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ON EXTENSION OF PRIME RADICAL IN 2-PRIMAL NEAR-RINGS

B. ELAVARASAN, K. PORSELVI [1], AND J. CATHERINE GRACE JOHN

ABSTRACT. In this paper, we study some characterizations of prime radical in 2-primal near-ring and introduce the notions of $\mathscr{P}_{\mathscr{N}}$-Baer ideals and strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideals in 2-primal near-ring. Some equivalent conditions are established for $\mathscr{P}_{\mathscr{N}}$-Baer ideal to be a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal.

1. PRELIMINARIES

Throughout this paper, $\mathscr{N}$ denotes a zero symmetric right near-ring and all prime ideals are assumed to be proper in $\mathscr{N}$. For any undefined concepts and notations, we refer to Pilz [6]. Let $\mathscr{P}_{\mathscr{N}}$ denote the prime radical of $\mathscr{N}$, for any ideal L of $\mathscr{N}$, $P(L)$ denote the prime radical of L and $\mathscr{N}(\mathscr{N})$ the set of nilpotent elements of $\mathscr{N}$. An ideal P_r of $\mathscr{N}$ is prime if for any ideals U, V of $\mathscr{N}$, $UV \subseteq P_r$ implies $U \subseteq P_r$ or $V \subseteq P_r$. An ideal M of $\mathscr{N}$ is semiprime ideal if for an ideal K of $\mathscr{N}$, $K^2 \subseteq M$ implies $K \subseteq M$. An ideal J of $\mathscr{N}$ is completely prime if for any $u', v' \in \mathscr{N}$, $u'v' \in J$ implies either $u' \in J$ or $v' \in J$. An ideal J of $\mathscr{N}$ is completely semiprime if for any $u \in \mathscr{N}$, $u^2 \in J$ implies $u \in J$. For any non-empty subsets R, S of $\mathscr{N}$, we denote the set $\{n \in \mathscr{N} \ : \ nS \subseteq R\}$ as $< R : S >$. For every ideal Q_i and $K \subseteq \mathscr{N}$, $< Q_i : K >$ is maximal element among $\{< Q_i : Q_1 > : \ Q_1 \subseteq \mathscr{N}, \ < Q_i : Q_1 > \neq \mathscr{N}\}$ if and only if $< Q_i : K > \neq \mathscr{N}$ and $< Q_i : K > \subseteq < Q_i : T > \neq \mathscr{N}$ implies that $< Q_i : K > = < Q_i : T >$ for any subset T of $\mathscr{N}$ [3]. If $< I_1 : K_1 >$ is the maximal element among

[1] *corresponding author*

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$\{< I_1 : K_1 >: K_1 \subseteq \mathscr{N}, < I_1 : K_1 >\neq \mathscr{N}\}$, then there is $c_1 \in \mathscr{N}$ with $c_1 \notin < I_1 : K_1 >$ which implies $c_1 k_1 \notin I_1$ and $< I_1 : K_1 >\subseteq< I_1 : k_1 >\neq \mathscr{N}$ for some $k_1 \in K_1 \backslash I_1$. So $< I_1 : K_1 >=< I_1 : k_1 >$. If $\mathscr{N}$ has only one nilpotent element 0, then $\mathscr{N}$ is reduced. If $\mathscr{P}_{\mathscr{N}} = \mathscr{N}(\mathscr{N})$, then $\mathscr{N}$ is called 2-primal, see [1]. Clearly, every reduced near-ring is 2-primal, but 2-primal near-rings are not necessarily to be reduced, see Example 1.1 of [4]. If $\mathscr{N}$ is 2-primal, then $\mathscr{P}_{\mathscr{N}}$ is a completely semiprime ideal.

In [7], T. P. Speed has introduced the notion of Baer ideals in a commutative baer ring and later in [5], C. Jayaram has generalized baer ideals to a commutative semiprime ring and investigated properties of baer rings, regular rings and quasiregular rings by using baer ideals.

