

1. INTRODUCTION

Here, graphs are finite and simple. A set $M \subseteq V(G)$ is called a *dominating set* if each vertex in $V - M$ is connected to at least one vertex in M. A set $N \subseteq V$ is said to be a *minimal dominating set* if $N - \{u\}$ is not a dominating set for any $u \in N$.

2. OVERVIEW OF THE PAPER

It has been investigated SSP graph's structure in cycle based graphs, like circulant graphs, wheel graphs, etc., [1, 2]. In this paper, it is discussed the concentric copies of cyclic graphs like web graphs.

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2.1. **Super Strongly Perfect Graph (SSPG).** A graph, if every induced subgraph H of G contains a minimal dominating set which covers all the maximal cliques of H then G is SSP. $SSPG$ and non$-SSPG$ are given in figures 1 and 2.

Illustration 1.

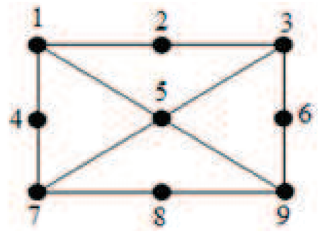

FIGURE 1. SSPG

Illustration 2.

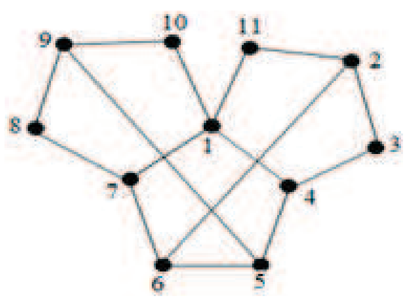

FIGURE 2. Non-SSPG

Theorem 2.1. *If G has a cycle odd length atleast five as an induced subgraph, then G is non $-SSP$.*

Theorem 2.2. *If a graph G has atleast one $K_n, n = 2, 3, ...,$ then G is $n-$ colourable iff it is SSP.*

3. Web Graph

A Web graph $W_{n,r}$ is a graph consisting of r concentric copies of the cycle graph C_n with corresponding vertices connected by spokes. It has nr vertices and $n(2r-1)$ edges [3]. Web graph, $W_{3,3}$ is illustrated in figure 3.

Illustration 3.

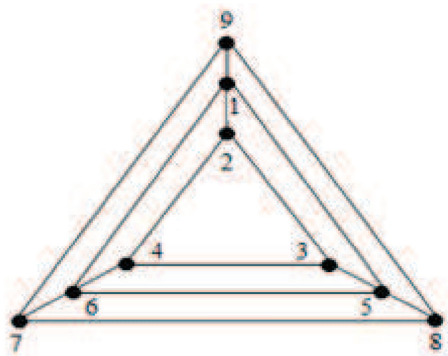

Figure 3. $W_{3,3}$

Theorem 3.1. *Every web graph $W_{3,r}, r \in N$ is SSP.*

Proof. Consider a web graph $W_{3,r}, r \in N$.
$\Rightarrow$ The selection of a vertex from each induced subgraph C_3 will intersect all K_3.
$\Rightarrow$ G is $SSPG$. □

Theorem 3.2. *Every web graph $W_{n,r}, r \in N, n \geq 4$, n is even, is SSP.*

Proof. Consider a web graph $W_{n,r}, r \in N$, $n \geq 4$, n is even.
$\Rightarrow$ G is bipartite.
$\Rightarrow$ G is SSPG. □

Theorem 3.3. *Every web graph $W_{n,r}, r \in N, n \geq 5$, n is odd, is non-SSP.*

Proof. Consider a web graph $W_{n,r}, r \in N, n \geq 5$, n is odd.
$\Rightarrow$ As n is odd, G is non- SSPG (theorem 2.1). □

Proposition 3.1. *Consider a web graph $W_{3,r}, r \in N$.*

(1) *It contains rK_3.*

(2) *It induces a minimal dominating set of r elements.*
(3) *It has colourability* 3.
(4) $W_{3,r}$ *is* $r+1$ *regular if* $r = 1, 2$.
(5) $W_{3,r}$ *is non-regular if* $r \geq 3$.

Proof.

(1) Consider a web graph $W_{3,r}, r \in N$.
$\Rightarrow G$ consists of r concentric copies of C_3 which is K_3.
$\Rightarrow G$ has r maximal cliques K_3.

