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INTERVAL VALUED INTUITIONISTIC FUZZY BI-IDEALS IN BOOLEAN LIKE SEMI RINGS

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ABSTRACT. The main motivation of this paper is to introduce the concept of Interval valued Intuitionistic Fuzzy Bi-ideals in Boolean like semi-rings. We discuss the results of all these newly introduced structures in detail. We also obtain some characterisations and complete theorems for Boolean like semi-rings.

1. INTRODUCTION

The concept of Fuzzy set was introduced by Zadeh. In 1986, Atanassov evaluated the concept of an Intuitionistic fuzzy set (IFS). In the sense of Atanassov, an IFS is characterized by a pair of functions valued in [0,1]: the membership function and the non-membership function. V. Vetrivel and P. Murugadas have created the concept of Interval valued Intuitionistic Fuzzy Bi-ideals in Gamma Near-rings [4] and R. Rajeswari, N. Meenakumari and S. Ragha [2] have introduced the concept of Interval valued Fuzzy Bi-ideals in Boolean like Semi rings. In this paper, we recreate the concept of Interval valued Intuitionistic Fuzzy Bi-ideals in Gamma Nearrings [4] into Interval valued intuitionistic Fuzzy Bi-ideals in Boolean like semi rings and investigate some of their properties.

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2. Preliminaries

Definition 2.1. *A non empty set R with two binary operations '+' and '$\cdot$' is called a **near-ring** [1] if the following hold:*

(i) $(R, +)$ *is a group;*
(ii) $(R, \cdot)$ *is a semigroup;*
(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ *for all* $x, y, z \in R$.

Definition 2.2. *A system $(R, +, \cdot)$ a **Boolean semi ring** [3] if and only if the following properties hold:*

(i) $(R, +)$ *is an additive (Abelian) group (whose 'zero' will be denoted by '0');*
(ii) $(R, \cdot)$ *is a semigroup of idempotents in the sense* $aa = a$, $va \in R$;
(iii) $a(b + c) = ab + ac$;
(iv) $abc = bac$ *for all* $a, b, c \in R$.

Definition 2.3. *A non-empty set R together with two binary operations + and $\cdot$ satisfying the following conditions is called a **Boolean like semi-ring** [3]:*

(i) $(R, +)$ *is an abelian group;*
(ii) $(R, \cdot)$ *is a semigroup;*
(iii) $a \cdot (b + c) = a \cdot b + a \cdot c$ *for all* $a, b, c \in R$;
(iv) $a + a = 0$, *for all* $a \in R$;
(v) $ab(a + b + ab) = ab$, *for all* $a, b \in R$.

Definition 2.4. *A subgroup B of $(N, +)$ is said to be **bi-ideal** of N if* $BNB \cap (BN) * B \subseteq B$.

In the case of zero symmetric near ring a subgroup B of $(N, +)$ is a bi-ideal [1] $BNB \subseteq B$.

Definition 2.5. *Let μ be a fuzzy set defined on R. Then μ is said to be a **fuzzy bi-ideal** [1] of R if:*

(i) $\mu(x - y) \geq min\{\mu(x), \mu(y)\}$, *for all* $x, y \in R$,
(ii) $\mu(xyz) \geq min\{\mu(x), \mu(z)\}$, *for all* $x, y, z \in R$.

Definition 2.6. *Let R and S be Boolean like semi-rings. A map $f : R \to S$ is called a **Boolean like semiring homomorphism** if $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in R$.*

Definition 2.7. *Let X be any set. A mapping $\widetilde{\eta}$: $X \to D[0,1]$ is called an interval valued fuzzy subset of X, where $D[0,1]$ denotes the family of closed subintervals of $[0,1]$ and $\widetilde{\eta}\ (x) = [\eta^-(x), \eta^+(x)]$ for all $x \in X$, where η^- and η^+ are fuzzy subsets of X such that $\eta^-(x) \leq \eta^+(x)$ for all $x \in X$.*