Following [5], an ideal J of $\mathscr{N}$ with $\mathscr{P}_{\mathscr{N}} \subseteq J$ is $\mathscr{P}_{\mathscr{N}}$-Baer ideal if $x \in J$ implies $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : x >>\subseteq J$. Also, an ideal J of $\mathscr{N}$ with $\mathscr{P}_{\mathscr{N}} \subseteq J$ is strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal if for any $a_1, b_1, c_1 \in \mathscr{N}$, $< \mathscr{P}_{\mathscr{N}} : a_1 > \cap < \mathscr{P}_{\mathscr{N}} : b_1 >=< \mathscr{P}_{\mathscr{N}} : c_1 >$ and $a_1, b_1 \in J$ imply $c_1 \in J$. A subset $M(\neq \phi)$ of $\mathscr{N}$ is a multiplicative subset if $0 \notin M$ and for $a_1, b_1 \in M$ implies $a_1 b_1 \in M$. Let $D = \{c_1 \in \mathscr{N} \ : < \mathscr{P}_{\mathscr{N}} : c_1 >= \mathscr{P}_{\mathscr{N}}\}$. Then $\mathscr{P}_{\mathscr{N}} \cap D = \phi$ and D is a multiplicative closed subset of $\mathscr{N}$. For any multiplicative closed subset S of $\mathscr{N}$, we define $O(S) = \{c_1 \in \mathscr{N} | c_1 s \in \mathscr{P}_{\mathscr{N}}$ for some $s \in S\}$. For each multiplicative closed subset S of $\mathscr{N}$, $O(S)$ is a $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$. An ideal J of $\mathscr{N}$ is an $\mathscr{P}_{\mathscr{N}}$-ideal if there exists a multiplicative subset M_1 of $\mathscr{N}$ such that $J = O(M_1)$. Let I and J be ideals of $\mathscr{N}$ with $J \subseteq I$, I is a $J-$ideal of $\mathscr{N}$ if $I = O(M_1)$ for some multiplicative subset M_1 of $\mathscr{N}$. If $\mathscr{N}$ is $2-$primal, then each minimal prime ideal is $\mathscr{P}_{\mathscr{N}}$-Baer ideal. Indeed, if $c_1 \in P_1$, where P_1 is minimal prime, then, by Theorem 3.5 of [4], $< \mathscr{P}_{\mathscr{N}} : c_1 >\nsubseteq P_1$ which implies $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>\subseteq P_1$, so P_1 is $\mathscr{P}_{\mathscr{N}}$-Baer ideal. Every $\mathscr{P}_{\mathscr{N}}-$ideal is a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal, and every strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$ is a $\mathscr{P}_{\mathscr{N}}$-Baer ideal, see Lemma 2.2. For any subset T of $\mathscr{N}$, $< \mathscr{P}_{\mathscr{N}} : T >$ is a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$.

Clearly intersection of strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideals (resp., $\mathscr{P}_{\mathscr{N}}$-Baer ideals) is again a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal (resp., $\mathscr{P}_{\mathscr{N}}$-Baer ideal). It should be noted that our definition of strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal($\mathscr{P}_{\mathscr{N}}$-Baer ideal) will coincide with that of Jayaram (1984) in a commutative semiprime ring.

2. Main Results

Theorem 2.1. *If $\mathscr{N}$ is zero-symmetric and if P'' is an ideal of $\mathscr{N}$, then the statements given below are equivalent:*

(i) *P'' is prime,*

(ii) *For any $c_1, c_2 \in \mathscr{N}$, $c_1 < c_2 >\subseteq P''$ implies $c_1 \in P''$ or $c_2 \in P''$,*

(iii) *If $A_1, A_2, ..., A_n$ are ideals of $\mathscr{N}$, then $A_1A_2...A_n \subseteq P''$ implies $A_i \subseteq P''$ for some i.*

Proof.

(i)$\Rightarrow$(ii) Suppose $c_1 < c_2 >\subseteq P''$ for some $c_1, c_2 \in \mathscr{N}$. Then $c_1 \in (P'' :< c_1 >)$. Since $(P'' :< c_1 >)$ is an ideal, $< c_1 >\subseteq (P'' :< c_2 >)$ and hence $< c_1 >< c_2 >\subseteq P''$ which implies $< c_1 >\subseteq P''$ or $< c_2 >\subseteq P''$. i.e., $c_1 \in P''$ or $c_2 \in P''$.

(ii)$\Rightarrow$(iii) Let $A_1, A_2, ..., A_n$ be ideals of $\mathscr{N}$ with $A_1A_2...A_n \subseteq P''$ and suppose that $A_n \nsubseteq P''$. We claim that $A_1A_2...A_{n-1} \subseteq P''$. Let $c_1 \in A_1A_2...A_{n-1}$ and let $c_2 \in A_n \backslash P''$. Then $c_1 < c_2 >\subseteq P''$. Since $c_2 \notin P''$ by (ii), we have $c_1 \in P''$. Thus $A_1A_2...A_{n-1} \subseteq P''$. Suppose $A_{n-1} \nsubseteq P''$. Then as eariler we can show that $A_1A_2...A_{n-2} \subseteq P''$. Proceeding in this way we get (iii).

(iii)$\Rightarrow$(i) It is obvious. □

Lemma 2.1. *If $\mathscr{N}$ is 2-primal, then for any $Z' \subseteq \mathscr{N}$; $c_1, c_2 \in \mathscr{N}$, we have*

(i) *$< \mathscr{P}_{\mathscr{N}} : Z' >=< \mathscr{P}_{\mathscr{N}} :< Z' >>$ and $Z' \subseteq< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : Z' >>$,*

(ii) *$< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1c_2 >>=< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >> \cap < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_2 >>$,*

(iii) *If $c_1c_2 \in \mathscr{P}_{\mathscr{N}}$, then $< \mathscr{P}_{\mathscr{N}} : c_1 + c_2 >=< \mathscr{P}_{\mathscr{N}} : c_1 > \cap < \mathscr{P}_{\mathscr{N}} : c_2 >$.*

Proof.

(i) For $t \in \mathscr{N}$, $t \in< \mathscr{P}_{\mathscr{N}} : Z' > \iff aZ' \subseteq \mathscr{P}_{\mathscr{N}} \iff a < Z' >\subseteq \mathscr{P}_{\mathscr{N}} \iff a \in< \mathscr{P}_{\mathscr{N}} :< Z' >>$. Also if $c_1 \in Z'$, then for any $t \in< \mathscr{P}_{\mathscr{N}} : Z' >$, we have $tc_1 \in \mathscr{P}_{\mathscr{N}}$ which implies $c_1 \in< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : Z' >>$.

