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ISOLATED EDGE DETOUR DOMINATION NUMBER OF SOME STANDARD GRAPHS

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ABSTRACT. Let G be a connected graph with at least two vertices. An edge detour dominating set S of G is called an isolated edge detour dominating set if <S > has an isolated vertex or $< V - S >$ is a single vertex. The isolated edge detour domination number $\gamma_{ied}(G)$ is the minimum cardinality taken over all isolated edge detour dominating sets of G. The minimum isolated edge detour dominating set is called $\gamma_{ied}-$ set of G. We say that G has infinite isolated edge detour domination number if there is no isolated edge detour dominating set in G. The isolated edge detour domination number of some standard graphs are determined. Total edge detour domination concept of graphs has several applications are determined.

1. INTRODUCTION

For a graph $G = (V, E)$ we mean an undirected graph without loops or multiple edges. The order and size of G is denoted by p and q respectively. We consider connected graph G with at least two vertices. For basic definition and terminology we refer to Buckley and Harary in [1].

For vertices u and v in a connected graph G, the detour distance $D(u, v)$ is the length of the largest $u - v$ path in G. A $u - v$ path of length $D(u, v)$ is called a

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$u - v$ detour. It is known that the detour distance is a metric and the vertex set $V(G)$. The detour eccentricity $e_d(v)$ of a vertex v in G is the maximum detour distance from v to a vertex of G, [2].

The detour radius, $rad_D(G)$ is the minimum detour eccentricity among the vertices of G while the detour diameter $diam_D(G)$ is the maximum detour eccentricity among the vertices of G. This concept was studied by Chartrand et al. in [2]. A vertex x is said to lie on a $u - v$ detor path P including the vertices u and v.

A set $S \subseteq V(G)$ is called a detour set if every vertex v in G lie on a detour joining a pair of vertices of S. The detour number $dn(G)$ of a G is the minimum order of a detour set and any detour set of order $dn(G)$ is called a minimum detour set of G. This concept was also studied by Chartrand et al. in [3].

Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S \subseteq V(G)$ is called a dominating set of G if every vertices in $V(G) - S$ is adjacent to some vertices in S. The domination number $\gamma(G)$ of G is the minimum order of its dominating and any dominating set of order $\gamma(G)$ is called $\gamma-$ set of G, [4].

Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S \subseteq V(G)$ is called a detour dominating set of G if S is both a detour and a dominating set of G. The detour number $\gamma_d(G)$ of G is the minimum order of its detour dominating set and any detour dominating set of order $\gamma_d(G)$ is called a γ_d-set of G, [5].

Theorem is used in the sequence.

2. ISOLATED EDGE DETOUR DOMINATION OF SOME STANDARD GRAPHS

The following definitions are given in [6].

Definition 2.1. *For a connected graph $G = (V, E)$, an edge detour dominating set S of G is called an isolated edge detour dominating set if $< S >$ has an isolated vertex or $< V - S >$ is a single vertex.*

Example 1. *In figure 1, $S_1 = \{v_1, v_3\}$ and $S2 = \{v_2, v_4\}$ are both edge detour dominating set and isolated edge detour dominating set. Hence, $\gamma_{ied}(G) = 2$.*

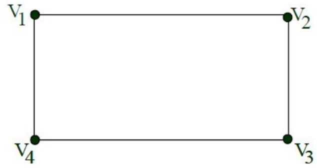

FIGURE 1

Example 2. *For a complete graph, we cannot find any isolated edge detour dominating set, since each edge is adjacent to other edge. Therefore the isolated edge detour domination number of K_n is infinite.*

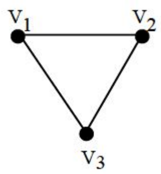

FIGURE 2. Complete graph K_3

Proposition 2.1. *For a connected graph G,*

$$2 \leq \gamma_{ed}(G) \leq \gamma_{ied}(G) \leq \gamma_{ced}(G) \leq n.$$

Definition 2.2. *A cycle C_n, is a circuit in which no vertex except the first (which is also the last) appears more than once. Alternatively, a cycle can be defined as a closed path.*

Proposition 2.2. *In cycle C_n, $n \neq 3, 5$, $\gamma_{ied}(C_n) = \lceil \frac{n}{3} \rceil$.*

Proof. **Case (i):** when $n \neq 3, 5$

Let $V(C_n) = \{v_1, v_2, ..., v_{i-1}, v_i, v_{i+1}, ..., v_n\}$.

The set $S = \{v_1, v_n\}$ is minimum edge detour set which dominates the vertex v_2 and v_{n-1}.

Now find the minimum isolated dominating set on C_{n-4} and $V(C_{n-4}) = \{v_3, v_4, ..., v_{i-1}, v_i, v_{i+1}, ..., v_{n-2}\}$.

Choose one vertex in every three vertices from $V(C_n)$.

Hence the set $S = \{v_4, v_6, ..., v_i, ..., v_{n-3}\}$ is both edge detour dominating and isolated edge detour dominating set.

Therefore, $\gamma_{ied}(C_n) = \lceil \frac{n}{3} \rceil$.

Case (ii): when $n = 3$

By the example 2, given above we cannot find isolated edge detour domination.

