ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.4, 1689–1697
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.25 Spec. Issue on NCFCTA-2020

MAXIMAL PRODUCT OF AN INTUITIONISTIC FUZZY IDEAL GRAPH OF $M\Gamma$ GROUP IN NEARRINGS

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ABSTRACT. The objective of this paper is to describe the product of two intuitionistic fuzzy (IF) graphs as maximal products in $M\Gamma$ group of near rings. Further we discuss the degree and total degree of a vertex in the maximal product of IF graph structure with their properties and few theorems.

1. INTRODUCTION

Piltz [8] introduced the abstract structure near rings as an extension of rings and studied various properties with theorems. In 1996, Zadeh [9] extended crisp set into a fuzzy set by defining a membership function which has many applications including medicine. In continuation Kim et al. [3] introduced fuzzy ideals of near rings and explained their characteristics with examples. Further Satyanarayana et al. [10] described these fuzzy ideals in near rings and its application as graph. In 1986 Atanassov [1] introduced Intuitionistic Fuzzy(IF) sets as an expansion of fuzzy sets with one more non-membership function.After the introduction of IF sets, Jianminget al. [2] introduced IF ideal in near rings and derived the conditions for the IF set to be IF ideals using level subsets. Karunambigaiet al. [4,5] introduced IF graph and discussed elaborately its various characteristics and types with many examples. Mala et al. [6] explained IF ideals in $M\Gamma$ group in near rings and discussed their properties with various theorems.

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2010 *Mathematics Subject Classification.* 03E72.

Key words and phrases. Maximal product of an intuitionistic fuzzy (IF) ideal graph, Degree and Total degree of a Vertex.

Further as an application, Mala et al. [7] developed IF ideals in $M\Gamma$ group in near rings as a graph and discussed their properties in detail. Muzzmal Sitara et al. [8] introduced maximal products of fuzzy graph structure and presented degree and total degree of a vertex in it.

2. Maximal Product Of Graph Of Intuitionistic Fuzzy Graph

Definition 2.1. *Let $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ be 2 graphs of the ideals I_1 and I_2. Then $G_1 * G_2 = (V_I, E_I, \mu_I, \gamma_I)$ is called maximal intuitionistic fuzzy graph with structure $V_I = V_{I1} * V_{I2}$ and $E_I = \{((u_1, v_1)(u_2, v_2))/u_1 = u_2$ and $v_1, v_2 \in E_{I2}$ (or) $v_1 = v_2$ and $u_1, u_2 \in E_{I1}\}$, with $\mu_I(u, v) = \mu_{I1}(u) \vee \mu_{I2}(v)$ for all $(u, v) \in V_I = V_{I1} * V_{I2}$, and $\gamma_I(u, v) = \gamma_{I1}(u) \wedge \gamma_{I2}(v)$ for all $(u, v) \in V_I$. Also,*

$$\mu_I((u_1, v_1)(u_2, v_2)) = \{\begin{array}{lll} \mu_{I1}(u_1) \vee \mu_{I2}(v_1v_2) & \text{where} & u_1 = u_2 \text{ and } v_1v_2 \in E_{I2}, \\ \mu_{I2}(v_2) \vee \mu_{I1}(u_1u_2) & \text{where} & v_1 = v_2 \text{ and } u_1u_2 \in E_{I1}, \end{array}$$

and

$$\gamma_I((u_1, v_1)(u_2, v_2)) = \{\begin{array}{lll} \gamma_{I1}(u_1) \wedge \gamma_{I2}(v_1v_2) & \text{where} & u_1 = u_2 \text{ and } v_1v_2 \in E_{I2}, \\ \gamma_{I2}(v_2) \wedge \gamma_{I1}(u_1u_2) & \text{where} & v_1 = v_2 \text{ and } u_1u_2 \in E_{I1}. \end{array}$$