(ii) Clearly $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >> \cap < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_2 >>\subseteq< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1c_2 >>$. If $t \in< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1c_2 >>$, then $t < \mathscr{P}_{\mathscr{N}} : c_1c_2 >\subseteq \mathscr{P}_{\mathscr{N}}$, so for $a \in< \mathscr{P}_{\mathscr{N}} : c_1 >\subseteq< \mathscr{P}_{\mathscr{N}} : c_1c_2 >$ and $b \in< \mathscr{P}_{\mathscr{N}} : c_2 >\subseteq< \mathscr{P}_{\mathscr{N}} : c_1c_2 >$, $ta \in \mathscr{P}_{\mathscr{N}}$ and $tb \in \mathscr{P}_{\mathscr{N}}$ imply $t \in< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >> \cap < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_2 >>$. So $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1c_2 >>=< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >> \cap < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_2 >>$.

(iii) If $t \in< \mathscr{P}_{\mathscr{N}} : c_1 > \cap < \mathscr{P}_{\mathscr{N}} : c_2 >$, then $tc_1 \in \mathscr{P}_{\mathscr{N}}$ and $tc_2 \in \mathscr{P}_{\mathscr{N}}$ which imply $t(c_1 + c_2) \in \mathscr{P}_{\mathscr{N}}$. Thus $t \in< \mathscr{P}_{\mathscr{N}} : c_1 + c_2 >$. If $s \in< \mathscr{P}_{\mathscr{N}} : c_1 + c_2 >$, then

$s(c_1+c_2) \in \mathscr{P}_{\mathscr{N}}$ implies $s(c_1+c_2)c_2 \in \mathscr{P}_{\mathscr{N}}$ and $s(c_1+c_2)c_1 \in \mathscr{P}_{\mathscr{N}}$. Since $c_1c_2 \in \mathscr{P}_{\mathscr{N}}$, we have $sc_2^2 \in \mathscr{P}_{\mathscr{N}}$ and $sc_1^2 \in \mathscr{P}_{\mathscr{N}}$. Since $\mathscr{P}_{\mathscr{N}}$ is completely semiprime, we have $sc_2 \in \mathscr{P}_{\mathscr{N}}$ and $sc_1 \in \mathscr{P}_{\mathscr{N}}$ which imply $s \in <\mathscr{P}_{\mathscr{N}} : c_1> \cap <\mathscr{P}_{\mathscr{N}} : c_2>$. Therefore $<\mathscr{P}_{\mathscr{N}} : c_1+c_2> = <\mathscr{P}_{\mathscr{N}} : c_1> \cap <\mathscr{P}_{\mathscr{N}} : c_2>$. □

Theorem 2.2. *If $\mathscr{N}$ is 2-primal, $T = \{<\mathscr{P}_{\mathscr{N}} : Q> : \ Q \subseteq \mathscr{N}, \ <\mathscr{P}_{\mathscr{N}} : Q> \neq \mathscr{N}\}$ and $S \subseteq \mathscr{N}$ with $S \nsubseteq \mathscr{P}_{\mathscr{N}}$, then the statements given below are equivalent:*

(i) *$<\mathscr{P}_{\mathscr{N}} : S>$ is maximal among T,*
(ii) *$<\mathscr{P}_{\mathscr{N}} : S>$ is completely prime,*
(iii) *$<\mathscr{P}_{\mathscr{N}} : S>$ is minimal prime.*

Proof.
$(i) \Rightarrow (ii)$ By assumption, $\exists y \in S \backslash \mathscr{P}_{\mathscr{N}}$: $<\mathscr{P}_{\mathscr{N}} : S> = <\mathscr{P}_{\mathscr{N}} : y>$. Since $<\mathscr{P}_{\mathscr{N}} : y> \subseteq <\mathscr{P}_{\mathscr{N}} : y^2>$ and $y^3 \notin \mathscr{P}_{\mathscr{N}}$, $<\mathscr{P}_{\mathscr{N}} : y> = <\mathscr{P}_{\mathscr{N}} : y^2>$. If $ab \in <\mathscr{P}_{\mathscr{N}} : y>$ and $a \notin <\mathscr{P}_{\mathscr{N}} : y>$ for some $a, b \in \mathscr{N}$, then $<\mathscr{P}_{\mathscr{N}} : ya> \neq \mathscr{N}$. By the maximality of $<\mathscr{P}_{\mathscr{N}} : y>$, we have $b \in <\mathscr{P}_{\mathscr{N}} : ya> = <\mathscr{P}_{\mathscr{N}} : y>$.

$(ii) \Rightarrow (iii)$ Suppose that Q is prime with $Q \subseteq <\mathscr{P}_{\mathscr{N}} : S>$. Let $y \in S \backslash \mathscr{P}_{\mathscr{N}}$ and $a \in <\mathscr{P}_{\mathscr{N}} : S>$. Then $<a><y> \in \mathscr{P}_{\mathscr{N}} \subseteq Q$. Since $y^2 \notin \mathscr{P}_{\mathscr{N}}$, we have $a \in Q$. So $Q = <\mathscr{P}_{\mathscr{N}} : S>$. $(iii) \Rightarrow (ii)$ It follows from Corollary 1.3 of [2].