(2) Consider a web graph $W_{3,r}, r \in N$.
$\Rightarrow G$ consists of r concentric copies of C_3.
$\Rightarrow$ From each copy of C_3, select one vertex to meet C_3.
Since G has r copies of C_3, the proof is done.
(3) Consider a web graph $W_{3,r}, r \in N$.
By part (1), G contains rK_3, the proof is done (theorem 2.2).
(4) Consider a web graph $W_{3,r}, r \in N,\ r = 1, r = 2$.
Case (1)
If $r = 1$, then G is a cycle graph C_3 which is $2-$ regular.
$\Rightarrow G$ is $2-$ regular.
$\Rightarrow G$ is $r+1$ regular.
Case (2)
If $r = 2$, then each vertex of G has third degree.
$\Rightarrow G$ is $3-$ regular.
$\Rightarrow G$ is $r+1$ regular.
$\Rightarrow$ in both cases, G is $r+1$ regular.
(5) Consider a web graph $W_{3,r}, r \in N, r \geq 3$.
$\Rightarrow$ In G, the vertices of the inner and outer cycle C_3 have third degree and all the remaining vertices have fourth degree.
$\Rightarrow G$ is non-regular.

□

This proposition is demonstrated below.

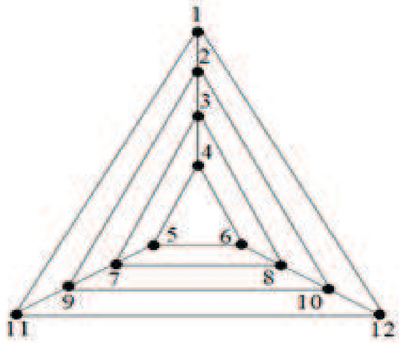

FIGURE 4. $W_{3,4}$

Here,

(1) $W_{3,4}$ contains 4 maximal cliques K_3.
(2) $W_{3,4}$ inducess a minimal dominating set of 4 elements.
(3) $W_{3,4}$ is $3-$ colourable.
(4) $W_{3,4}$ is non-regular.

Proposition 3.2. *Consider a web graph $W_{n,r}, r \in N, n \geq 4$, n is even.*

(1) *It contains n times $(2r-1)$ maximal cliques K_2.*
(2) *It contains a minimal dominating set of $\frac{nr}{2}$ elements.*
(3) *It has colourability 2.*
(4) *It is $r+1-$ regular if $r = 1, 2$.*
(5) *It is non-regular if $r \geq 3$.*

Proof.

(1) Consider a web graph $W_{n,r}, r \in N, n \geq 4$ and n is even.
$\Rightarrow$ As G has r copies of C_n, there exist nr edges (cycle C_n has n edges). Since the corresponding vertices of the cycle are connected by the spokes, there exist $n(r-1)$ edges.
$\Rightarrow$ G has $nr + n(r-1) = nr + nr - n = 2nr - n = n(2r-1)$ maximal cliques, each of which is a K_2.
Hence the proof.
(2) Consider a web graph $W_{n,r}, r \in N, n \geq 4$ and n is even.
$\Rightarrow$ G has nr vertices which is bipartite.
$\Rightarrow$ G has a minimal dominating set of $\frac{nr}{2}$ elements.

(3) Consider a web graph $W_{n,r}, r \in N, n \geq 4$ and n is even.
By part (1), G contains rK_2, the proof is done (theorem 2.2).

(4) Consider a web graph $W_{n,r}, r \in N, n \geq 4$ and n is even, $r = 1, 2$.
Case (1) If $r = 1$, then G is an even cycle graph C_n which is 2-regular.
$\Rightarrow G$ is 2- regular.
$\Rightarrow G$ is $r + 1$ regular.
Case (2) If $r = 2$, each vertex of G has degree 3. $\Rightarrow G$ is 3 regular.
$\Rightarrow G$ is $r + 1$ regular.
Hence in both cases, G is $r + 1$ regular.

(5) Consider a web graph $W_{n,r}, r \in N, n \geq 4$ and n is even, $r \geq 3$.
$\Rightarrow$ In G, the vertex of the inner and outer cycle C_n have third degree and all the remaining vertices have fouth degree.
$\Rightarrow G$ is non-regular.

□

This proposition is demonstrated in figure 5. In this graph,

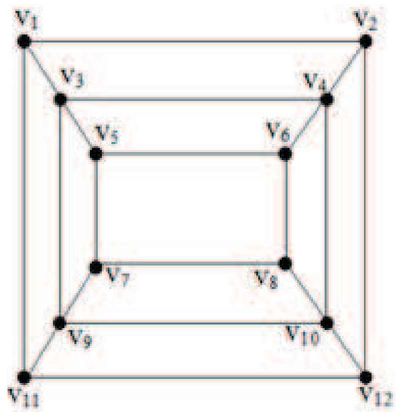

FIGURE 5. $W_{4,3}$

(1) $W_{4,3}$ contains 20 maximal cliques K_4.
(2) $W_{4,3}$ contains a minimal dominating set of 6 vertices.
(3) $W_{4,3}$ is 2-colourable.
(4) $W_{4,3}$ is non-regular.

4. Conclusion

The cyclic structure of web graphs are analysed here, using super strongly perfect graph's structure. Also, the SSP parameters of web graphs are given.

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