Example 1. *Consider intuitionistic fuzzy graph of the ideal $I_1 = \{0\}$ and $I_2 = \{0, 1, 2\}$ of Z_3 as G_{I1} and G_{I2} then we get $G_{I1} * G_{I2}$ as a maximal intuitionistic graph as to follows: $G_{I1} * G_{I2}$ has vertex set*

$$\begin{aligned} V_I = V_{I1} \times V_{I2} &= \{0, 1, 2\} \times \{0, 1, 2\} \\ &= \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2)\} \end{aligned}$$

and edges set E_I has edges with either first co-ordinate same or second co-ordinate same in V_I. Thus $G_{I1} * G_{I2}$ is represented as follows:

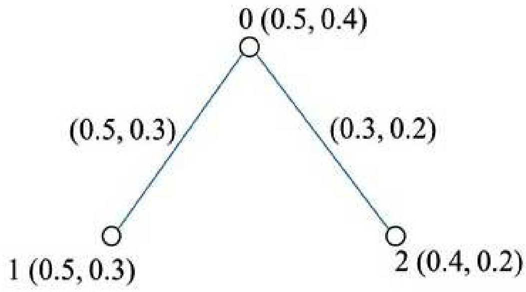

Figure 1 : Graph G_{I1}

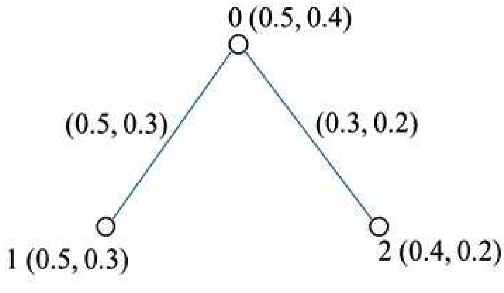

Figure 2 : Graph G_{I2}

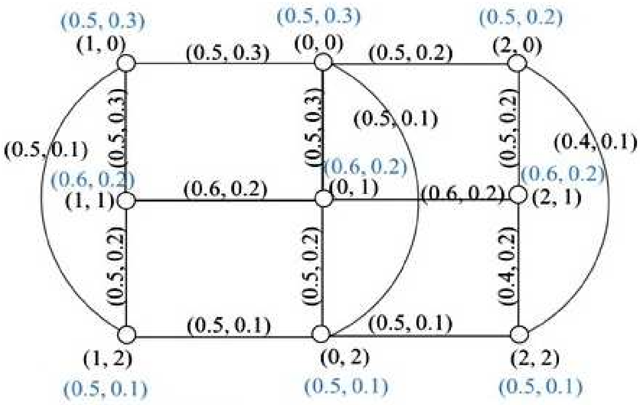

Figure 3 : Graph $G_{I1} * G_{I2}$

Remark 2.1. *Maximal product $G = G_1 * G_2$ may be of a strong intuitionistic fuzzy graph when G_1 and G_2 are not strong intuitionistic fuzzy graph.*

Case (i): $u_1 = u_2$ *and* $v_1v_2 \in E_{I2}$. *Then*

$$\mu_I((u_1, v_1)(u_2, v_2)) = \mu_{I1}(u_1) \wedge \mu_{I2}(v_1v_2) = \mu_{I1}(u_1) \wedge [\mu_{I2}(v_1) \vee \mu_{I2}(v_2)]$$
$$= [\mu_{I1}(u_1) \wedge \mu_{I2}(v_1)] \vee [\mu_{I1}(u_2) \wedge \mu_{I2}(v_2)] = \mu_I(u_1v_1) \vee \mu_I(u_2v_2).$$

Case (ii): $v_1 = v_2$ *and* $u_1u_2 \in E_{I1}$. *Then*

$$\mu_I((u_1, v_1)(u_2, v_2)) = \mu_{I2}(v_2) \wedge \mu_I 1(u_1u_2) = \mu_{I2}(v_2) \wedge [\mu_{I1}(u_1) \vee \mu_{I1}(u_2)]$$
$$= [\mu_{I2}(v_2) \wedge \mu_{I1}(u_1)] \vee [\mu_{I2}(v_2) \wedge \mu_{I1}(u_2)] = \mu_I(u_1v_1) \vee \mu_I(u_2v_2).$$

For all the edges of the maximal product.