$(ii) \Rightarrow (i)$ Suppose that $<\mathscr{P}_{\mathscr{N}} : S> \subseteq <\mathscr{P}_{\mathscr{N}} : T> \neq \mathscr{N}$. Then there is $y \in T$ such that $y \notin \mathscr{P}_{\mathscr{N}}$. Let $a \in <\mathscr{P}_{\mathscr{N}} : T>$. Then $ay \in \mathscr{P}_{\mathscr{N}} \subseteq <\mathscr{P}_{\mathscr{N}} : S>$, so $a \in <\mathscr{P}_{\mathscr{N}} : S>$ or $y \in <\mathscr{P}_{\mathscr{N}} : S>$. Since $y^2 \notin \mathscr{P}_{\mathscr{N}}$, we have $y \notin <\mathscr{P}_{\mathscr{N}} : S>$. Thus $a \in <\mathscr{P}_{\mathscr{N}} : S>$ and hence $<\mathscr{P}_{\mathscr{N}} : S> = <\mathscr{P}_{\mathscr{N}} : T>$. □

Theorem 2.3. *If $\mathscr{N}$ is 2-primal, then the maximality and minimality conditions of the elements on the set $T = \{<\mathscr{P}_{\mathscr{N}} : A_1> : \ A_1 \subseteq \mathscr{N}, \ <\mathscr{P}_{\mathscr{N}} : A_1> \neq \mathscr{N}\}$ are coincide.*

Proof. Suppose a.c.c. holds on the set T and let $<\mathscr{P}_{\mathscr{N}} : X_1> \supseteq <\mathscr{P}_{\mathscr{N}} : X_2> \supseteq$ ... be a descending chain on the set T. Then $<\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : X_1>> \subseteq <\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : X_2>> \subseteq$ ... is an ascending chain, which ends after finite steps.

Since $<\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : X_i>>> = <\mathscr{P}_{\mathscr{N}} : X_i>$, so the descending chain $<\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : X_1>>> \supseteq <\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : <\mathscr{P}_{\mathscr{N}} : X_2>>> \supseteq$ ... or the chain ends after finite steps. Similarly we can prove the converse part. □

Theorem 2.4. *For any $\mathscr{N} \neq \mathscr{P}_{\mathscr{N}}$, the statements given below are equivalent:*

•

(i) *$\mathscr{N}$ is 2-primal and, for every $c_1 \in \mathscr{N} \backslash \mathscr{P}_{\mathscr{N}}$, $< \mathscr{P}_{\mathscr{N}} : c_1 >$ is contained in some maximal element among $T = \{< \mathscr{P}_{\mathscr{N}} : X_n > : \ X_n \subseteq \mathscr{N}, < \mathscr{P}_{\mathscr{N}} : X_n > \neq \mathscr{N}\}$,*

(ii) *The number of minimal completely prime ideals P_i, $i = 1, 2, ..., n$ with $\bigcap_{i=1}^{n} P_i = \mathscr{P}_{\mathscr{N}}$ of $\mathscr{N}$ is finite.*

Proof.

$i) \Rightarrow ii)$ Assume that $c_1c_2 \in \mathscr{P}_{\mathscr{N}}$ for some $c_1 \in \mathscr{N} \backslash \mathscr{P}_{\mathscr{N}}$ and $c_2 \in \mathscr{N} \backslash \mathscr{P}_{\mathscr{N}}$. Then there is a maximal element $< \mathscr{P}_{\mathscr{N}} : c_3 >$ in T such that $c_3 \in \mathscr{N} \backslash \mathscr{P}_{\mathscr{N}}$ and $c_1 \in < \mathscr{P}_{\mathscr{N}} : c_2 > \subseteq < \mathscr{P}_{\mathscr{N}} : c_3 >$. By Theorem 2.2, $< \mathscr{P}_{\mathscr{N}} : c_3 >$ is completely prime ideal. Consider the set of all distinct minimal completely prime ideals P_α of $\mathscr{N}$ where $P_\alpha =< \mathscr{P}_{\mathscr{N}} : z_\alpha >$ $(\alpha \in I)$ and $z_\alpha \in \mathscr{N} \backslash \mathscr{P}_{\mathscr{N}}$. Let $P = \bigcap_{\alpha \in I} P_\alpha$. Then $z_\alpha \in < \mathscr{P}_{\mathscr{N}} : P_\alpha >$ and $< \mathscr{P}_{\mathscr{N}} : P_\alpha > \subseteq < \mathscr{P}_{\mathscr{N}} : P >$ for all $\alpha \in I$.

