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FEEBLY GENERALIZED LOCALLY CLOSED SETS IN BITOPOLOGICAL SPACES

S. V. VANI[1], K. BALA DEEPA ARASI, AND S. JACKSON

ABSTRACT. The aim of this present paper is to introduce the concepts of Feebly generalized locally closed sets, fg-submaximal spaces and study their basic properties in bitopological spaces.

1. INTRODUCTION

The study of generalization of closed sets has been found to ensure some new separation axioms which have been very useful in the study of certain objects of digital topology. In recent years many generalizations of closed sets have been developed by various authors.

K. Bala Deepa Arasi et.al. [1] introduced the concept of feebly generalized closed sets in bitopological spaces. Bourbaki [2] defined a subset of a topological space to be a locally closed set if it is the intersection of an open set and a closed set. Ganster and Reilly [3] introduced locally closed sets in topological spaces and stone called locally closed sets as fg sets. Maki et.al. [6] introduced the concept of generalized locally closed sets and obtain several different notions of generalized continuities.

[1]*corresponding author*

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In this paper we introduce the concept of feebly generalized locally closed sets, feebly generalized submaximal spaces and study their basic properties in bitopological spaces.

2. Preliminaries

Through this paper X represent bitopological space (X, τ_1, τ_2), $i, j \in \{1, 2\}$ and $i \neq j$.

Definition 2.1. *A subset A of a topological space (X, τ) is called:*

(1) *a semi-open set [4] if $A \subseteq cl(int(A))$.*
(2) *a semi closed set [4] if $(int(cl(A))) \subseteq A$.*
(3) *a feebly open set [5] if $A \subseteq scl(int(A))$.*
(4) *a feebly closed set [5] if $sint(cl(A)) \subseteq A$.*

Definition 2.2. *A subset A of a bitopological space (X, τ_1, τ_2) is called:*

(1) *(i, j)-feebly generalized closed (briefly (τ_i, τ_j)-fg-closed) [1] set if $\tau_j - fcl(A) \subseteq U$ whenever $A \subseteq U$ and U is feebly open in (X, τ_i).*
(2) *(i, j)-feebly generalized open (briefly (τ_i, τ_j)-fg-open) set if $X - A$ is (τ_i, τ_j)-fg-closed.*

3. Feebly generalized locally closed sets in bitopological spaces

Definition 3.1. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be:*

(1) *$(\tau_i\tau_j)$-feebly generalized locally closed set if $A = U \cap V$ where U is τ_i-fg open set and V is τ_j-fg closed set in X.*
(2) *$(\tau_i\tau_j)$-feebly generalized locally closed* set if $A = U \cap V$ where U is τ_i-fg open set and V is τ_j-feebly closed set in X.*
(3) *$(\tau_i\tau_j)$-feebly generalized locally closed** set if $A = U \cap V$ where U is τ_i-feebly open set and V is τ_j-fg closed set in X.*

Remark 3.1.

(i) *The class of all $(\tau_i\tau_j)$-feebly generalized locally closed sets in (X, τ_1, τ_2) is denoted by $(\tau_i\tau_j)$-$fgLC$ (X, τ_1, τ_2).*
(ii) *The class of all $(\tau_i\tau_j)$-feebly generalized locally closed* sets in (X, τ_1, τ_2) is denoted by $(\tau_i\tau_j)$-$fgLC^*$ (X, τ_1, τ_2).*

(iii) *The class of all* $(\tau_i\tau_j)$*-feebly generalized locally closed*** *sets in* (X, τ_1, τ_2) *is denoted by* $(\tau_i\tau_j)$*-*fg*LC*** (X, τ_1, τ_2).