Remark 2.2. *Converse of theorem 2.1 is not true. Maximal product $G = G_1 * G_2$ may be a strong intuitionistic fuzzy structure where G_1 and G_2 are not strong intuitionistic fuzzy structure.*

Example 2. *Maximal product of two strong intuitionistic fuzzy graph is also a strong fuzzy graph.*

Definition 2.2. *An intuitionistic fuzzy graph $G(V_I, E_I, \mu_I, \gamma_I)$ is $\mu_I - \gamma_I$ strong if*

$$\mu_I(v_1v_2) = min\{\mu_{I1}(v_1), \mu_{I2}(v_2)\} v_1 \in V_{I1} \text{ and } v_2 \in V_{I2} \quad \text{and}$$
$$\gamma_I(v_1v_2) = min\{\gamma_{I1}(v_1), \gamma_{I2}(v_2)\} v_1 \in V_{I1} \text{ and } v_2 \in V_{I2}$$

and it is same for all $v_1, v_2 \in E_I$ in the $\mu_I - \gamma_I$ strong intuitionistic fuzzy graph is called strong intuitionistic fuzzy graph.

Theorem 2.1. *Maximal product of two strong intuitionistic fuzzy graph is also a strong intuitionistic fuzzy graph.*

Proof. Let $G_1 = (V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G_2 = (V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ be two strong intuitionistic fuzzy graph structure. Then

$$\mu_I((u_1, v_1)(u_2, v_2)) = \{\mu_{I1}(u_1) \vee \mu_{I2}(v_1v_2) \quad \text{where} \quad u_1 = u_2 \text{ and } v_1v_2 \in E_{I2}$$
$$\mu_{I2}(v_2) \vee \mu_{I1}(u_1u_2) \quad \text{where} \quad v_1 = v_2 \text{ and } u_1u_2 \in E_{I1},$$

and

$$\gamma_I((u_1, v_1)(u_2, v_2)) = \{\gamma_{I1}(u_1) \wedge \gamma_{I2}(v_1v_2) \quad \text{where} \quad u_1 = u_2 \text{ and } v_1v_2 \in E_{I2}$$
$$\gamma_{I2}(v_2) \wedge \gamma_{I1}(u_1u_2) \quad \text{where} \quad v_1 = v_2 \text{ and } u_1u_2 \in E_{I1}.$$

Let $G_1(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G_2(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ be two graphs of the ideals I_1 and I_2 then $G_1 * G_2 = (V_I, E_I, \mu_I, \gamma_I)$ is called maximal product of intuitionistic fuzzy graph with the structure $V_I = V_{I1} * V_{I2}$ and $E_I = \{(u_1, v_1)(u_2, v_2)/u_1 = u_2, v_1v_2 \in E_{I2}$ or $v_1 = v_2, u_1u_2 \in E_{I1}\}$.

With $\mu_I(u, v) = \mu_{I1}(u) \vee \mu_{I2}(v)$ for all $u, v \in V_I = V_{I1} * V_{I2}$, $\gamma_I(u, v) = \gamma_{I1}(u) \wedge \mu_{I2}(v)$ for all $u, v \in V_I = V_{I1} * V_{I2}$ Also,

$$\mu_I((u_1, v_1)(u_2v_2)) = \{\mu_{I1}(u_1) \vee \mu_{I2}(v_1v_2) \quad \text{where} \quad u_1 = u_2 \text{ and } v_1v_2 \in E_{I2}$$
$$\mu_{I2}(v_2) \vee \mu_{I1}(u_1u_2) \quad \text{where} \quad v_1 = v_2 \text{ and } u_1u_2 \in E_{I1} \quad \text{and}$$
$$\gamma_I((u_1, v_1)(u_2, v_2)) = \{\gamma_{I1}(u_1) \wedge \gamma_{I2}(v_1v_2) \quad \text{where} \quad u_1 = u_2 \text{ and } v_1v_2 \in E_{I2}$$
$$\gamma_{I2}(v_2) \wedge \gamma_{I1}(u_1u_2) \quad \text{where} \quad v_1 = v_2 \text{ and } u_1u_2 \in E_{I1}.$$