We now claim that $\mathscr{P}_{\mathscr{N}} = P$. If not, then there is a maximal element $< \mathscr{P}_{\mathscr{N}} : z_\beta >$ in T with $z_\beta \notin \mathscr{N} \backslash \mathscr{P}_{\mathscr{N}}$ and $< \mathscr{P}_{\mathscr{N}} : P > \subseteq < \mathscr{P}_{\mathscr{N}} : z_\beta >$. So $P_\beta =< \mathscr{P}_{\mathscr{N}} : z_\beta >$ for some $\beta \in I$, but $z_\beta \in < \mathscr{P}_{\mathscr{N}} : P >$, we have ${z_\beta}^2 \in z_\beta < \mathscr{P}_{\mathscr{N}} : P > \subseteq z_\beta < \mathscr{P}_{\mathscr{N}} : z_\beta >= z_\beta P_\beta \subseteq \mathscr{P}_{\mathscr{N}}$, a contradiction. So $P = \bigcap_{\alpha \in I} P_\alpha = \mathscr{P}_{\mathscr{N}}$. We now prove that $|I|$ is finite. If not, then for some $\alpha_1 \in I$, $< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} >$ is not contained in all $< \mathscr{P}_{\mathscr{N}} : z_\alpha >$ which implies $z_{\alpha_1} z_\alpha$ for all $\alpha (\neq 1) \in I$. Take some $\alpha_2 \in I$, $< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \nsubseteq < \mathscr{P}_{\mathscr{N}} : z_{\alpha_2} >$ which implies $< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \subset < \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \cap < \mathscr{P}_{\mathscr{N}} : z_{\alpha_2} >$. If $< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \cap < \mathscr{P}_{\mathscr{N}} : z_{\alpha_2} > \neq \mathscr{P}_{\mathscr{N}}$, then we have a descending chain $< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \supset < \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \cap < \mathscr{P}_{\mathscr{N}} : z_{\alpha_2} > \supset ....$ If $t(z_{\alpha_1} + z_{\alpha_2}) \in \mathscr{P}_{\mathscr{N}}$, then $tz_{\alpha_1} \in \mathscr{P}_{\mathscr{N}}$ and $tz_{\alpha_2} \in \mathscr{P}_{\mathscr{N}}$. This shows that $< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \cap < \mathscr{P}_{\mathscr{N}} : z_{\alpha_2} >=< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} + z_{\alpha_2} >$. So the obtained descending chain $< \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} > \supset < \mathscr{P}_{\mathscr{N}} : z_{\alpha_1} + z_{\alpha_2} > \supset ...$ not terminated, a contradiction to Theorem 2.3. Hence $|I|$ is finite.

$(ii) \Rightarrow (i)$ It is trivial as $\mathscr{N} / \mathscr{P}_{\mathscr{N}}$ is reduced. □

Lemma 2.2. *Let N_1 be an ideal of a 2-primal near-ring $\mathscr{N}$ with $\mathscr{P}_{\mathscr{N}} \subseteq N_1$. If N_1 is strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal, then N_1 is a $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$.*

Proof. Let N_1 be a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$. For $c_1 \in N_1$ and let $c_2 \in < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>$. We now prove that $< \mathscr{P}_{\mathscr{N}} : c_1 >=< \mathscr{P}_{\mathscr{N}} : c_1c_2 >$. Clearly $< \mathscr{P}_{\mathscr{N}} : c_1 > \subseteq < \mathscr{P}_{\mathscr{N}} : c_1c_2 >$. Let $a \in < \mathscr{P}_{\mathscr{N}} : c_1c_2 >$. Then $ac_2 \in <$

$\mathscr{P}_{\mathscr{N}} : c_1 >$. Since $c_2 < \mathscr{P}_{\mathscr{N}} : c_1 > \subseteq \mathscr{P}_{\mathscr{N}}$, we get $ac_2^2 \in \mathscr{P}_{\mathscr{N}}$ implies $ac_2 \in \mathscr{P}_{\mathscr{N}}$, so $a \in < \mathscr{P}_{\mathscr{N}} : c_2 >$. Thus $< \mathscr{P}_{\mathscr{N}} : c_2 > = < \mathscr{P}_{\mathscr{N}} : c_1c_2 >$ and hence $c_2 \in N_1$. □

Lemma 2.3. *If Q is an ideal of a 2-primal near-ring $\mathscr{N}$ with $\mathscr{P}_{\mathscr{N}} \subseteq Q$, then the statements given below are equivalent:*

(i) *Q is $\mathscr{P}_{\mathscr{N}}$-Baer ideal,*

(ii) *For any $c_1, c_2 \in \mathscr{N}$, $< \mathscr{P}_{\mathscr{N}} : c_1 > = < \mathscr{P}_{\mathscr{N}} : c_2 >$ and $c_1 \in Q$ imply $c_2 \in Q$,*

(iii) *$Q = \bigcup_{c_1 \in Q} < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>$.*

Proof.

$(i) \Rightarrow (ii)$ and $(iii) \Rightarrow (i)$ are evident.

$(ii) \Rightarrow (iii)$ For any $c_1 \in Q$ and $c_2 \in < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>$, we have $< \mathscr{P}_{\mathscr{N}} : c_1 > \subseteq < \mathscr{P}_{\mathscr{N}} : c_2 >$ and $< \mathscr{P}_{\mathscr{N}} : c_2 > = < \mathscr{P}_{\mathscr{N}} : c_1 > \cup < \mathscr{P}_{\mathscr{N}} : c_2 > = < \mathscr{P}_{\mathscr{N}} : c_1c_2 >$ as $c_2 < \mathscr{P}_{\mathscr{N}} : c_1 > \subseteq \mathscr{P}_{\mathscr{N}}$. Since $c_1c_2 \in Q$, we have $c_2 \in Q$. So, $\bigcup_{c_1 \in Q} < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >> \subseteq Q$. Since for any $c_1 \in \mathscr{N}$, we have $c_1 \in < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>$. Thus $Q \subseteq \bigcup_{c_1 \in Q} < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>$ and hence $Q = \bigcup_{c_1 \in Q} < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>$. □

Lemma 2.4. *If Q is a $\mathscr{P}_{\mathscr{N}}$-Baer ideal of a 2-primal near-ring $\mathscr{N}$, then $Q = \mathscr{P}_{\mathscr{N}}(Q)$.*

