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# FIBONACCI MEAN ANTI-MAGIC LABELING OF SOME GRAPHS

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ABSTRACT. Let $G = (V(G), E(G))$ be a simple, finite, connected and undirected graph. A Fibonacci Mean Anti-Magic labeling of a graph $G$ is an injective function $g : V(G) \to \{f_2, f_3, ..., f_{n+1}\}$ where $f_n$ is the $n_{th}$ Fibonacci number with the induced function $g^* : E(G) \to N$ defined by,

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u) + g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u) + g(v) \text{ is odd} \end{cases}$$

and all these edge labelings are distinct. The graph which admits Fibonacci Mean Anti-Magic Labeling is a Fibonacci Mean Anti-Magic graph. In this paper, we investigate this labeling for some special graphs.

## 1. INTRODUCTION

Let $G = (V(G), E(G))$ with $p$ vertices and $q$ edges be a simple, finite, connected and undirected graph. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling can be found in J.A.Gallian(2014). S.Somasundaram and R.Ponraj in [4] introduced the notion of mean labeling of graphs. The concept of Fibonacci Mean Anti-Magic Labeling in graphs was introduced by Ameenal Bibi and T. Ranjani in [1]. In 1994, N .Hartsfield and G. Ringel introduced the concept of Anti-Magic labeling, [2].

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## 2. PRELIMINARIES

**Definition 2.1.** *[5] Each vertex labeling $f$ of a graph $G$ be a $(p, q)$ graph from $\{0, 1, 2, ..., q\}$ induces a edge labeling $gf$ where $g^*(e)$ is sum the labels of end vertices of an edge $e$. Labeling $f$ is called anti-magic if and only if all the edge labelings are pair wise distinct.*

**Definition 2.2.** *By an edge anti-magic vertex labeling we mean a one-to-one mapping $V(G)$ into $\{0, 1, 2, ..., q\}$ such that the set of edge weights of all edges in $G$ is $\{1, 2, ..., q\}$.*

Different kinds of anti-magic graphs were studied by T. Nicholas, S. Somasundaram and V. Vilfred in [3].

**Definition 2.3.** *A graph $G$ with $p$ vertices and $q$ edges is a mean graph if there is an injective function $f$ from the vertices of $G$ to $\{1, 2, ..., q\}$ such that when each edge labeled with $(f(u) + f(v))/2$ if $f(u) + f(v)$ is even and $(f(u) + f(v) + 1)/2$ if $f(u) + f(v)$ is odd, the resulting edges are distinct.*

**Definition 2.4.** *The Fibonacci numbers can be defined by the linear recurrence relation $F_n = F_(n-1) + F_(n-2);\ n \geq 3$. This generates the infinite sequence of integers 1,2,3,5,8,13,21,34,55,89,144,...*

**Definition 2.5.** *A walk in which no vertex is repeated is called path. A path with $n$ vertices is denoted as $P_n$. A path from $v_0$ to $v_n$ is denoted as $v_0 - v_n$ path.*

**Definition 2.6.** *A closed path is called a cycle. A cycle with $n$ vertices is denoted by $C_n$.*

**Definition 2.7.** *A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. It is denoted by $K_n$*

**Definition 2.8.** *The Cartesian product $G = G_1 \times G_2$ sometimes simply called the graph product and denoted by $G_1 \times G_2$ of graphs $G_1$ and $G_2$ with disjoint point sets $V_1$ and $V_2$ and edge sets $X_1$ and $X_2$ is the graph with point set $V_1 \times V_2$ and $u = (u_1, u_2)$ adjacent with $v = (v_1, v_2)$ whenever $[u_1 = v_1$ and $u_2\ adj\ v_2]$ or $[u_2 = v_2$ and $u_1\ adj\ v_1]$.*

## 3. Main Results

**Definition 3.1.** *Book graph is a cartesian product of a star and a single edge, denoted by $B_n$. The $n$ -book graph is defined as the cartesian product $S_{n+1} \times P_2$, where $S_{n+1}$ is a star graph and $P_2$ is the path graph.*