Example 1. *Let* $X = \{a, b, c\}$. $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

(1) τ_1*-feebly open sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b, c\}\}$.
(2) τ_2*-feebly closed sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$.
(3) τ_1*-*fg *open sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$.
(4) τ_2*-*fg *closed sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$.
(5) $(\tau_1\tau_2)$*-feebly generalized locally closed sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$.
(6) $(\tau_1\tau_2)$*-feebly generalized locally closed** *sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$.
(7) $(\tau_1\tau_2)$*-feebly generalized locally closed*** *sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b, c\}\}$.

Remark 3.2. *Every* τ_j*-feebly closed set is not* $(\tau_i\tau_j)$*-feebly generalized locally closed set in* (X, τ_1, τ_2) *in general, as can be seen from the following example.*

Example 2. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{b\}, \{a, b\}\}$. *Then* τ_2*-feebly closed sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ *and* $(\tau_1\tau_2)$*-feebly generalized locally closed sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{c\}\}$. *Here* $A = \{a, c\}$ *is* τ_2*-feebly closed sets in* (X, τ_1, τ_2) *but not* $(\tau_1\tau_2)$*-feebly generalized locally closed sets in* (X, τ_1, τ_2).

Remark 3.3. *Every* τ_1*-feebly open set is not* $(\tau_1\tau_2)$*-feebly generalized locally closed set in* (X, τ_1, τ_2) *in general as can be seen from the following example.*

Example 3. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$, $\tau_2 = \{\emptyset, X, \{c\}\}$. *Then* τ_1*-feebly open sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ *and* $(\tau_1\tau_2)$*-feebly generalized locally closed sets in* (X, τ_1, τ_2) *are* $\{\emptyset, X, \{b\}\}$. *Here* $A = \{a, b\}$ *is* τ_1*-feebly open sets in* (X, τ_1, τ_2) *but not* $(\tau_1\tau_2)$*-feebly generalized locally closed sets in* (X, τ_1, τ_2).

Proposition 3.1. *In any bitopologicalspace* (X, τ_1, τ_2)

(a) $A \in (\tau_i\tau_j)$*-*fg*LC** $(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_i\tau_j)$*-*$fg$*LC*$(X, \tau_1, \tau_2)$.
(b) $A \in (\tau_i\tau_j)$*-*fg*LC*** $(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_i\tau_j)$*-*$fg$*LC*$(X, \tau_1, \tau_2)$.

Proof.

(a) Since A is $(\tau_i\tau_j)$-fgLC* (X, τ_1, τ_2), then we have $A = U \cap V$ where U is τ_i-fg open set and V is τ_j-fg closed set. Since every τ_j-feebly closed sets are τ_j-fg closed set in (X, τ_1, τ_2), $A = U \cap V$ where U is τ_i-fg open and V is τ_j-fg closed in (X, τ_1, τ_2). Therefore $A \in (\tau_i\tau_j)$-fgLC(X, τ_1, τ_2).

(b) Since A is $(\tau_i\tau_j)$-fgLC$^{**}(X, \tau_1, \tau_2)$, then we have $A = U \cap V$ where U is τ_i-feebly open set and V is τ_j-fg closed set. Since every τ_i-feebly open sets are τ_i-fg open set in (X, τ_1, τ_2), $A = U \cap V$ where U is τ_i-fg open and V is τ_j-fg closed in (X, τ_1, τ_2). Therefore $A \in (\tau_i\tau_j)$-fgLC(X, τ_1, τ_2).

□

Remark 3.4. *The converse of (a) and (b) of the above Theorem need not be true as seen from the following example*

Example 4. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, τ_2=$\{\emptyset, X, \{a\}, \{b, c\}\}$

$(\tau_1\tau_2)$–*fgLC*(X, τ_1, τ_2) = $\{\tau, X, \{a\}, \{a, b\}, \{a, c\}\}$

$(\tau_1\tau_2)$–*fgLC*$^{**}(X, \tau_1, \tau_2)$ = $\{\tau, X, \{a\}\}$.

Here $A = \{a, b\}$ *is* $(\tau_1\tau_2)$-fg *locally closed but not* $(\tau_1\tau_2)$-fg *locally closed** *in* (X, τ_1, τ_2).