Proof. Let $c_1 \in \mathscr{P}_{\mathscr{N}}(Q)$. Then, by Proposition 2.94 of [6], we can find a positive integer n such that $c_1^n \in Q$. Since $\mathscr{N}$ is 2-primal, we get $< \mathscr{P}_{\mathscr{N}} : c_1 > = < \mathscr{P}_{\mathscr{N}} : c_1^n >$. By Lemma 2.3, we have $c_1 \in Q$. Thus $\mathscr{P}_{\mathscr{N}}(Q) \subseteq Q$ and hence $Q = \mathscr{P}_{\mathscr{N}}(Q)$. □

Corollary 2.1. *For every strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal Q of a 2-primal near-ring $\mathscr{N}$, we have $Q = \mathscr{P}_{\mathscr{N}}(Q)$.*

Proof. It follows from Lemma 2.2 and Lemma 2.4. □

Theorem 2.5. *If I_1 is a reflexive ideal of $\mathscr{N}$ and P'' is prime with $I_1 \subseteq P''$, then the statements given below are equivalent:*

(i) *P'' is a minimal prime,*

(ii) *For every $a \in P''$, there exist $x_i \in \mathscr{N} \backslash P''$ such that $a^{t_0}x_1a^{t_1}x_2a^{t_2}x_3...x_na^{t_n} \in I_1$, where $t_i's$ are positive integers with t_0 and t_n allowed to be zero.*

Proof.

$(i) \Rightarrow (ii)$ Let $a \in P''$ and $T = \{a^{t_0}x_1a^{t_1}x_2a^{t_2}x_3...x_na^{t_n}$, where $x_i \in \mathscr{N}\backslash P''$ and $t_i's$ are the positive integers with t_0 and t_n allowed to be zero$\}$. Then $F = T \cup (\mathscr{N}\backslash P'')$ is a multiplicative closed subset of $\mathscr{N}$. If $I_1 \cap F = \phi$, then, by Proposition 2.1.6 of [1], there exists a proper maximal ideal M_1 with $M_1 \cap F = \phi$. Since $a \notin M_1$, we have $M_1 + < a >= \mathscr{N}$ which implies $b + c = 1$ for some $b \in M_1$ and $c \in < a >$. Since $a \in P''$, we have $b \in \mathscr{N}\backslash P''$. So $b \in M_1 \cap F \neq \{\phi\}$, a contradiction. Thus $I_1 \cap F = \phi$ and hence $I_1 \cap T \neq \{\phi\}$.

$(ii) \Rightarrow (i)$ Suppose that K is a prime ideal with $I_1 \subseteq K \subseteq P''$. Then for any $a \in P''$, there are $x_i \in \mathscr{N}\backslash P''$ such that $a^{t_0}x_1a^{t_1}x_2a^{t_2}x_3...x_na^{t_n} \in I_1$ where $t_i's$ are positive integers with t_0 and t_n allowed to be zero. Since I_1 is reflexive ideal, we have $< a >^{t_0} < x_1 >< a >^{t_1} < x_2 > ... < x_n >< a >^{t_n} \subseteq I_1 \subseteq K$ which implies $a \in K$. Thus $P'' \subseteq K$ and hence P'' is a minimal prime. □

Lemma 2.5. *Let $\mathscr{N}$ be a 2-primal near-ring and K a $\mathscr{P}_{\mathscr{N}}$-Baer ideal (resp., strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal) of $\mathscr{N}$. Then each minimal prime ideal P of $\mathscr{N}$ containing K is a $\mathscr{P}_{\mathscr{N}}$-Baer ideal (resp., strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal) of $\mathscr{N}$.*

Proof. Suppose that K is $\mathscr{P}_{\mathscr{N}}$-Baer ideal and P is minimal prime containing K. Let $< \mathscr{P}_{\mathscr{N}} : c_1 >=< \mathscr{P}_{\mathscr{N}} : c_2 >$ and $c_1 \in P$. Then by Theorem 2.5, there exist $x_i \in \mathscr{N}\backslash P$ such that $c_1^{t_0}x_1c_1^{t_1}x_2c_1^{t_2}x_3...x_nc_1^{t_n} \in K$ where $t_i's$ are positive integers with t_0 and t_n allowed to be zero. Since $< \mathscr{P}_{\mathscr{N}} : x_1x_2...x_nc_1 >=< \mathscr{P}_{\mathscr{N}} : x_1x_2...x_nc_2 >=< \mathscr{P}_{\mathscr{N}} : c_1^{t_0}x_1c_1^{t_1}x_2c_1^{t_2}x_3...x_nc_1^{t_n} >$ and K is $\mathscr{P}_{\mathscr{N}}$-Baer ideal, we have $x_1x_2...x_nc_2 \in K$ and so $x_1x_2...x_nc_2 \in P$. Consequently $c_2 \in P$ as x_i's are not in P. Therefore P is a $\mathscr{P}_{\mathscr{N}}$-Baer ideal. □

Corollary 2.2. *Let $\mathscr{N}$ be a 2-primal near-ring. Then every $\mathscr{P}_{\mathscr{N}}$-Baer ideal (resp., strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal) of $\mathscr{N}$ is the intersection of every prime $\mathscr{P}_{\mathscr{N}}$-Baer ideals (resp., prime strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideals) containing it.*