**Theorem 3.1.** *The Book graph Bn is a Fibonacci Mean Anti-magic graph for $n \geq 2$.*

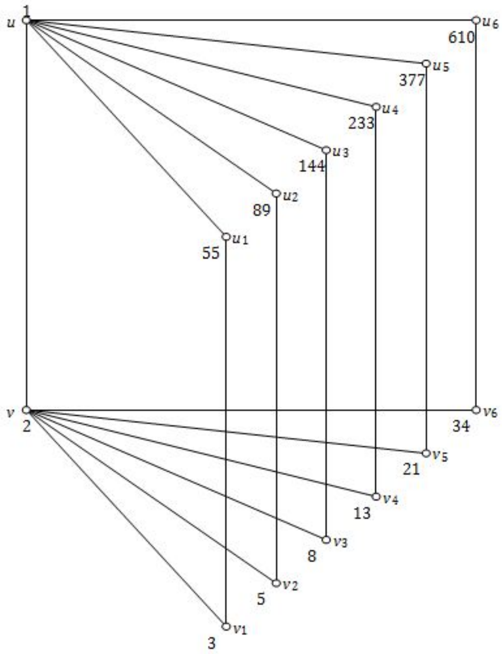


FIGURE 1. $B_6$

*Proof.* Let $G = B_n$ be a book graph. Let $V(G) = \{u, v, u_i, v_i/1 \leq i \leq n\}$ be the vertex set and $E(G) = \{uv\} \cup \{uu_i/1 \leq i \leq n\} \cup \{vv_i/1 \leq i \leq n\} \cup \{u_iv_i/1 \leq i \leq n\}$ be the edge set.

Define $g : V(G) \to \{f_2, f_3, ..., f_{n+1}\}$ by

$$\begin{aligned} g(u) &= f_2, \\ g(v) &= f_3, \\ g(v_i) &= f_{i+3},\ 1 \leq i \leq n, \\ g(u_i) &= f_{n+i+3},\ 1 \leq i \leq n \end{aligned}$$

Then the induced function $g^* : E(G) \to N$ given by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u)+g(v) \text{ is odd} \end{cases}$$

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-magic labeling and so $B_n$ $(n \geq 2)$ is a Fibonacci Mean Antimagic graph. □

**Illustration 3.1.** *The illustration of the book graph $B_n$ when $n = 6$ is shown in figure 1.*

**Theorem 3.2.** *Complete graph $K_n$ $(n \geq 3)$ are not Fibonacci Mean $A_n$-timagic graphs.*

*Proof.* Let $G = K_n$ be the complete graph on $n$ vertices (say) $v_1, v_2, ..., v_n$.
They must receive labels $f_2, f_3, ..., f_{(n+1)}$ in some order. Let $v_i,\ v_j,\ v_k$ receive 1,2,3 as labels respectively, i.e. $g(v_i) = 1,\ g(v_j) = 2,\ g(v_k) = 3$.
Then $v_i, v_j$ and $v_k$ have the same edge label 2. Thus, the labels induced by the function $g^* : E(G) \to N$ are not distinct.
Therefore, the graph $K_n$ is not a Fibonacci Mean Antimagic graph. □

**Definition 3.2.** *The corona of $G$ with $H$, $G \odot H$ is the graph obtained by taking one copy of $G$ and $P$ copies of $H$ and joining the $i^{th}$ vertex of $G$ with an edge to every vertex in the $i^{th}$ copy of $H$. $P_n \odot K_1$ is called the comb and $P_n \odot 2K_1$ is called the double comb.*

**Theorem 3.3.** *The graph $P_n \odot 2K_1$ is a Fibonacci Mean Antimagic graph.*

*Proof.* Let $G = P_n \odot 2K_1$ be a graph. Let $v_i$ $(1 \leq i \leq n)$ be the vertices of the path $P_n$. Let $V(G) = \{u_i, v_i, w_i/1 \leq i \leq n\}$ be the vertex set and $E(G) = \{u_iv_i/1 \leq i \leq n\} \cup \{v_iv_{n+1}/1 \leq i \leq n-1\} \cup \{v_iw_i/1 \leq i \leq n\}$ be the edge set.