Definition 2.8. *An interval number $\bar{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ and where a^- and a^+ are the lower and upper limits of $\tilde{a}$ respectively. We also identify the interval $[a, a]$ by the number $a \in [0,1]$.*

For any interval numbers $\bar{a}_j = [a_j^-, a_j^+], \bar{b}_j = [b_j^-, b_j^+] \in D[0,1],\ j \in \Omega$ (where Ω is index set) we define:

$$\begin{aligned} max^i\{\bar{a}_j, \bar{b}_j\} &= [max\{a_j^-, b_j^-\}, max\{a_j^+, b_j^+\}] \\ min^i\{\bar{a}_j, \bar{b}_j\} &= [min\{a_j^-, b_j^-\}, min\{a_j^+, b_j^+\}] \\ inf^i \bar{a}_j &= [\cap_{j\in\Omega} a_j^-,\ \cap_{j\in\Omega} a_j^+] \\ sup^i \bar{a}_j &= [\cup_{j\in\Omega} a_j^-,\ \cup_{j\in\Omega} a_j^+] \end{aligned}$$

and let

(i) $\bar{a} \leq \bar{b}$ iff $a^- \leq b^-$ and $a^+ \leq b^+$;
(ii) $\bar{a} = \bar{b}$ iff $a^- = b^-$ and $a^+ = b^+$;
(iii) $\bar{a} < \bar{b}$ iff $a^- \leq b^-$ and $a^+ \neq b^+$;
(iv) $k\bar{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Notation 1. *We shall use the notation μ_t called a level subset of μ for $\{x \in M/\mu(x) \geq t\}$ where $t \in [0,1]$.*

Definition 2.9. *Let $A =< \mu_A, v_A >$ be an IFS in M and let $t \in [0,1]$. Then the sets $U(\bar{\mu}_A; t) = \{x \in R/\bar{\mu}_A \geq t\}$ and $L(\bar{\gamma}_A; t) = \{x \in R/\bar{\gamma}_A \leq t\}$ are called upper level and lower level set of A [4], respectively.*

Definition 2.10. *An interval valued fuzzy subset $\bar{\eta}$ in a Boolean like semi-ring R is called an **Interval Valued (i.v) Fuzzy Bi-ideal** [2] of R if:*

(i) $\bar{\eta}(x - y) \geq min^i\{\bar{\eta}(x), \bar{\eta}(y)\}$ *for all $x, y \in R$;*
(ii) $\bar{\eta}(xyz) \geq min^i\{\bar{\eta}(x), \bar{\eta}(z)\}$ *for all $x, y, z \in R$.*

Definition 2.11. *An intuitionistic fuzzy set $A =< \mu_A, v_A >$ of R is called an **Intuitionistic Fuzzy bi-ideal** of R if:*

(i) $\mu_A(x - y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$;

(ii) $\mu_A(xyz) \geq \{\mu_A(x) \wedge \mu_A(z)\}$ *for all* $x, y, z \in R$*;*
(iii) $\gamma_A(x - y) \leq \{\gamma_A(x) \vee \gamma_A(y)\}$*;*
(iv) $\gamma_A(xyz) \leq \{\gamma_A(x) \vee \gamma_A(z)\}$ *for all* $x, y, z \in R$*.*

Notation 2. μ_A *- Membership function and* γ_A *- Non-Membership function.*

3. Interval valued intuitionistic fuzzy bi-ideals in Boolean like semi rings

Definition 3.1. *An interval valued intuitionistic fuzzy set* $A =< \bar{\mu}_A, \bar{\gamma}_A >$ *of* R *is called an Interval Valued Intuitionistic Fuzzy bi-ideal of* R *if:*

(i) $\bar{\mu}_A(x - y) \geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(y)\}$*;*
(ii) $\bar{\mu}_A(xyz) \geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(z)\}$ *for all* $x, y, z \in R$*;*
(iii) $\bar{\gamma}_A(x - y) \leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\}$*;*
(iv) $\bar{\gamma}_A(xyz) \leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(z)\}$ *for all* $x, y, z \in R$*.*

