ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.5, 2595–2605
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.5.22

ASCENDING PENDENT DOMINATION DECOMPOSITION OF GRAPHS

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ABSTRACT. Let $G = (V, E)$ be a simple connected graph. A subset S of vertices in a graph G is called a dominating set if every vertex $v \in V$ is either in S or adjacent to some vertex in S. A dominating set S in G is called a dendant dominating set if $< S >$ contains atleast one pendant vertex. If $G_1, G_2, G_3, ..., G_n$ are connected subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) ... \cup E(G_n)$, then $(G_1, G_2, G_3, ..., G_n)$ is said to be a decomposition of G. In this paper, we introduce ascending dendent domination decomposition of graphs. Also we obtain $P_{(n+1)^2}, C_{n(n+2)}$ and $K_{\frac{m(m+1)}{2},n}$ admits ascending pendent domination decomposition.

1. INTRODUCTION

Let $G = (V, E)$ be a simple connected graph. A vertex of degree zero is called an isolated vertex and a vertex of degree one is called a pendent vertex. An edge incident with a pendent vertex is called a pendent edge. Pendent domination in some generalised graphs was introduced by Nayaka, Puttaswamy and Purushothama in [5]. Ascending domination decomposition of subdivision of graphs was introduced by Lakshmiprabha and Nagarajan in [3] and Maheswari and Nagarajan in [4]. This motivates us to define ascending pendent domination decomposition, which is researched in [1, 2]. In this paper, we obtain the graphs namely $P_{(n+1)^2}, C_{n(n+2)}$ and $K_{\frac{m(m+1)}{2},n}$ admits ascending pendent domination decomposition.

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2010 *Mathematics Subject Classification.* Primary 05C50, 05C69, 05C70.

Key words and phrases. Dominating set, pendent dominating set, decomposition.

Definition 1.1. *If $G_1, G_2, G_3, ..., G_n$ are edge disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) ... \cup E(G_n)$, then $(G_1, G_2, G_3, ..., G_n)$ is said to be a decomposition of G.*

Definition 1.2. *A subset S of vertices in a graph G is called a dominating set if every vertex $v \in V$ is either in S or adjacent to some vertex in S. The least cardinality of a dominating set in G is called the domination number of G and is usually denoted by $\gamma(G)$.*

Definition 1.3. *A dominating set S in G is called a pendant dominating set if $< S >$ contains at least one pendant vertex. The minimum cardinality of a pendent dominating set is called the pendant domination number denoted by $\gamma_{pe}(G)$.*

Definition 1.4. *The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.*

Definition 1.5. *The graph $C_n \odot K_1$ is called a crown. The graph $P_n \odot K_1$ is called a comb.*

Definition 1.6. *For a vertex $v \in V$, the open neighbourhood of v is the set $N(v)$ containing all the vertices u adjacent to v and the closed neighbourhood of v is the set $N[v]$ containing v and all the vertices u adjacent to v.*

2. MAIN RESULTS

Definition 2.1. *A decomposition $(G_1, G_2, ..., G_n)$ of G is said to be ascending pendent domination decomposition (APDD) if*

(i) *Each G_i is connected*
(ii) $\gamma_{pe}(G_i) = i + 1, \ 1 \leq i \leq n.$

Example 1. *The following example shows that the graph G admits APDD into 3 - parts.*

Here $\gamma_{pe}(G_1) = 2$, $\gamma_{pe}(G_2) = 3$ and $\gamma_{pe}(G_3) = 4$.