Example 5. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, τ_2=$\{\emptyset, X, \{a\}, \{b, c\}\}$

$(\tau_1\tau_2)$–*fgLC*(X, τ_1, τ_2) = $\{\tau, X, \{a\}, \{a, b\}, \{a, c\}\}$

$(\tau_1\tau_2)$–*fgLC*$^{**}(X, \tau_1, \tau_2)$ = $\{\tau, X, \{a\}\}$.

Here $A = \{a, b\}$ *is* $(\tau_1\tau_2)$-FG *locally closed but not* $(\tau_1\tau_2)$-fg *locally closed*** *in* (X, τ_1, τ_2).

Proposition 3.2. *In any bitopological space* (X, τ_1, τ_2)

(a) $A \in (\tau_i\tau_j)$–*fgLC*$(X, \tau_1, \tau_2) \Rightarrow A \in \tau_j$–*fgC*$(X, \tau_1, \tau_2)$.

(b) $A \in (\tau_i\tau_j)$–*fgLC*$(X, \tau_1, \tau_2) \Rightarrow A \in \tau_i$–*fgO*$(X, \tau_1, \tau_2)$.

Proof.

(a) Since $A = A \cap X$ and A is τ_i-FG open set in (X, τ_1, τ_2) and X is τ_j-fg closed in (X, τ_1, τ_2), we have $A \in \tau_j - fgC(X, \tau_1, \tau_2)$.

(b) Since $A = A \cap X$ and A is τ_i-fg open set in (X, τ_1, τ_2) and X is τ_j-fg closed in (X, τ_1, τ_2), we have $A \in \tau_j - fgO(X, \tau_1, \tau_2)$.

□

Remark 3.5. *The converse of (a) and (b) of the above Theorem need not be true as seen from the following example.*

Example 6. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$

$(\tau_1\tau_2)$–*fg*$LC(X,\tau_1,\tau_2) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$
τ_2–*fg*$C(X,\tau_1,\tau_2) = \{$*All the subsets of* $X\}$.
Here $A = \{b,c\}$ *is* $\tau_2 - fg$ *closed but not* $(\tau_1\tau_2)$-fg *locally closed in* (X,τ_1,τ_2).

Example 7. *Let* $X = \{a,b,c\}$. $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{a,b\}\}$
$(\tau_1\tau_2)$–*fg*$LC(X,\tau_1,\tau_2) = \{\emptyset, X, \{b\}\}$
τ_1–*fg*$O(X,\tau_1,\tau_2) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$.
Here $A = \{a,b\}$ *is* $(\tau_1\tau_2)$-fg *open but not* $(\tau_1\tau_2)$-fg *locally closed in* (X,τ_1,τ_2).

Proposition 3.3. *For a subset* A *of a bitopological space* (X,τ_1,τ_2)*, the following are equivalent:*

(a) $A \in (\tau_i\tau_j)$–*fg*LC^*
(b) $A = U \cap [\tau_j - fcl(A)]$ *for some* $\tau_i - fg$*-open set* U.
(c) $A \cup \{X - [\tau_j - fcl(A)]\}$ *is* $\tau_i - fg$*-open.*
(d) $[\tau_j - fcl(A)] - A$ *is* $\tau_i - fg$*-closed.*

Proof.

(a)⇒(b) Since A is $(\tau_i\tau_j) - fg$ locally closed* set in (X,τ_1,τ_2), we have $A = U \cap V$, where U is $\tau_i - fg$ open set and V is $\tau_j - fg$ closed set in X. Since $A \subseteq \tau_j - fcl(A)$ and $A \subseteq U$, we have

$$A \subseteq U \cap [\tau_j - fcl(A)] \tag{3.1}$$

Since $A \subseteq V$ and V is τ_j-feebly closed in X, we have $\tau_j - fcl(A) \subseteq A$. Therefore $U \cap [\tau_j - fcl(A)] \subseteq U \cap V = A$. Hence

$$U \cap [\tau_j - fcl(A)] \subseteq A \tag{3.2}$$

From (3.1) and (3.2) we have $A = U \cap [\tau_j - fcl(A)]$ for some τ_i-fg open set U in (X,τ_1,τ_2).