Lemma 3.1. *If* B *is a bi-ideal of a Boolean like Semi ring* R*, then for any* $0 < t$*,* $s < 1$*, there exists an interval valued intuitionistic fuzzy bi-ideal* $C =< \bar{\mu}_A, \bar{\gamma}_A >$ *of* R *such that* $C_{<t,s>} = B$*.*

Proof. Let $C \to [0, 1]$ be a function defined by:

$$\bar{\mu}_B(x) = \begin{cases} t & if \quad x \in B \\ 0 & if \quad x \notin B \end{cases} \quad \text{and} \quad \bar{\gamma}_B(x) = \begin{cases} s & if \quad y \in B \\ 1 & if \quad y \notin B \end{cases}$$

for all $x \in R$ and $s, t \in [0, 1]$. Then $C_{<t,s>} = B$ is an interval valued intuitionistic fuzzy bi-ideal of R with $t + s \leq 1$.

Now, suppose that B is a bi-ideal of R. For all $x, y \in B$ such that $x - y \in B$ we have:

$$\begin{aligned} \bar{\mu}_c(x - y) \geq t &= \{\bar{\mu}_c(x) \wedge \bar{\mu}_c(y)\} \\ \bar{\gamma}_c(x - y) \leq s &= \{\bar{\gamma}_c(x) \vee \bar{\gamma}_c(y)\} \end{aligned}$$

Now, for all $x, y, z \in B$ we have:

$$\begin{aligned} \bar{\mu}_c(xyz) \geq t &= \{\bar{\mu}_c(x) \wedge \bar{\mu}_c(z)\} \\ \bar{\gamma}_c(xyz) \leq s &= \{\bar{\gamma}_c(x) \vee \bar{\gamma}_c(z)\} \end{aligned}$$

$\therefore C_{<t,s>}$ is an interval valued intuitionistic fuzzy bi-ideal of R. □

Lemma 3.2. *Let B be a non-empty subset of a Boolean like semi ring R. Then B is a bi-ideal of R iff an intuitionistic fuzzy set $\bar{B} =< \bar{\delta}_B, \bar{\delta}'_B >$ is an interval valued intuitionistic fuzzy bi-ideal of R.*

Proof. Let $x, y \in B$ as in the hypothesis, $x - y \in B$.

(i) If $x, y \in B$ then $\bar{\delta}_B(x) = \bar{1}$, $\bar{\delta}'_B(x) = \bar{0}$, $\bar{\delta}_B(y) = \bar{1}$ and $\bar{\delta}'_B(y) = \bar{0}$.
Now,

$$\begin{aligned} \bar{\delta}_B(x-y) &= \bar{1} \geq \{\bar{\delta}_B(x) \wedge \bar{\delta}_B(y)\} \\ \bar{\delta}'_B(x-y) &= \bar{0} \leq \{\bar{\delta}'_B(x) \vee \bar{\delta}'_B(y)\} \end{aligned}$$

(ii) If $x \in B$, $y \notin B$ then $\bar{\delta}_B(x) = \bar{1}$, $\bar{\delta}'_B(x) = \bar{0}$, $\bar{\delta}_B(y) = \bar{0}$ and $\bar{\delta}'_B(y) = \bar{1}$.
Thus,

$$\begin{aligned} \bar{\delta}_B(x-y) &= \bar{0} \geq \{\bar{\delta}_B(x) \wedge \bar{\delta}_B(y)\} \\ \bar{\delta}'_B(x-y) &= \bar{1} \leq \{\bar{\delta}'_B(x) \vee \bar{\delta}'_B(y)\} \end{aligned}$$

(iii) If $x \notin B$, $y \in B$ then $\bar{\delta}_B(x) = \bar{0}$, $\bar{\delta}'_B(x) = \bar{1}$, $\bar{\delta}_B(y) = \bar{1}$ and $\bar{\delta}'_B(y) = \bar{0}$.
Thus,

$$\begin{aligned} \bar{\delta}_B(x-y) &= \bar{0} \geq \{\bar{\delta}_B(x) \wedge \bar{\delta}_B(y)\} \\ \bar{\delta}'_B(x-y) &= \bar{1} \leq \{\bar{\delta}'_B(x) \vee \bar{\delta}'_B(y)\} \end{aligned}$$

Let $x, y, z \in B$. From the hypothesis, $xyz \in B$.