Proof. It is evident from Lemma 2.4 and Lemma 2.5. □

Lemma 2.6. *Let $\mathscr{N}$ be a 2-primal near-ring and Q be a $\mathscr{P}_{\mathscr{N}}$-ideal of $\mathscr{N}$. Then every minimal prime ideal belonging to Q is a minimal prime ideal of $\mathscr{N}$.*

Proof. Let Q be a $\mathscr{P}_{\mathscr{N}}$-ideal and P a minimal prime ideal belonging to Q. Then $Q = O(K)$ for some multiplicative subset K of $\mathscr{N}$ and Q is reflexive. By Theorem 2.5, we claim that for each $q \in P$, there exist $x_i \in \mathscr{N}\backslash P$ such that

$q^{t_0}x_1q^{t_1}x_2q^{t_2}x_3...x_nq^{t_n} \in \mathscr{P}_{\mathscr{N}}$, where $t_i's$ are positive integers. Let $q \in P$. Then, by Theorem 2.5, there exist $x_i \in \mathscr{N}\backslash P$ such that $q^{t_0}x_1q^{t_1}x_2q^{t_2}x_3...x_nq^{t_n} \in Q = O(K)$, so $q^{t_0}x_1q^{t_1}x_2q^{t_2}x_3...x_nq^{t_n}d \in \mathscr{P}_{\mathscr{N}}$ for some $d \in K$. Since $\mathscr{N}$ is 2-primal, we have $q < d >< x_1 >< x_2 > ... < x_n >\subseteq \mathscr{P}_{\mathscr{N}}$. Clearly $P \cap K = \phi$ and therefore $< d >< x_1 >< x_2 > ... < x_n >\nsubseteq P$. So there exists $y \in< d >< x_1 >< x_2 > ... < x_n > \backslash P$ such that $qy \in \mathscr{P}_{\mathscr{N}}$. Hence P is minimal prime. □

Lemma 2.7. *Suppose that for each $u \in \mathscr{N}$, there is $v \in \mathscr{N}$ such that $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : u >>=< \mathscr{P}_{\mathscr{N}} : v >$. Then every $\mathscr{P}_{\mathscr{N}}$-Baer ideal is strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal.*

Proof. Assume that for each $u \in \mathscr{N}$, there is $v \in \mathscr{N}$ with $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : u >>=< \mathscr{P}_{\mathscr{N}} : v >$. Let Q be a $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$ and $< \mathscr{P}_{\mathscr{N}} : c_1 > \cap < \mathscr{P}_{\mathscr{N}} : c_2 >=< \mathscr{P}_{\mathscr{N}} : c_3 >$ for $c_1, c_2 \in Q$. By assumption, there exist $c_1', c_2' \in \mathscr{N}$ with $< \mathscr{P}_{\mathscr{N}} < \mathscr{P}_{\mathscr{N}} : c_1 >>=< \mathscr{P}_{\mathscr{N}} : c_1' >$ and $< \mathscr{P}_{\mathscr{N}} < \mathscr{P}_{\mathscr{N}} : c_2 >>=< \mathscr{P}_{\mathscr{N}} : c_2' >$. Since $c_1 \in< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>$ and $c_2 \in< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_2 >>$, we have $c_1c_1', c_2\alpha_2' \in \mathscr{P}_{\mathscr{N}}$ and $c_1 + c_1', c_2 + c_2' \in D$. Suppose $c_3 \notin Q$. By Lemma 2.4, there is a prime $\mathscr{P}_{\mathscr{N}}$-Baer ideal P of $\mathscr{N}$ such that $Q \subseteq P$ and $c_3 \notin P$. Since $c_3c_1'c_2' \in \mathscr{P}_{\mathscr{N}}$, we get $c_1' \in P$ or $c_2' \in P$. But in either case we have $P \cap D \neq \phi$, as P is a $\mathscr{P}_{\mathscr{N}}$-Baer ideal. Thus $c_3 \in Q$ and hence Q is a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal. □

Theorem 2.6. *If $\mathscr{N}$ is a 2-primal with identity, then the statements given below are equivalent:*

(i) *Every ideal of $\mathscr{N}$ containing $\mathscr{P}_{\mathscr{N}}$ is a $\mathscr{P}_{\mathscr{N}}$-ideal,*
(ii) *Every ideal of $\mathscr{N}$ containing $\mathscr{P}_{\mathscr{N}}$ is strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal,*
(iii) *Every ideal of $\mathscr{N}$ containing $\mathscr{P}_{\mathscr{N}}$ is $\mathscr{P}_{\mathscr{N}}$-Baer ideal,*
(iv) *For any $s, t \in \mathscr{N}$, $< \mathscr{P}_{\mathscr{N}} : s >=< \mathscr{P}_{\mathscr{N}} : t >$ implies $< s >=< t >$,*
(v) *For any $s \in \mathscr{N}$, we have $s + s^2 \in \mathscr{P}_{\mathscr{N}}$.*

Proof.
$(i) \Rightarrow (ii)$ Let K' be an ideal of $\mathscr{N}$. Then $K' = O(R')$ for some multiplicative subset R' of $\mathscr{N}$. Let $c_1, c_2 \in K'$ with $< \mathscr{P}_{\mathscr{N}} : c_1 > \cap < \mathscr{P}_{\mathscr{N}} : c_2 >=< \mathscr{P}_{\mathscr{N}} : z >$ for some $z \in \mathscr{N}$. Then $c_1s_1, c_2s_2 \in \mathscr{P}_{\mathscr{N}}$ for some $s_1, s_2 \in R'$. Since $s_1, s_2 \in R'$ and $s_1s_2 \in< \mathscr{P}_{\mathscr{N}} : c_1 > \cap < \mathscr{P}_{\mathscr{N}} : c_2 >$, we have $(s_1s_2)z \in \mathscr{P}_{\mathscr{N}}$. Thus $z = O(R') = K'$ and hence K' is a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$.