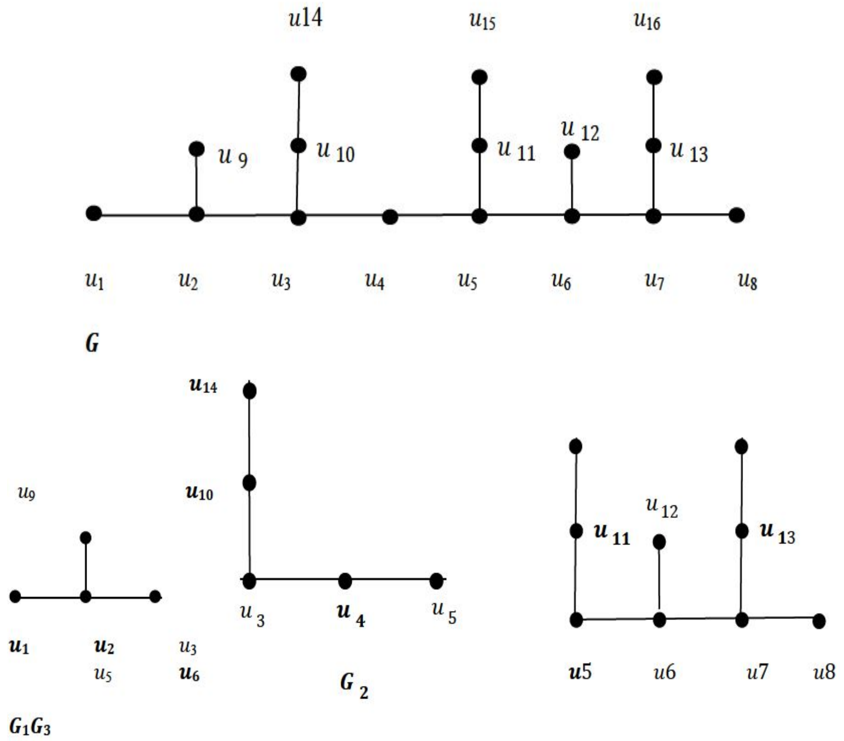

FIGURE 1

Theorem 2.1. *The path $P_{(n+1)^2}$ admits APDD into n - parts.*

Proof. Let $\{u_1, u_2, ..., u_{(n+1)^2}\}$ be the set of vertices of $P_{(n+1)^2}$.
Define

$$
\begin{aligned}
G_1 &= \langle N[(u_2, u_3)]\rangle \\
G2 &= \langle N[(u_5, u_6), u_8]\rangle \\
G3 &= \langle N[(u_{10}, u_{11}), u_{13}, u_{15}]\rangle \\
G4 &= \langle N[(u_{17}, u_{18}), u_{20}, u_{22}, u_{24}]\rangle \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
G_n &= \langle N[(u_q, u_{q+1}), u_{q+3}, u_{q+5}, ..., u_s]\rangle
\end{aligned}
$$

where q and s can be calculated by using Newton's Divided Difference Formula.

To find: q

n	q	Δq	$\Delta^2 q$	$\Delta^3 q$	$\Delta^4 q$
1	2				
		3			
2	5		2		
		5		0	
3	10		2		0
		7		0	
4	17		2		
		9			
5	26				

$$\begin{aligned} n &= n_0 + xh \\ n &= 1 + x(1) \Rightarrow x = n - 1. \end{aligned}$$

$$\begin{aligned} q &= q_0 + x\frac{\Delta q_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 q_0 + \cdots \\ &= 2 + (n-1)3 + \frac{n(n-1)}{2}2 \\ &= n^2 + 1 \end{aligned}$$

Hence $q = n^2 + 1$ and $q + 1 = n^2 + 2$.
To find : s

n	s	Δs	$\Delta^2 s$	$\Delta^3 s$	$\Delta^4 s$
2	8				
		7			
3	15		2		
		9		0	
4	24		2		0
		11		0	
5	35		2		
		13			
6	48				

$$\begin{aligned} n &= n_0 + xh \\ n &= 2 + x(1) \Rightarrow x = n - 2. \end{aligned}$$

$$\begin{aligned} s &= s_0 + x\frac{\Delta s_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 s_0 + \cdots \\ &= 8 + (n-2)7 + \frac{(n-2)(n-3)}{2}2 \\ &= n(n+2) \end{aligned}$$

Clearly, path $P_{(n+1)^2}$ can be decomposed into $G_1, G_2, ..., G_n$ and $\gamma_{pe}(G_i) = i+1,\ 1 \leq i \leq n$.