(i) If $x, z \in B$ then $\bar{\delta}_B(x) = \bar{1}$, $\bar{\delta}'_B(x) = \bar{0}$, $\bar{\delta}_B(z) = \bar{1}$ and $\bar{\delta}'_B(z) = \bar{0}$.
Thus,

$$\begin{aligned} \bar{\delta}_B(xyz) &= \bar{1} \geq \{\bar{\delta}_B(x) \wedge \bar{\delta}_B(z)\} \\ \bar{\delta}'_B(xyz) &= \bar{0} \leq \{\bar{\delta}'_B(x) \vee \bar{\delta}'_B(z)\} \end{aligned}$$

(ii) If $x \in B$, $z \notin B$ then $\bar{\delta}_B(x) = \bar{1}$, $\bar{\delta}'_B(x) = \bar{0}$, $\bar{\delta}_B(z) = \bar{0}$ and $\bar{\delta}'_B(z) = \bar{1}$.
Thus,

$$\begin{aligned} \bar{\delta}_B(xyz) &= \bar{0} \geq \{\bar{\delta}_B(x) \wedge \bar{\delta}_B(z)\} \\ \bar{\delta}'_B(xyz) &= \bar{1} \leq \{\bar{\delta}'_B(x) \vee \bar{\delta}'_B(z)\} \end{aligned}$$

(iii) If $x \notin B$, $z \in B$ then $\bar{\delta}_B(x) = \bar{0}$, $\bar{\delta}'_B(x) = \bar{1}$, $\bar{\delta}_B(z) = \bar{1}$ and $\bar{\delta}'_B(z) = \bar{0}$.
Thus,

$$\begin{aligned} \bar{\delta}_B(xyz) &= \bar{0} \geq \{\bar{\delta}_B(x) \wedge \bar{\delta}_B(z)\} \\ \bar{\delta}'_B(xyz) &= \bar{1} \leq \{\bar{\delta}'_B(x) \vee \bar{\delta}'_B(z)\} \end{aligned}$$

(iv) If $x \notin B$, $z \notin B$ then $\bar{\delta}_B(x) = \bar{0}$, $\bar{\delta}'_B(x) = \bar{1}$, $\bar{\delta}_B(z) = \bar{0}$ and $\bar{\delta}'_B(z) = \bar{1}$. Thus,

$$\begin{aligned} \bar{\delta}_B(xyz) &= \bar{0} \geq \{\bar{\delta}_B(x) \wedge \bar{\delta}_B(z)\} \\ \bar{\delta}'_B(xyz) &= \bar{1} \leq \{\bar{\delta}'_B(x) \vee \bar{\delta}'_B(z)\} \end{aligned}$$

Hence the condition holds.

Conversely, suppose that IFS $\bar{B} =< \bar{\delta}_B, \bar{\delta}'_B >$ is an interval valued intuitionistic fuzzy bi-ideal of R. Then by previous lemma, $\bar{\delta}_B$ is two-valued. Hence B is a bi-ideal of R. □

Theorem 3.1. *Let R be a Boolean like semi ring and $\{\bar{A}_i\}_{i\in A}$ is a family of interval valued intuitionistic fuzzy bi-ideals of R, then $\cap A_i$ is an interval valued intuitionistic fuzzy bi-ideals of R, where $\cap A_i = \{\wedge\bar{\mu}_{A_i}, \vee\bar{\gamma}_{A_i}\}$ and*

$$\begin{aligned} \wedge\bar{\mu}_{A_i}(x) &= inf\{\bar{\mu}_{A_i}(x)/i \in \wedge,\ x \in R\} \\ \vee\bar{\gamma}_{A_i}(x) &= sup\{\bar{\gamma}_{A_i}(x)/i \in \vee,\ x \in R\} \end{aligned}$$