Define $g : V(G) \to \{f_2, f3, ..., f_{n+1}\}$ by

$$g(u_i) = f_{i+1},\ 1 \leq i \leq n,$$
$$g(v_i) = f_{n+i+1},\ 1 \leq i \leq n,$$
$$g(w_i) = f_{2n+i+1},\ 1 \leq i \leq n$$

Then the induced function $g^* : E(G) \to N$ given by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u) + g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u) + g(v) \text{ is odd.} \end{cases}$$

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-magic labeling and so $P_n \odot 2K_1$ is a Fibonacci Mean Antimagic graph. □

**Illustration 3.2.** *The illustration of the graph $P_6 \odot 2k_1$ is given in figure 2.*

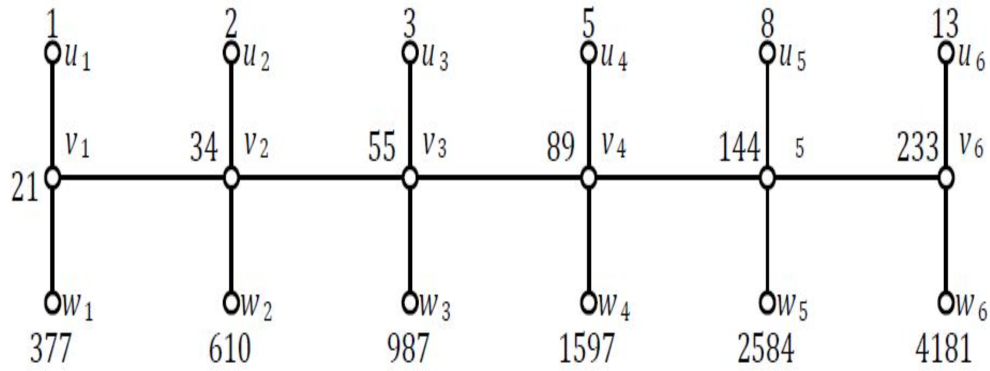


FIGURE 2. $P_6 \odot 2k_1$

**Definition 3.3.** *An $(n, m)$ balloon tree is a graph obtained by connecting one leaf of each of $n$-copies of an $m$-star graph. Let us denote it by $BL_{n,m}$.*

**Theorem 3.4.** *The balloon tree graph $BL_{n,m}$ is a Fibonacci Mean Antimagic graph.*

*Proof.* Let $G = BL_{n,m}$ be a balloon tree graph. Let $V(G) = \{u_{00}, u_{ij}/1 \leq i \leq n,\ 1 \leq j \leq m\}$ be the vertex set and $E(G) = \{u_{00}u_{i0}/1 \leq i \leq n\} \cup \{u_{i0}u_{ij}/1 \leq i \leq n,\ 1 \leq j \leq m\}$ be the edge set.
Define $g : V(G) \to \{f_2, f_3, ...f_{n+1}\}$ by

$$g(u_{ij}) = \begin{cases} f_2 & if\ i = 0,\ j = 0 \\ f_{i(m+1)-m+j+2} & if\ 1 \leq i \leq n,\ 0 \leq j \leq m \end{cases}$$

Then the induced function $g^* : E(G) \to N$ given by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u) + g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u) + g(v) \text{ is odd.} \end{cases}$$

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-magic labeling and so $BL_{n,m}$ is a Fibonacci Mean Antimagic graph. □

**Illustration 3.3.** *The illustration of the balloon tree graph $BL_{2,7}$ is given in figure 3.*

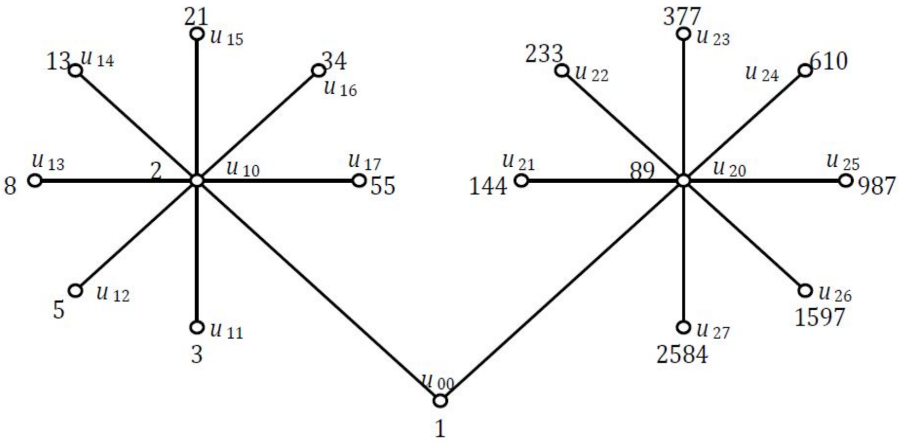


FIGURE 3. $BL_{2,7}$

**Definition 3.4.** *An $(n, m)$ banana tree is a graph obtained by connecting one leaf of each of $n$-copies of an $m$-star graph with a single root vertex that is distinct from all the stars and we denote it by $Ba_{n,m}$.*