Hence the path $P_{(n+1)^2}$ admits APPD into n- parts. □

Illustration 2.1. *For $n = 3$, P_{16} admits APPD into 3 - parts.*

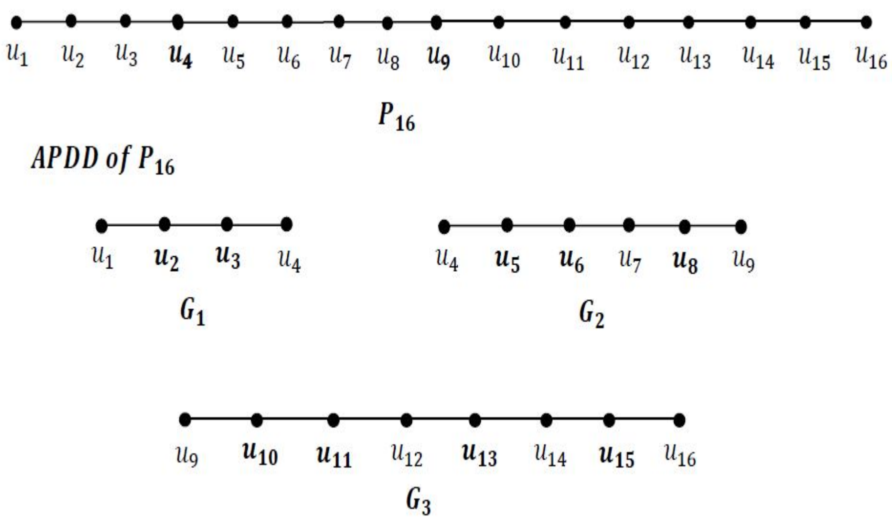

FIGURE 2

Here $\gamma_{pe}(G_1) = 2,\ \gamma_{pe}(G_2) = 3$ *and* $\gamma_{pe}(G_3) = 4$.

Theorem 2.2. *The cycle $C_{n(n+2)}$ admits APDD into $n-$ parts.*

Proof. Let $\{u_1, u_2, ..., u_{n(n+2)}\}$ be the set of vertices of $C_{n(n+2)}$.

Define

$$
\begin{aligned}
G_1 &= \langle N[(u_2, u_3)]\rangle \\
G2 &= \langle N[(u_5, u_6), u_8]\rangle \\
G3 &= \langle N[(u_{10}, u_{11}), u_{13}, u_{15}]\rangle \\
G4 &= \langle N[(u_{17}, u_{18}), u_{20}, u_{22}, u_{24}]\rangle \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots \\
G_n &= \langle N[(u_q, u_{q+1}), u_{q+3}, u_{q+5}, ..., u_s]\rangle
\end{aligned}
$$

where q and s can be calculated by using Newton's Divided Difference Formula.

To find: q

n	q	Δq	$\Delta^2 q$	$\Delta^3 q$	$\Delta^4 q$
1	2				
		3			
2	5		2		
		5		0	
3	10		2		0
		7		0	
4	17		2		
		9			
5	26				

$$
\begin{aligned}
n &= n_0 + xh \\
n &= 1 + x(1) \Rightarrow x = n - 1.
\end{aligned}
$$

$$
\begin{aligned}
q &= q_0 + x\frac{\Delta q_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 q_0 + \cdots \\
&= 2 + (n-1)3 + \frac{n(n-1)}{2}2 \\
&= n^2 + 1
\end{aligned}
$$

Hence $q = n^2 + 1$ and $q + 1 = n^2 + 2$.