Proof. Let $x, y, z \in R$. We have:

$$\begin{aligned} (i)\ \wedge\bar{\mu}_{A_i}(x-y) &= inf\{\bar{\mu}_{A_i}(x-y)/i \in \wedge\, x, y \in R\} \\ &\geq inf\{\bar{\mu}_{A_i}(x) \wedge \bar{\mu}_{A_i}(y)/i \in \wedge\, x, y \in R\} \\ &= \{\{inf(\bar{\mu}_{A_i}(x)) \wedge inf(\bar{\mu}_{A_i}(y)\}/i \in \wedge\, x, y \in R\} \\ &= \{\{inf(\bar{\mu}_{A_i}(x))/i \in \wedge\, x \in R\} \wedge \{inf(\bar{\mu}_{A_i}(y)/i \in \wedge\, y \in R\}\} \\ &= \{\wedge\bar{\mu}_{A_i}(x) \wedge \wedge\bar{\mu}_{A_i}(y)\} \end{aligned}$$

$$\begin{aligned} (ii)\ \wedge\bar{\mu}_{A_i}(xyz) &= inf\{\bar{\mu}_{A_i}(xyz)/i \in \wedge\, x, z \in R\} \\ &\geq inf\{\bar{\mu}_{A_i}(x) \wedge \bar{\mu}_{A_i}(z)/i \in \wedge\, x, z \in R\} \\ &= \{\{inf(\bar{\mu}_{A_i}(x)) \wedge inf(\bar{\mu}_{A_i}(z)\}/i \in \wedge\, x, z \in R\} \\ &= \{\{inf(\bar{\mu}_{A_i}(x))/i \in \wedge\, x \in R\} \wedge \{inf(\bar{\mu}_{A_i}(z)/i \in \wedge\, z \in R\}\} \\ &= \{\wedge\bar{\mu}_{A_i}(x) \wedge \wedge\bar{\mu}_{A_i}(z)\} \end{aligned}$$

$$
\begin{aligned}
(iii)\ \vee \bar{\gamma}_{A_i}(x-y) &= sup\{\bar{\gamma}_{A_i}(x-y)/i \in \vee\ x, y \in R\} \\
&\geq sup\{\bar{\gamma}_{A_i}(x) \vee \bar{\gamma}_{A_i}(y)/i \in \vee\ x, y \in R\} \\
&= \{\{sup(\bar{\gamma}_{A_i}(x)) \vee sup(\bar{\gamma}_{A_i}(y)\}/i \in \vee\ x, y \in R\} \\
&= \{\{sup(\bar{\gamma}_{A_i}(x))/i \in \vee\ x \in R\} \vee \{sup(\bar{\gamma}_{A_i}(y)/i \in \vee\ y \in R\}\} \\
&= \{\vee\bar{\gamma}_{A_i}(x) \vee \vee\bar{\gamma}_{A_i}(y)\}
\end{aligned}
$$

$$
\begin{aligned}
(iv)\ \vee \bar{\gamma}_{A_i}(xyz) &= sup\{\bar{\gamma}_{A_i}(xyz)/i \in \vee\ x, z \in R\} \\
&\geq sup\{\bar{\gamma}_{A_i}(x) \vee \bar{\gamma}_{A_i}(z)/i \in \vee\ x, z \in R\} \\
&= \{\{\vee(\bar{\gamma}_{A_i}(x)) \vee sup(\bar{\gamma}_{A_i}(z)\}/i \in \vee\ x, z \in R\} \\
&= \{\{sup(\bar{\gamma}_{A_i}(x))/i \in \vee\ x \in R\} \vee \{sup(\bar{\gamma}_{A_i}(z)/i \in \vee\ z \in R\}\} \\
&= \{\vee\bar{\gamma}_{A_i}(x) \vee \vee\bar{\gamma}_{A_i}(z)\}
\end{aligned}
$$