**Theorem 3.5.** *The banana tree graph $Ba_{n,m}$ is a Fibonacci Mean Antimagic graph.*

*Proof.* Let $G = Ba_{n,m}$ be a banana tree graph. Let $V(G) = \{u_{00}, u_{ij}/1 \leq i \leq n,\ 1 \leq j \leq m\}$ be the vertex set and $E(G) = \{u_{00}u_{i1}/1 \leq i \leq n\} \cup \{u_{i2}u_{i1}/1 \leq i \leq n\} \cup \{u_{i2}u_{ij}/1 \leq i \leq n,\ 1 \leq j \leq m\}$ be the edge set.
Define $g : V(G) \to \{f_2, f_3, ..., f_{n+1}\}$ by,

$$g(u_{ij}) = \begin{cases} f_2 & if\ i = 0,\ j = 0 \\ f_{i(m+1)-m+j+2} & if\ 1 \leq i \leq n,\ 0 \leq j \leq m. \end{cases}$$

Then the induced function $g^* : E(G) \to N$ is given by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u) + g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u) + g(v) \text{ is odd}. \end{cases}$$

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-magic labeling and so $Ba_{n,m}$ is a Fibonacci Mean Antimagic graph. □

**Illustration 3.4.** *The illustration of the banana tree graph $Ba_{4,4}$ is given in figure 4.*

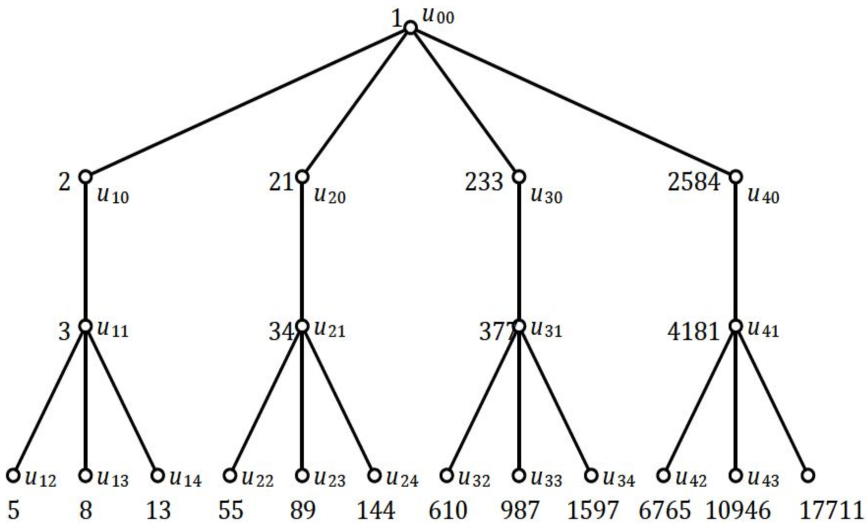


FIGURE 4. $Ba_{4,4}$

**Definition 3.5.** *The $H$-graph of path $P_n$ is the graph obtained from two copies of $P_n$ with vertices $v_1, v_2, ..., v_n$ and $u_1, u_2, ..., u_n$ by joining the vertices $U\frac{n+1}{2}$ and $U\frac{n+1}{2}$ if $n$ is odd and the vertices $U\frac{n}{2}+1$ and $U\frac{n}{2}$ if $n$ is even*

**Theorem 3.6.** *The $H$-graph of path $P_n$ is a Fibonacci Mean Antimagic graph.*

*Proof.* Let $G = H$ of path $P_n$ be a graph. $Let V(G) = \{u_i, v_i/1 \leq i \leq n\}$ be the vertex set and
$E(G) = \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{u\lceil\frac{n}{2}\rceil v\lceil\frac{n}{2}\rceil\}$
be the edge set when $n$ is odd and $E(G) = \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{u\frac{n}{2}v\frac{n}{2}+1\}$ be the edge set when $n$ is even.