□

Example 3. $\{0,1,2\} * \{0,1,2\}$, $(0,0)(0,1)(0,2)(1.0)(1,1)(1,2)(2,0)(2,1)(2,2)$

$$\begin{aligned}
\mu((0,0)(0,1)) &= \max\{0.6, 0.6\} = 0.6 = \mu I \\
\gamma((0,0)(0,1)) &= \min\{0.2, 0.1\} = 0.1 = \gamma I \\
\mu((0,0)(0,2)) &= \max\{0.6, 0.6\} = 0.6 \\
\gamma((0,0)(0,2)) &= \min\{0.2, 0.2\} = 0.2
\end{aligned}$$

The degree of a vertex in maximal product $G_1 * G_2$ *of two intuitionistic fuzzy graph structure* $G_1(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ *and* $G_2(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ *is given by*

$$d_{G_1 * G_2}\mu(u_j, v_j) = \sum \mu_{I1}(u_i u_k) \vee \mu_{I2}(v_j) + \sum \mu_{I2}(v_j v_i) \vee \mu_{I1}(ui)$$

and

$$\begin{aligned}
d_{G_1 * G_2}\gamma(u_i v_j) &= \sum \gamma_{I1}(u_i u_k) \wedge \gamma_{I2}(v_j) + \sum \gamma_{I2}(v_j v_i) \wedge \gamma_{I1}(u_i) \\
\deg_{G_{I1} * G_{I2}} \mu(u_1 v_1) &= \mu_{I1}(u_1 u_2) \vee \mu_{I2}(v_1) + \mu_{I1}(u_1 u_3) \vee \mu_{I2}(v_1) \\
&\quad + \mu_{I2}(v_1 v_2) \vee \mu_{I1}(u_1) \\
&= (0.4 \vee 0.3) + (0.4 \vee 0.3) + (0.2 \vee 0.5) \\
&= 0.4 + 0.4 + 0.5 \\
&= 1.3 \\
\deg_{G_{I1} * G_{I2}} \mu(u_1 v_2) &= \mu_{I1}(u_1 u_2) \vee \mu_{I2}(v_2) + \mu_{I1}(u_1 u_3) \vee \mu_{I2}(v_2) \\
&\quad + \mu_{I2}(v_1 v_2) \vee \mu_{I1}(u_1) \\
&= (0.4 \vee 0.5) + (0.4 \vee 0.5) + (0.2 \vee 0.5) \\
&= 0.5 + 0.5 + 0.5 \\
&= 1.5 \\
\deg_{G_{I1} * G_{I2}} \mu(u_2 v_1) &= \mu_{I1}(u_2 u_3) \vee \mu_{I2}(v_1) + \mu_{I1}(u_2 u_1) \vee \mu_{I2}(v_1) \\
&\quad + \mu_{I2}(v_1 v_2) \vee \mu_{I1}(u_2) \\
&= (0.3 \vee 0.3) + (0.4 \vee 0.3) + (0.2 \vee 0.6) \\
&= 0.3 + 0.4 + 0.6 \\
&= 1.3
\end{aligned}$$