□

Theorem 3.2. *Let $\bar{A}$ be an interval valued intuitionistic fuzzy bi-ideal of Boolean like semi ring R, then $\bar{A}'$ is also an interval valued intuitionistic fuzzy bi-ideal of R.*

Proof. Let $x, y, z \in R$.

$$
\begin{aligned}
(i) \qquad \bar{\mu}'_A(x-y) &= 1 - \bar{\mu}_A(x-y) \\
&\geq 1 - \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(y)\} \\
&= \{1 - \bar{\mu}_A(x)\} \wedge \{1 - \bar{\mu}_A(y)\} \\
&= \{\bar{\mu}'_A(x) \wedge \bar{\mu}'_A(y)\}
\end{aligned}
$$

$$
\begin{aligned}
(ii) \qquad \bar{\mu}'_A(xyz) &= 1 - \bar{\mu}_A(xyz) \\
&\geq 1 - \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(z)\} \\
&= \{1 - \bar{\mu}_A(x)\} \wedge \{1 - \bar{\mu}_A(z)\} \\
&= \{\bar{\mu}'_A(x) \wedge \bar{\mu}'_A(z)\}
\end{aligned}
$$

$$
\begin{aligned}
(iii) \qquad \bar{\gamma}'_A(x-y) &= 1 - \bar{\gamma}_A(x-y) \\
&\geq 1 - \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\} \\
&= \{1 - \bar{\gamma}_A(x)\} \vee \{1 - \bar{\gamma}_A(y)\} \\
&= \{\bar{\gamma}'_A(x) \vee \bar{\gamma}'_A(y)\}
\end{aligned}
$$

$$(iv) \qquad \bar{\gamma}'_A(xyz) = 1 - \bar{\gamma}_A(xyz)$$
$$\geq 1 - \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(z)\}$$
$$= \{1 - \bar{\gamma}_A(x)\} \vee \{1 - \bar{\gamma}_A(z)\}$$
$$= \{\bar{\gamma}'_A(x) \vee \bar{\gamma}'_A(z)\}$$

□

Theorem 3.3. *Let R be a Boolean like semi ring. An interval valued intuitionistic fuzzy set $\bar{A}$ of R is an interval valued intuitionistic fuzzy bi-ideal of R iff the level sets $U(\bar{\mu}_A; t) = \{x \in R / \bar{\mu}_A \geq t\}$ and $L(\bar{\gamma}_A; t) = \{x \in R / \bar{\gamma}_A \leq t\}$ are a bi-ideals of R.*

Proof. Let $\bar{A}$ be an interval valued intuitionistic fuzzy bi-ideal of R. Then:

(i) $\bar{\mu}_A(x-y) \geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(y)\}$
Let $x, y \in U(\bar{\mu}_A; t) \Rightarrow \bar{\mu}_A(x) \geq t,\ \bar{\mu}_A(y) \geq t$
$\bar{\mu}_A(x-y) \geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(y)\} \geq t$
$\therefore x - y \in U(\bar{\mu}_A; t)$

(ii) $\bar{\mu}_A(xyz) \geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(z)\}$
Let $x, y, z \in U(\bar{\mu}_A; t) \Rightarrow \bar{\mu}_A(x) \geq t,\ \bar{\mu}_A(y) \geq t,\ \bar{\mu}_A(z) \geq t$
$\bar{\mu}_A(xyz) \geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(z)\} \geq t$
$\therefore xyz \in U(\bar{\mu}_A; t)$

According to (i) and (ii), we conclude $U(\bar{\mu}_A; t)$ is a bi-ideal of R.