We define $g : V(G) \to \{f_2, f_3, ...f_{n+1}\}$ by

$$f(u_i) = F_{i+1}, \quad 1 \leq i \leq n$$
$$f(v_i) = F_{n+i+1}, \quad 1 \leq i \leq n.$$

Then the induced function $g^* : E(G) \to N$ is given by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u) + g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u) + g(v) \text{ is odd.} \end{cases}$$

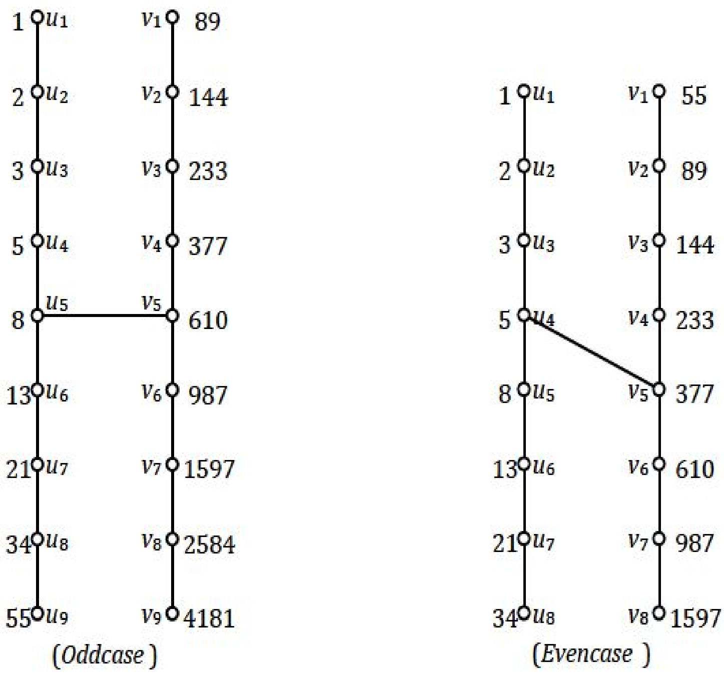


FIGURE 5. $H$ graph

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-Magic labeling and so $H$-graph is a Fibonaci Mean Antimagic graph. □

**Illustration 3.5.** *The illustration of $H$-graph is given in figure 5.*

**Definition 3.6.** *A two-dimensional grid graph, also known as a rectangular grid graph or two-dimensional lattice graph is an $m \times n$ lattice graph that is the graph cartesian product $P_m \times P_n$ of path graphs on $m$ and $n$ vertices. The $m \times n$ grid graph is denoted as $L_{m,n}$.*

**Theorem 3.7.** *The grid graph $L_{m,n}$ is a Fibonacci Mean Antimagic graph.*

*Proof.* Let $G = L(m,n)$ be a graph. Let $V(G) = \{u_{ij}/1 \leq i \leq m,\ 1 \leq j \leq n\}$ be the vertex set and $E(G) = \{u_{ij}u_{i(j+1)}/1 \leq i \leq m,\ 1 \leq j \leq n-1\} \cup \{u_{ij}u_{(i+1)j}/1 \leq i \leq m-1,\ 1 \leq j \leq n\}$ be the edge set.
Define $g : V(G) \to \{f_2, f_3, ...f_{n+1}\}$ by,

$$g(u_{ij}) = \begin{cases} f_{n(i-1)+j+1} & if \quad i \ is \ odd,\ 1 \leq j \leq n \\ f_{ni-j+2} & if \quad i \ is \ even,\ 1 \leq j \leq n. \end{cases}$$

Then the induced function $g^* : E(G) \to N$ is given by,

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u)+g(v) \text{ is odd}. \end{cases}$$

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-magic labeling and so $L_{m,n}$ is a Fibonacci Mean Antimagic graph.

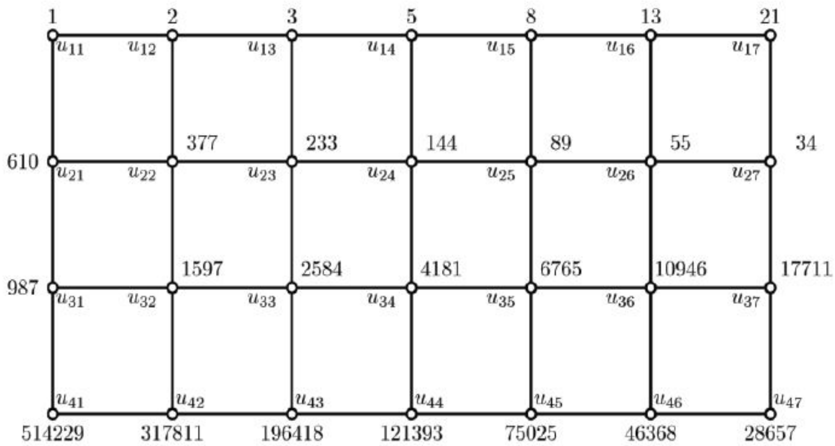


FIGURE 6. L(4,7)