$$\begin{aligned}
\deg_{G_{I1}*G_{I2}} \mu(u_2v_2) &= \mu_{I1}(u_2u_3) \vee \mu_{I2}(v_2) + \mu_{I1}(u_2u_1) \vee \mu_{I2}(v_2) \\
&\quad + \mu_{I2}(v_1v_2) \vee \mu_{I1}(u_2) \\
&= (0.3 \vee 0.5) + (0.4 \vee 0.5) + (0.2 \vee 0.6) \\
&= 0.5 + 0.5 + 0.6 \\
&= 1.6 \\
\deg_{G_{I1}*G_{I2}} \mu(u_3v_1) &= \mu_{I1}(u_3u_1) \vee \mu_{I2}(v_1) + \mu_{I1}(u_3u_2) \vee \mu_{I2}(v_1) \\
&\quad + \mu_{I2}(v_1v_2) \vee \mu_{I1}(u_3) \\
&= (0.4 \vee 0.3) + (0.3 \vee 0.3) + (0.2 \vee 0.4) \\
&= 0.4 + 0.3 + 0.4 \\
&= 1.1 \\
\deg_{G_{I1}*G_{I2}} \mu(u_1v_2) &= \mu_{I1}(u_3u_1) \vee \mu_{I2}(v_2) + \mu_{I1}(u_3u_2) \vee \mu_{I2}(v_2) \\
&\quad + \mu_{I2}(v_1v_2) \vee \mu_{I1}(u_3) \\
&= (0.4 \vee 0.5) + (0.3 \vee 0.5) + (0.2 \vee 0.4) \\
&= 0.5 + 0.5 + 0.4 \\
&= 1.4 \\
\deg_{G_{I1}*G_{I2}} \mu(u_1v_1) &= \mu_{I1}(u_1u_2) \vee \mu_{I2}(v_1) + \mu_{I1}(u_1u_3) \vee \mu_{I2}(v_1) \\
&\quad + \mu_{I2}(v_1v_2) \vee \mu_{I1}(u_1) \\
&= (0.4 \vee 0.3) + (0.4 \vee 0.3) + (0.2 \vee 0.5) \\
&= 0.4 + 0.4 + 0.5 \\
&= 1.3 \\
\deg_{G_{I1}*G_{I2}} \gamma(u_1v_1) &= \gamma_{I1}(u_1u_2) \wedge \gamma_{I2}(v_1) + \gamma_{I1}(u_1u_3) \wedge \gamma_{I2}(v_1) \\
&\quad + \gamma_{I2}(v_2v_1) \wedge \gamma_{I1}(u_1) \\
&= 0.2 \wedge 0.1 + 0.3 \wedge 0.1 + 0.2 \wedge 0.3 \\
&= 0.1 + 0.1 + 0.2 \\
&= 0.4
\end{aligned}$$

$$\begin{aligned}
\deg_{G_{I1}*G_{I2}}\gamma(u_1v_2) &= \gamma_{I1}(u_1u_2)\wedge\gamma_{I2}(v_2)+\gamma_{I1}(u_1u_3)\wedge\gamma_{I2}(v_2)\\
&\quad+\gamma_{I2}(v_1v_2)\wedge\gamma_{I1}(u_1)\\
&= 0.2\wedge 0.2+0.3\wedge 0.2+0.1\wedge 0.3\\
&= 0.2+0.2+0.1\\
&= 0.5\\
\deg_{G_{I1}*G_{I2}}\gamma(u_2v_1) &= \gamma_{I1}(u_2u_3)\wedge\gamma_{I2}(v_1)+\gamma_{I1}(u_2u_1)\wedge\gamma_{I2}(v_1)\\
&\quad+\gamma_{I2}(v_2v_1)\wedge\gamma_{I1}(u_2)\\
&= 0.1\wedge 0.1+0.2\wedge 0.1+0.1\wedge 0.1\\
&= 0.1+0.1+0.1\\
&= 0.3\\
\deg_{G_{I1}*G_{I2}}\gamma(u_2v_2) &= \gamma_{I1}(u_2u_3)\wedge\gamma_{I2}(v_2)+\gamma_{I1}(u_2u_1)\wedge\gamma_{I2}(v_2)\\
&\quad+\gamma_{I2}(v_1v_2)\wedge\gamma_{I1}(u_2)\\
&= 0.1\wedge 0.2+0.2\wedge 0.2+0.1\wedge 0.2\\
&= 0.1+0.2+0.1\\
&= 0.4\\
\deg_{G_{I1}*G_{I2}}\gamma(u_3v_1) &= \gamma_{I1}(u_3u_1)\wedge\gamma_{I2}(v_1)+\gamma_{I1}(u_3u_2)\wedge\gamma_{I2}(v_1)\\
&\quad+\gamma_{I2}(v_2v_1)\wedge\gamma_{I1}(u_3)\\
&= 0.3\wedge 0.1+0.1\wedge 0.1+0.1\wedge 0.2\\
&= 0.1+0.1+0.1\\
&= 0.3\\
\deg_{G_{I1}*G_{I2}}\gamma(u_3v_2) &= \gamma_{I1}(u_3u_1)\wedge\gamma_{I2}(v_2)+\gamma_{I1}(u_3u_2)\wedge\gamma_{I2}(v_2)\\
&\quad+\gamma_{I2}(v_1v_2)\wedge\gamma_{I1}(u_3)\\
&= 0.3\wedge 0.2+0.1\wedge 0.2+0.1\wedge 0.1\\
&= 0.2+0.1+0.2\\
&= 0.4
\end{aligned}$$