(iii) $\bar{\gamma}_A(x-y) \leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\}$
Let $x, y \in L(\bar{\gamma}_A; t) \Rightarrow \bar{\gamma}_A(x) \geq t,\ \bar{\gamma}_A(y) \geq t$
$\bar{\gamma}_A(x-y) \leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\} \leq t$
$\therefore x - y \in L(\bar{\gamma}_A; t)$

(iv) $\bar{\gamma}_A(xyz) \leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(z)\}$
Let $x, y, z \in L(\bar{\gamma}_A; t) \Rightarrow \bar{\gamma}_A(x) \leq t,\ \bar{\gamma}_A(y) \leq t,\ \bar{\gamma}_A(z) \leq t$
$\bar{\gamma}_A(xyz) \leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(z)\} \leq t$
$\therefore xyz \in L(\bar{\gamma}_A; t)$

According to (iii) and (iv) we conclude that $L(\bar{\gamma}_A; t)$ is a bi-ideal of R.

Conversely, If $U(\bar{\mu}_A; t)$ is a bi-ideal of R. Let $t = \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(y)\}$.

Now, $x, y \in U(\bar{\mu}_A; t) \Rightarrow x - y \in U(\bar{\mu}_A; t)$

$$\begin{aligned} \Rightarrow \bar{\mu}_A(x-y) &\geq t \\ \Rightarrow \bar{\mu}_A(x-y) &\geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(y)\} \end{aligned}$$

If $L(\bar{\gamma}_A; t)$ is a bi-ideal of R, let $t = \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\}$.
Then $x, y \in L(\bar{\gamma}_A; t) \Rightarrow x - y \in L(\bar{\gamma}_A; t)$

$$\begin{aligned} \Rightarrow \bar{\gamma}_A(x-y) &\leq t \\ \Rightarrow \bar{\gamma}_A(x-y) &\leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\} \end{aligned}$$

Define $t = \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(z)\}$. Let $x, y, z \in U(\bar{\mu}_A; t) \Rightarrow (xyz) \in U(\bar{\mu}_A; t)$

$$\begin{aligned} \Rightarrow \bar{\mu}_A((xyz)) &\geq t \\ \Rightarrow \bar{\mu}_A(xyz) &\geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(y)\} \end{aligned}$$

Define $t = \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\}$. Let $x, y, z \in L(\bar{\gamma}_A; t) \Rightarrow (xyz) \in L(\bar{\gamma}_A; t)$

$$\begin{aligned} \Rightarrow \bar{\gamma}_A(xyz) &\leq t \\ \Rightarrow \bar{\gamma}_A(xyz) &\leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(z)\} \end{aligned}$$

$\therefore$ $\bar{A}$ is an i.v intuitionistic fuzzy bi-ideal of R. □

Theorem 3.4. *If $\bar{A}$ is an interval valued intuitionistic fuzzy bi-ideal of Boolean like semi-ring R, then the Boolean like semi ring homomorphic pre-image of an interval valued intuitionistic fuzzy bi-ideal is again a interval valued intuitionistic fuzzy bi-ideal.*

Proof. Let $f : R \to R'$ be a Boolean like semi ring homomorphism and $\bar{A}$ is an interval valued intuitionistic fuzzy bi-ideal of R.

Then the pre-image of $\bar{\mu}$ under f is denoted with $f^{-1}(\bar{A})$ and is defined by $f^{-1}(\bar{\mu}_A(x)) = \bar{\mu}_A(f(x))$ and $f^{-1}(\bar{\gamma}_A(x)) = \bar{\gamma}A(f(x))$, for all $x \in R$.

$$\begin{aligned} (i) \qquad f^{-1}(\bar{\mu}_A(x-y)) &= \bar{\mu}_A(f(x-y)) \\ &= \bar{\mu}_A(f(x) - f(y)) \\ &\geq \{\bar{\mu}_A(f(x)) \wedge \bar{\mu}_A(f(y))\} \\ &= \{f^{-1}(\bar{\mu}_A(x)) \wedge f^{-1}(\bar{\mu}_A(y))\} \end{aligned}$$

$$(ii) \qquad \begin{aligned} f^{-1}(\bar{\mu}_A(xyz)) &= \bar{\mu}_A(f(xyz)) \\ &\geq \{\bar{\mu}_A(f(x)) \wedge \bar{\mu}_A(f(z))\} \\ &= \{f^{-1}(\bar{\mu}_A(x)) \wedge f^{-1}(\bar{\mu}_A(z))\} \end{aligned}$$