To find : s

n	s	Δs	$\Delta^2 s$	$\Delta^3 s$	$\Delta^4 s$
2	8				
		7			
3	15		2		
		9		0	
4	24		2		0
		11		0	
5	35		2		
		13			
6	48				

$$n = n_0 + xh$$
$$n = 2 + x(1) \Rightarrow x = n - 2.$$

$$\begin{aligned} s &= s_0 + x\frac{\Delta s_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 s_0 + \cdots \\ &= 8 + (n-2)7 + \frac{(n-2)(n-3)}{2}2 \\ &= n(n+2) \end{aligned}$$

Clearly, path $C_{n(n+2)}$ can be decomposed into $G_1, G_2, ..., G_n$ and $\gamma_{pe}(G_i) = i+1,\ 1 \le i \le n$.

Hence the path $C_{n(n+2)}$ admits APPD into n- parts. □

Illustration 2.2. *For $n = 3$, C_{15} admits APDD into 3- parts.*

At the figure 3 $\gamma_{pe}(G_1) = 2,\ \gamma_{pe}(G_2) = 3$ and $\gamma_{pe}(G_3) = 4$.

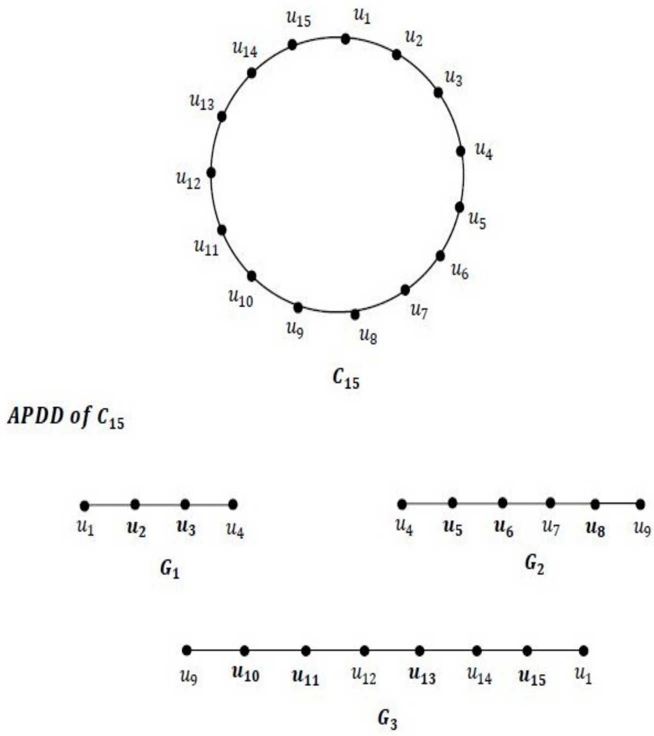

FIGURE 3

Theorem 2.3. *A complete bipartite* $K_{\frac{m(m+1)}{2},n}$ *admits APDD into* $m-$ *parts and* $\gamma_{pe}\left(K_{\frac{m(m+1)}{2},n}\right) = \sum_{i=1}^{m} \gamma_{pe}(G_i) - m + 1.$

Proof. Let $\{u_1, u_2, ..., u_{\frac{m(m+1)}{2}}\}$ be the set of vertices of degree n in $K_{\frac{m(m+1)}{2},n}$.

Let $\{v_1, v_2, ..., v_n\}$ be the set of vertices of degree $\frac{m(m+1)}{2}$ in $K_{\frac{m(m+1)}{2},n}$. Define

$$\begin{aligned} G_1 &= \langle N[u_1] \rangle \\ G_2 &= \langle N[u_2, u_3] \rangle \\ G_3 &= \langle N[u_4, u_5, u_6] \rangle \\ &\quad \ldots\ldots\ldots\ldots\ldots\ldots \\ G_m &= \langle N[u_r, u_{r+1}, ..., u_s] \rangle \end{aligned}$$

where r and s can be calculated by using Newton's Divided Difference Formula.