□

**Illustration 3.6.** *The illustration of the grid-graph $L(4,7)$ is given in figure 6.*

**Definition 3.7.** *An armed crown is a graph in which path $P_m$ is attached at each vertex of cycle $C_n$. This graph is denoted by $C_m \oplus P_m$.*

**Theorem 3.8.** *The graph $C_n \oplus P_m$ is a Fibonacci Mean Antimagic graph.*

*Proof.* Let $C_n \oplus P_m$ be a graph. Let $V(G) = \{u_{ij}/1 \leq i \leq n,\ 1 \leq j \leq m\}$ be the vertex set and $E(G) = \{u_{ij}u_{i(j+1)}/1 \leq i \leq n,\ 1 \leq j \leq m-1\} \cup \{u_{i3}u_{(i+1)3}/1 \leq i \leq n-1\}$ be the edge set.
Define $g : V(G) \to \{f_2, f_3, ...f_{n+1}\}$ by $g(u_{ij}) = f_{n(j-1)+i+1},\ 1 \leq i \leq n,\ 1 \leq j \leq m$. Then the induced function $g^* : E(G) \to N$ is given by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u)+g(v) \text{ is odd.} \end{cases}$$

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-magic labeling and so $C_n \oplus P_m$ is a Fibonacci Mean Antimagic graph.

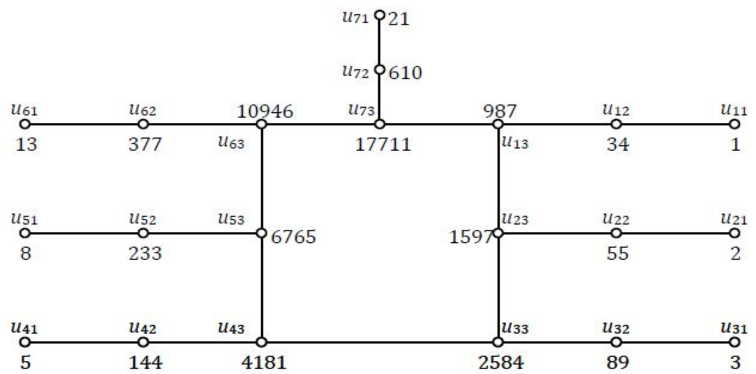


FIGURE 7. $C_7 \oplus P_3$

□

**Illustration 3.7.** *The illustration of the armed crown $C_7 \oplus P_3$ is given in figure 7.*

**Definition 3.8.** *Umbrella is the graph obtained from fan by joining a path $P_m$ to a middle vertex of path $P_n$ in fan $F_n$. It is denoted by $U_{m,n}$.*

**Theorem 3.9.** *The umbrella graph $U_{m,n}$ is a Fibonacci Mean Antimagic graph.*

*Proof.* Let $u_{(m,n)}$ be a graph. Let $V(G) = \{u_i, v_i/1 \leq i \leq m,\ 1 \leq i \leq n\}$ be the vertex set and $E(G) = \{u_iu_{i+1}/1 \leq i \leq m-1\} \cup \{u_iv_n/1 \leq i \leq m\} \cup \{v_iv_{i+1}/1 \leq i \leq n-1\}$ be the edge set.

Define $g : V(G) \to \{f_2, f_3, ...f_{n+1}\}$ by

$$\begin{aligned} g(u_i) &= F_{n+i+1},\ 1 \leq i \leq m \\ g(v_i) &= F_{n-i+2},\ 1 \leq i \leq n. \end{aligned}$$

Then the induced function $g^* : E(G) \to N$ is given by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & if\ g(u) + g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & if\ g(u) + g(v) \text{ is odd.} \end{cases}$$

We observe that the labels are all distinct. Hence the function $g$ is a Fibonacci Mean Anti-magic labeling and so $U(m, n)$ is a Fibonacci Mean Antimagic graph.

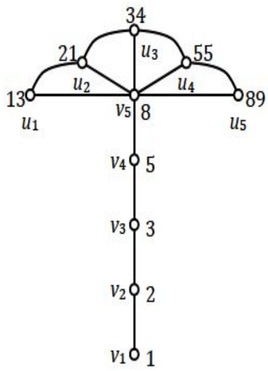


FIGURE 8. $U(5, 5)$

□

**Illustration 3.8.** *The illustration of the umbrella graph $U(5, 5)$ is given in figure 8.*

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