(b)⇒(a) Suppose that $A = U \cap [\tau_j - fcl(A)]$ for some τ_i-fg open set U in X. Since $\tau_j - fcl(A)$ is τ_j-feebly closed in (X,τ_i,τ_j) and U is τ_i-fg open set in (X,τ_i,τ_j), we have $A \in (\tau_i\tau_j) - fgLC^*(X,\tau_i,\tau_j)$.

(b)⇒(c) Since $A = U \cap [\tau_j - fcl(A)]$ for some τ_i-fg open set U in (X,τ_i,τ_j), we have $A \cup \{X - [\tau_j - fcl(A)]\} = \{U \cap [\tau_j - fcl(A)]\} \cup \{X - [\tau_j - fcl(A)]\} = U$. Therefore $A \cup \{X - [\tau_j - fcl(A)]\}$ is τ_i-fg open.

(c)⇒(b) Suppose that $A \cup \{X - [\tau_j - fcl(A)]\}$ is τ_i-fg open in (X, τ_1, τ_2). Let $U = A \cup \{X - [\tau_j - fcl(A)]\}$. Then U is τ_i-fg open in (X, τ_1, τ_2). Now,

$$\begin{aligned} U \cap [\tau_j - fcl(A)] &= [A \cup \{X - [\tau_j - fcl(A)]\}] \cap [\tau_j - fcl(A)] \\ &= \{A \cup [\tau_j - fcl(A)]^c\} \cap [\tau_j - fcl(A)] \\ &= \{A \cap [\tau_j - fcl(A)]\} \cup \{[\tau_j - fcl(A)]^c\} \cap [\tau_j - fcl(A)] \\ &= A \cup \emptyset = A \end{aligned}$$

Therefore $A = U \cap [\tau_j - fcl(A)]$ for some τ_i-fg open set U in (X, τ_1, τ_2).

(c)⇒(d) Suppose that $A \cup \{X - [\tau_j - fcl(A)]\}$ is τ_i-fg open in (X, τ_1, τ_2). Let $U = A \cup \{X - [\tau_j - fcl(A)]\}$. Since U is $\tau_i - fg$ open in (X, τ_1, τ_2), we have $X - U$ is U is τ_i-fg closed in (X, τ_1, τ_2). Now

$$\begin{aligned} X - U &= X - [A \cup \{X - [\tau_j - fcl(A)]\}] \\ &= (X - A) \cap \{X - [\tau_j - fcl(A)]\} \\ &= [\tau_j - fcl(A)] - A \end{aligned}$$

Therefore $[\tau_j - fcl(A)] - A$ is τ_i-fg closed in (X, τ_1, τ_2).

(d) ⇒(c) Suppose that $[\tau_j - fcl(A)] - A$ is $\tau_i - fg$ closed in (X, τ_1, τ_2). Let $V = [\tau_j - fcl(A)] - A$. Then V is τ_i-fg closed in $(X, \tau_1, \tau_2) \Rightarrow X - V$ is τ_i-fg open in (X, τ_1, τ_2). Now

$$\begin{aligned} X - V &= X - \{[\tau_j - fcl(A)] - A\} \\ &= X \cap \{[\tau_j - fcl(A)] - A\}^c \\ &= X \cap \{[\tau_j - fcl(A)] \cap A^c\}^c \\ &= X \cap \{[\tau_j - fcl(A)]^c \cup (A^c)^c\} \\ &= X \cap \{[\tau_j - fcl(A)]^c \cup A\} \\ &= \{X \cap [\tau_j - fcl(A)]^c\} \cup \{X \cap A\} \\ &= [\tau_j - fcl(A)]^c \cup A \end{aligned}$$

Hence $A \cup \{X - [\tau_j - fcl(A)]\}$ is τ_1-fg closed (X, τ_1, τ_2).