$(ii) \Rightarrow (iii)$ It is evident from the fact that each $\mathscr{P}_{\mathscr{N}}$-ideal of $\mathscr{N}$ is a strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$.

$(iii) \Rightarrow (iv)$ It is trivial as every strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$ is $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$.

$(iv) \Rightarrow (v)$ For each $s \in \mathscr{N}$, $< \mathscr{P}_{\mathscr{N}} : s >=< \mathscr{P}_{\mathscr{N}} : s^2 >$. By (iv), $< s >=< s^2 >$ which implies $s + s^2 \in < s > + < s^2 > \subseteq \mathscr{P}_{\mathscr{N}}$.

$(v) \Rightarrow (i)$ Let I_1 be an ideal of $\mathscr{N}$ with $\mathscr{P}_{\mathscr{N}} \subseteq I_1$ and let $t \in I_1$. Then $(1+t)t \in \mathscr{P}_{\mathscr{N}}$. Take $I_* = \{x \in \mathscr{N} \ : < \mathscr{P}_{\mathscr{N}} : z > \subseteq < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : x > \ for\ some\ z \in I_1\}$. Then I_* is a multiplicative closed subset of $\mathscr{N}$ and $< \mathscr{P}_{\mathscr{N}} : t > \subseteq < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : 1 + t >>$ which imply $1 + t \in I_*$ and $t \in O(I_*)$, so $I_1 \subseteq O(I_*)$. Let $r \in O(I_*)$. Then $rs \in \mathscr{P}_{\mathscr{N}}$ for some $s \in I_*$ with $< \mathscr{P}_{\mathscr{N}} : z > \subseteq < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : s >$ for some $z \in I_1$. By (v), $(1 + z)z \in \mathscr{P}_{\mathscr{N}}$ implies $1 + z \in < \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : s >>$. Since $r \in < \mathscr{P}_{\mathscr{N}} : s >$, we have $(1 + z)r = r + zr \in \mathscr{P}_{\mathscr{N}} \subseteq I_1$ which implies $r \in I_1$.

Thus $O(I_*) \subseteq I_1$ and hence I_1 is $\mathscr{P}_{\mathscr{N}}$-ideal. □

Theorem 2.7. *If $\mathscr{N}$ is 2-primal, then the statements given below are equivalent:*

(i) *For any $c_1 \in \mathscr{N}$, there is $c_2 \in \mathscr{N}$ such that $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : c_1 >>=< \mathscr{P}_{\mathscr{N}} : c_2 >$,*

(ii) *Every $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$ containing $\mathscr{P}_{\mathscr{N}}$ is an $\mathscr{P}_{\mathscr{N}}$-ideal,*

(iii) *Every strongly $\mathscr{P}_{\mathscr{N}}$-Baer ideal of $\mathscr{N}$ containing $\mathscr{P}_{\mathscr{N}}$ is an $\mathscr{P}_{\mathscr{N}}$-ideal,*

(iv) *For $X \subseteq \mathscr{N}$, $< \mathscr{P}_{\mathscr{N}} : X >$ is an $\mathscr{P}_{\mathscr{N}}-$ideal.*

Proof.

$(i) \Rightarrow (ii)$ It is evident from Lemma 2.7 and Theorem 2.6. $(ii) \Rightarrow (iii) \Rightarrow (iv)$ are obvious. $(iv) \Rightarrow (i)$ Let $n \in I$. Then by (iv), $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : n >>= O(S)$ for some multiplicative subset S of $\mathscr{N}$ and $ns \in \mathscr{P}_{\mathscr{N}}$ for some $s \in S$ which imply $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : n >> \subseteq < \mathscr{P}_{\mathscr{N}} : y >$. Also $< \mathscr{P}_{\mathscr{N}} : y > \subseteq O(S) =< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : n >>$. Therefore $< \mathscr{P}_{\mathscr{N}} :< \mathscr{P}_{\mathscr{N}} : n >>=< \mathscr{P}_{\mathscr{N}} : y >$. □

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DEPARTMENT OF MATHEMATICS
KARUNYA INSTITUTE OF TECHNOLOGY AND SCIENCES
COIMBATORE - 641 114, INDIA
Email address: `belavarasan@gmail.com`

DEPARTMENT OF MATHEMATICS
KARUNYA INSTITUTE OF TECHNOLOGY AND SCIENCES
COIMBATORE - 641 114, INDIA
Email address: `porselvi94@yahoo.co.in`

DEPARTMENT OF MATHEMATICS
KARUNYA INSTITUTE OF TECHNOLOGY AND SCIENCES
COIMBATORE - 641 114, INDIA
Email address: `catherinegracejohn@gmail.com`