The e_i degree of a vertex of maximal product $G_1 * G_2$ is given by

$$e_i - d_{G_1*G_2}\mu(u_jv_j) = \sum \mu_{I1}(u_iu_k) \vee \mu_{I2}(v_j) + \sum \mu_{I2}(v_jv_i) \vee \mu_{I1}(ui) \quad \textit{and}$$
$$e_i - d_{G_1*G_2}\gamma(u_iv_j) = \sum \gamma_{I1}(u_iu_k) \wedge \gamma_{I2}(v_j) + \sum \gamma_{I2}(v_jv_i) \wedge \gamma_{I1}(u_i)$$

$$\begin{aligned}
e_1 - \deg_{G_1*G_2} \mu(u_1v_1) &= \mu_{I1}(u_1u_2) \vee \mu_{I2}(v_1) + \mu_{I2}(v_1v_2) \vee \mu_{I2}(u_1) \\
&= (0.4 \vee 0.3) + (0.2 \vee 0.5) \\
&= 0.4 + 0.5 \\
&= 0.9 \\
e_1 - \deg_{G_1*G_2} \mu(u_1v_2) &= \mu_{I1}(u_1u_2) \vee \mu_{I2}(v_2) + \mu_{I2}(v_1v_2) \vee \mu_{I2}(u_1) \\
&= (0.4 \vee 0.5) + (0.2 \vee 0.5) \\
&= 0.5 + 0.5 \\
&= 1.0 \\
e_2 - \deg_{G_1*G_2} \mu(u_2v_1) &= \mu_{I1}(u_2u_3) \vee \mu_{I2}(v_1) + \mu_{I2}(v_1v_2) \vee \mu_{I1}(u_2) \\
&= (0.3 \vee 0.3) + (0.2 \vee 0.6) \\
&= 0.3 + 0.6 \\
&= 0.9 \\
e_2 - \deg_{G_1*G_2} \mu(u_2v_2) &= \mu_{I1}(u_2u_3) \vee \mu_{I2}(v_2) + \mu_{I2}(v_2v_1) \vee \mu_{I1}(u_2) \\
&= (0.3 \vee 0.5) + (0.2 \vee 0.6) \\
&= 0.5 + 0.6 \\
&= 1.1 \\
e_3 - \deg_{G_1*G_2} \mu(u_3v_1) &= \mu_{I1}(u_3u_1) \vee \mu_{I2}(v_1) + \mu_{I2}(v_1v_2) \vee \mu_{I1}(u_3) \\
&= (0.4 \vee 0.2) + (0.2 \vee 0.4) \\
&= 0.5 + 0.3 \\
&= 0.8 \\
e_3 - \deg_{G_1*G_2} \mu(u_3v_2) &= \mu_{I1}(u_3u_1) \vee \mu_{I2}(v_2) + \mu_{I2}(v_2v_1) \vee \mu_{I1}(u_3) \\
&= (0.4 \vee 0.5) + (0.2 \vee 0.3) \\
&= 0.5 + 0.3 \\
&= 0.8
\end{aligned}$$

3. Conclusion

The concept of maximal product of graph of an IF ideal of $M\Gamma$ group in near rings has been introduced and explained its characteristics with examples. As graph has many applications in areas like networking, logistics, using IF graphs and maximal product of IF graphs further applications can be explored in future.

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