□

Proposition 3.4. *In bitopological spaces (X, τ_1, τ_2) the following are equivalent:*

(a) *$A - [\tau_i - fint(A)]$ is τ_j-fg open in (X, τ_1, τ_2).*
(b) *$[\tau_i - fint(A)] \cup [X - A]$ is τ_j- fg closed in (X, τ_1, τ_2).*
(c) *$U \cup [\tau_i - fint(A)] = A$ for some τ_j-fg open set U in (X, τ_1, τ_2).*

Proof.

(a) $\Rightarrow$ (b)

$$\begin{aligned} X - \{A - [\tau_i - fint(A)]\} &= X \cap \{A - [\tau_i - fint(A)]\}^c \\ &= X \cap [A \cap \{\tau_i - fint(A)\}^c]^c \\ &= X \cap \{A^c \cup [\{\tau_i - fint(A)\}^c]^c\} \\ &= X \cap \{A^c \cup [\tau_i - fint(A)]\} \\ &= \{A^c \cup [\tau_i - fint(A)]\} \\ &= [\tau_i - fint(A)] \cup [X - A] \end{aligned}$$

Since $A - [\tau_i - fint(A)]$ is τ_j-fg open, we have $X - \{A - [\tau_i - fint(A)\} = [\tau_i - fint(A)] \cup [X - A]$ is τ_j- fg closed in (X, τ_1, τ_2).

(b)$\Rightarrow$(a) Suppose that $[\tau_i - fint(A)] \cup [X - A]$ is τ_2- fg closed in (X, τ_1, τ_2). Since we have $X - [\tau_i - fint(A)] \cup [X - A]$ is τ_j- fg open. Now

$$\begin{aligned} X - \{[\tau_i - fint(A)] \cup [X - A]\} &= X \cap \{[\tau_i - fint(A)] \cup [X - A]\}^c \\ &= X \cap \{[\tau_i - fint(A)] \cup A^c\}^c \\ &= X \cap \{[\tau_i - fint(A)]^c \cap (A^c)^c\} \\ &= X \cap \{[\tau_i - fint(A)]^c \cap A\} \\ &= A \cap [\tau_i - fint(A)]^c \\ &= A - [\tau_i - fint(A)] \end{aligned}$$

Therefore $A - [\tau_i - fint(A)]$ is τ_j- fg closed in (X, τ_1, τ_2).

(b)$\Rightarrow$(c) Suppose that $\{[\tau_i - fint(A)] \cup [X - A]\}$ is τ_j- fg closed in (X, τ_1, τ_2). Let $W = \{[\tau_i - fint(A)] \cup [X - A]\}$. Then W is τ_j- fg closed. Then W^c is τ_j-

fg open. Now ,

$$\begin{aligned} W^c \cup [\tau_i - fint(A)] &= \{[[\tau_i - fint(A)] \cup [X - A]]\}^c \cup [\tau_i - fint(A)] \\ &= \{[\tau_i - fint(A)]^c \cap (A^c)^c\} \cup [\tau_i - fint(A)] \\ &= \{[\tau_i - fint(A)]^c \cap A\} \cup [\tau_i - fint(A)] \\ &= \{[\tau_i - fint(A)]^c \cup [\tau_i - fint(A)]\} \cap \{A \cup [\tau_i - fint(A)]\} \\ &= X \cap A = A. \end{aligned}$$

Take $U = W^c$. Then $A = U \cup [\tau_i - fint(A)] = A$ for some τ_j- fg open in (X, τ_1, τ_2).