To find: r

n	r	Δr	$\Delta^2 r$	$\Delta^3 r$	$\Delta^4 r$
1	1				
		1			
2	2		1		
		2		0	
3	4		1		0
		3		0	
4	7		1		
		4			
5	11				

$$\begin{aligned} m &= m_0 + xh \\ m &= 1 + x(1) \Rightarrow x = m - 1. \end{aligned}$$

$$\begin{aligned} r &= r_0 + x\frac{\Delta r_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 r_0 + \cdots \\ &= 1 + (m-1)3 + \frac{(m-1)(m-2)}{2}1 \\ &= \frac{m^2 - m + 1}{2} \end{aligned}$$

To find : s

n	s	Δs	$\Delta^2 s$	$\Delta^3 s$	$\Delta^4 s$
1	1				
		2			
2	3		1		
		3		0	
3	6		1		0
		4		0	
4	10		1		
		5			
5	15				

$$\begin{aligned} m &= m_0 + xh \\ m &= 1 + x(1) \Rightarrow x = m - 1. \end{aligned}$$

$$\begin{aligned} s &= s_0 + x\frac{\Delta s_0}{1!} + \frac{x(x-1)}{2!}\Delta^2 s_0 + \cdots \\ &= 1 + (m-1)2 + \frac{(m-1)(m-2)}{2}1 \\ &= \frac{m(m+1)}{2} \end{aligned}$$

Clearly, path $K_{\frac{m(m+1)}{2},n}$ can be decomposed into $G_1, G_2, ..., G_m$ and $\gamma_{pe}(G_i) = i+1,\ 1 \leq i \leq n$.

Hence the path $K_{\frac{m(m+1)}{2},n}$ admits APPD into m- parts.

Also, the pendent dominating set in $K_{\frac{m(m+1)}{2},n}$ is $\{u_1, u_2, ..., u_{\frac{m(m+1)}{2}}\}$ and we can choose anyone of the vertices in $\{v_1, v_2, ..., v_n\}$.

Therefore, $\gamma_{pe}\left(K_{\frac{m(m+1)}{2},n}\right) = \frac{m(m+1)}{2} + 1$.

Now,

$$\begin{aligned} \sum_{i=1}^{m} \gamma_{pe}(G_i) &= \gamma_{pe}(G_1) + \gamma_{pe}(G_2) + \cdots + \gamma_{pe}(G_m) \\ &= 2 + 3 + \cdots + m + 1 \\ &= (1 + 2 + 3 + \cdots + m + m + 1) - 1 \\ &= (1 + 2 + 3 + \cdots + m) + m \\ &= \frac{m(m+1)}{2} + m \\ &= \gamma_{pe}\left(K_{\frac{m(m+1)}{2},n}\right) - 1 + m \end{aligned}$$

Hence $\gamma_{pe}\left(K_{\frac{m(m+1)}{2},n}\right) = \sum_{i=1}^{m} \gamma_{pe}(G_i) - m + 1$. □

Illustration 2.3. *For m= 3 and n= 3, $K_{6,3}$ admits APDD into 3 - parts.*

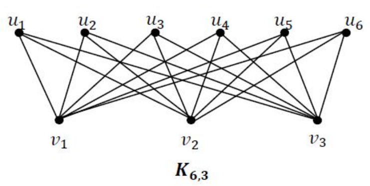

FIGURE 4

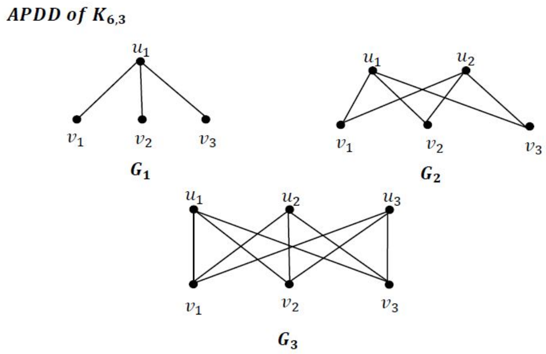

FIGURE 5

Here $\gamma_{pe}(G_1) = 2,\ \gamma_{pe}(G_2) = 3$ and $\gamma_{pe}(G_3) = 4$.

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