$$(iii) \qquad \begin{aligned} f^{-1}(\bar{\gamma}_A(x-y)) &= \bar{\gamma}_A(f(x-y)) \\ &= \bar{\gamma}_A(f(x)-f(y)) \\ &\geq \{\bar{\gamma}_A(f(x)) \vee \bar{\gamma}_A(f(y))\} \\ &= \{f^{-1}(\bar{\gamma}_A(x)) \vee f^{-1}(\bar{\gamma}_A(y))\} \end{aligned}$$

$$(iv) \qquad \begin{aligned} f^{-1}(\bar{\gamma}_A(xyz)) &= \bar{\gamma}_A(f(xyz)) \\ &\geq \{\bar{\gamma}_A(f(x)) \vee \bar{\gamma}_A(f(z))\} \\ &= \{f^{-1}(\bar{\gamma}_A(x)) \vee f^{-1}(\bar{\gamma}_A(z))\} \end{aligned}$$

Hence $f^{-1}(\bar{A})$ is an interval valued intuitionistic fuzzy bi-ideal of R. □

Theorem 3.5. *Let R be a Boolean like semi-ring and $< \bar{\mu}_A, \bar{\gamma}_A >$ be an interval valued intuitionistic fuzzy bi-ideal of R. Then the sets $R\bar{\mu}_A = \{x \in R/\bar{\mu}_A(x) = \bar{\mu}_A(0)\}$ and $R\bar{\gamma}_A = \{x \in R/\bar{\gamma}_A(x) = \bar{\gamma}_A(0)\}$ are interval valued intuitionistic fuzzy bi-ideals of R.*

Proof. Let $< \bar{\mu}_A, \bar{\gamma}_A >$ be an interval valued intuitionistic fuzzy bi-ideal of R. Let $x, y, z \in R\bar{\mu}_A \Rightarrow \bar{\mu}_A(x) = \bar{\mu}_A(0),\ \bar{\mu}_A(y) = \bar{\mu}_A(0)$ and $\bar{\mu}_A(z) = \bar{\mu}_A(0)$. Then

$$(i) \qquad \begin{aligned} \bar{\mu}_A(x-y) &\geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(x)\} \\ &= \{\bar{\mu}_A(0) \wedge \bar{\mu}_A(0)\} \\ &= \bar{\mu}_A(0) \end{aligned}$$

$$\therefore x - y \in R\bar{\mu}_A$$

$$(ii) \qquad \begin{aligned} \bar{\mu}_A(xyz) &\geq \{\bar{\mu}_A(x) \wedge \bar{\mu}_A(z)\} \\ &= \{\bar{\mu}_A(0) \wedge \bar{\mu}_A(0)\} \\ &= \bar{\mu}_A(0) \end{aligned}$$

$$\therefore xyz \in R\bar{\mu}_A$$

Let $x, y, z \in R\bar{\gamma}_A$, $\bar{\gamma}_A(x) = \bar{\gamma}_A(0)$, $\bar{\gamma}_A(y) = \bar{\gamma}_A(0)$ and $\bar{\gamma}_A(z) = \bar{\gamma}_A(0)$. Then

$$
\begin{aligned}
(iii) \qquad \bar{\gamma}_A(x-y) &\leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(y)\} \\
&= \{\bar{\gamma}_A(0) \vee \bar{\gamma}_A(0)\} \\
&= \bar{\gamma}_A(0)
\end{aligned}
$$

$$\therefore x - y \in R\bar{\gamma}_A$$

$$
\begin{aligned}
(iv) \qquad \bar{\gamma}_A(xyz) &\leq \{\bar{\gamma}_A(x) \vee \bar{\gamma}_A(z)\} \\
&= \{\bar{\gamma}_A(0) \vee \bar{\gamma}_A(0)\} \\
&= \bar{\gamma}_A(0)
\end{aligned}
$$

$$\therefore xyz \in R\bar{\gamma}_A$$

□

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