(c)⇒(b) Suppose that $A = U \cup [\tau_i - fint(A)] = A$ for some τ_j- fg open set in (X, τ_i, τ_j). Now,

$$\begin{aligned} \{[\tau_i - fint(A)] \cup [X - A]\} &= [\tau_i - fint(A)] \cup A^c \\ &= [\tau_i - fint(A)] \cup \{U \cup [\tau_i - fint(A)]\}^c \\ &= [\tau_i - fint(A)] \cup \{U^c \cap [\tau_i - fint(A)]^c\} \\ &= \{[\tau_i - fint(A)] \cup U^c\} \cap \{[\tau_i - fint(A)] \\ &\qquad \cup [\tau_1 - fint(A)]^c\} \\ &= \{[\tau_i - fint(A)] \cup U^c\} \cap \{X\} \\ &= \{[\tau_i - fint(A)] \cup U^c\} \\ &= X - U \end{aligned}$$

Since U is τ_j- fg open set in (X, τ_1, τ_2), we have $X - U$ is τ_j- fg closed in (X, τ_1, τ_2). Therefore $\{[\tau_i - fint(A)] \cup [X - A]\}$ is τ_j- fg closed in (X, τ_1, τ_2).

□

4. fg- DENSE SETS AND fg-SUBMAXIMAL SPACES IN BITOPOLOGICAL SPACES

We introduce following definitions:

Definition 4.1. *A subset A of a space (X, τ_1, τ_2) is called $(\tau_i\tau_j)$-fg- dense if $(\tau_i\tau_j)$–fg$cl(A) = X$.*

Example 8. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{b, c\}\}$, $\tau_2 = \{\emptyset, X, \{a, b\}\}$.
$(\tau_1\tau_2)$*–fg*$C(X, \tau_1, \tau_2) = \{\emptyset, X, \{c\}, \{a, c\}\}$.
$(\tau_1\tau_2) - fg$ *dense sets are* $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$.

Definition 4.2. *A bitopological space* (X, τ_1, τ_2) *is called*

(1) $(\tau_i\tau_j)$-fg *submaximal space if every* τ_i*-feebly dense subset of* X *is* τ_j-fg *open in* X.
(2) $(\tau_i\tau_j)$-fg *submaximal space if every* τ_j*-feebly dense subset of* X *is* τ_i-fg *open in* X.

Proposition 4.1. *Every* $(\tau_i\tau_j)$-fg *dense set in* (X, τ_i, τ_j) *is* τ_i*-feebly dense.*

Proof. Let A be an $(\tau_i\tau_j)$-fg dense set in (X, τ_i, τ_j). Then $(\tau_i\tau_j) - fgcl(A) = X$. Since $(\tau_i\tau_j) - fgcl(A) \subseteq \tau_i - fcl(A)$, we have $\tau_i - fcl(A) = X$. Therefore A is τ_i-feebly dense. □

Remark 4.1. *The converse of the above Theorem need not be true, as seen from the following example.*

Example 9. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{a\}\}$.
$(\tau_1\tau_2)$-fg *dense set in* $(X, \tau_1, \tau_2) = \{\emptyset, X, \{a, c\}\}$.
τ_1*-feebly dense sets are* $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$.
Here $A = \{b, c\}$ *is* τ_1*-feebly dense set but not* $(\tau_1\tau_2)$-fg *dense set.*

Proposition 4.2. *Every* $(\tau_i\tau_j)$-fg *dense set in* (X, τ_1, τ_2) *is* τ_j*-feebly dense.*

Proof. Let A be an $(\tau_i\tau_j)$-fg dense set in (X, τ_i, τ_j). Then $(\tau_i\tau_j) - fgcl(A) = X$. Since $(\tau_i\tau_j)$–fg$cl(A) \subseteq \tau_j - fcl(A)$, we have $\tau_j - fcl(A) = X$. Therefore A is τ_j-feebly dense. □

Remark 4.2. *The converse of the above proposition need not be true as seen from the following example.*

Example 10. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{a\}\}$.
$(\tau_1\tau_2)$-fg *dense set in* $(X, \tau_1, \tau_2) = \{\emptyset, X, \{a, c\}\}$.
τ_2*-feebly dense sets are* $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$.
Here $A = \{a, b\}$ *is* τ_2*-feebly dense set but not* $(\tau_1\tau_2)$-fg *dense set.*

Proposition 4.3. *If a bitopological space* X *is* $(\tau_1\tau_2)$*- submaximal space then* X *is* $(\tau_1\tau_2)$-fg *submaximal space.*

Proof. Since X is $(\tau_1\tau_2)$- submaximal space. We have every τ_1-dense subset of X is τ_2-open in X. Since every τ_2- feebly open set X is τ_2-fg open in X, we have every τ_1- dense subset of X is τ_2-fg open in X. Therefore (X, τ_i, τ_j) is $(\tau_1\tau_2)$-fg submaximal space. $\square$

Proposition 4.4. *A bitopological space (X, τ_1, τ_2) is $(\tau_i\tau_j)$-fg submaximal space iff $\tau_2\tau_1$–fg$LC^*(X, \tau_1, \tau_2) = \mathcal{P}(X)$.*

Proof. Suppose that (X, τ_1, τ_2) is $(\tau_1\tau_2)$-fg submaximal space. Obviously $\tau_j\tau_i - fgLC^*(X, \tau_i, \tau_j) \subseteq \mathcal{P}(X)$. Let $A \in \mathcal{P}(X)$ and $Y = A \cup \{X - [\tau_i - fcl(A)]\}$. Since $[\tau_i - fcl(A)] = X$, we have Y is τ_i-feebly dense subset of X. Since (X, τ_i, τ_j) is $(\tau_i\tau_j)$-fg submaximal space, we have Y is τ_j-fg open in X. Since every $\tau_j - fg$ open set in X is $\tau_j\tau_i - fgLC^*$ set in (X, τ_i, τ_j), we have $Y \in \tau_j\tau_i - fgLC^*(X, \tau_i, \tau_j)$. Therefore $\mathcal{P}(X) \subseteq \tau_j\tau_i-fgLC^*(X, \tau_i, \tau_j)$. According to (3.1) and (3.2), we have $\tau_j\tau_i-$fg$LC^*(X, \tau_i, \tau_j) = \mathcal{P}(X)$.

Conversely, suppose that $\tau_j\tau_i-$fg$LC^*(X, \tau_1, \tau_2) = \mathcal{P}(X)$. Let A be the τ_1-feebly dense subset of X. Then $A \cup \{X - [\tau_i - fcl(A)]\} = A \cup [\tau_i - fcl(A)]^c = A$. Therefore $A \in \tau_j\tau_i-$fg$LC^*(X, \tau_i, \tau_j) \Rightarrow A$ is τ_j-fg open in (X, τ_i, τ_j) [by Theorem door space]. Hence X is $(\tau_i\tau_j)$-fg submaximal space. $\square$

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PG AND RESEARCH DEPARTMENT OF MATHEMATICS
A.P.C. MAHALAXMI COLLEGE FOR WOMEN, THOOTHUKUDI
Affiliated to Manonmaniam Sundaranar University, Tirunelveli. TN, INDIA
E-mail address: vanikathir16@gmail.com

PG AND RESEARCH DEPARTMENT OF MATHEMATICS
A.P.C. MAHALAXMI COLLEGE FOR WOMEN, THOOTHUKUDI
Affiliated to Manonmaniam Sundaranar University, Tirunelveli. TN, INDIA
E-mail address: baladeepa@apcmcollege.ac.in

DEPARTMENT OF MATHEMATICS
V.O. CHIDAMBARAM COLLEGE, THOOTHUKUDI
Affiliated to Manonmaniam Sundaranar University, Tirunelveli. TN, INDIA