When $n = 5$

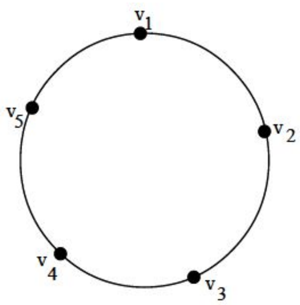

FIGURE 3. Cycle C_5

Here the set $S = \{v_1, v_3, v_5\}$ is an edge detour dominating set but not isolated edge detour dominating set, since either one of the edges are connected. □

Definition 2.3. *The Actinia graph $A(m_i, n)$ is obtained by identifying $m_i (1 \leq i \leq n)$ pendent edges to the vertices of a cycle C_n.*

Proposition 2.3. *For a Actinia graph $A(m_i, n)$, $\gamma_{ied} A(m_i, n) = mn$.*

Proof. Let $\{v_1, v_2, ..., v_n\}$ be a vertices of a cycle C_n. Let $\{u^i_m : 1 \leq i \leq n\}$ be a pendant edges to the vertices of the cycle C_n. The set $S = \{u^i_j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$ will be a edge detour dominating set. Clearly the set S is an isolated edge detour dominating set.

Therefore, $\gamma_{ied} A(m_i, n) = mn$. □

Definition 2.4. *The Jewel graph J_n is a graph with vertex set $V(J_n) = \{u, x, v, y, v_i : 1 \leq i \leq n\}$ and the edge set $E(J_n) = \{ux, uy, vx, vy, xy, uv_i, vv_i : 1 \leq i \leq n\}$.*

Proposition 2.4. *For a Jewel graph J_n, $\gamma_{ied}(J_n) = 3$.*

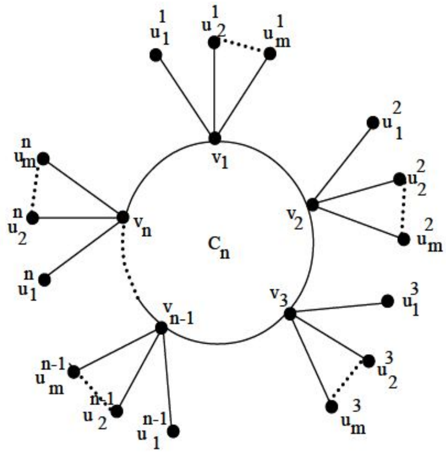

FIGURE 4. Actinia graph $A(m_i, n)$

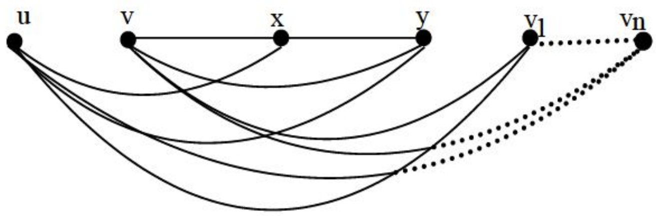

FIGURE 5. Jewel graph J_n

Proof. Let $V(J_n) = \{u, x, v, y, v_i : 1 \leq i \leq n\}$.

Consider the set $S = \{v_1, v_2, x\}$.

The set S is both edge detour dominating and isolated edge detour dominating set.

Therefore, $\gamma_{ied}(J_n) = 3$. □

Definition 2.5. *The Helm H_n is a graph obtained from a wheel by attaching a pendant vertex at each vertex of the n-cycle.*

Proposition 2.5. *For a Helm graph H_n, $\gamma_{ied}(H_n) = n + 1$.*

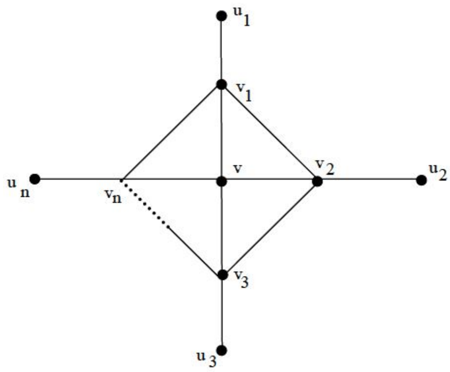

FIGURE 6. Helm graph H_n

Proof. Let $V(H_n) = \{v, v_1, v_2, ..., v_n, u_1, u_2, u_3, ..., u_n\}$, where v is a central vertex. Consider all the pendent edges to form a edge detour set. Here the set $S = \{u_1, u_2, u_3, ..., u_n\}$ forms a edge detour set.

The set $\{S\} \cup v$ forms the edge detour dominating set whose edges are all isolated.

Hence, $\gamma_{ied}(H_n) = n + 1$. □

Definition 2.6. *A Flower graph Fl_n is a graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.*

Proposition 2.6. *For a flower graph Fl_n, $\gamma_{ied}(Fl_n) = n$.*

Proof. Let $V(Fl_n) = \{v, v_1, v_2, ..., v_n, u_1, u_2, u_3, ..., u_n\}$, where v is the central vertex. The set $S = \{u_1, u_2, u_3, ..., u_n\}$ forms an isolated edge detour dominating set. Therefore, $\gamma_{ied}(Fl_n) = n$. □

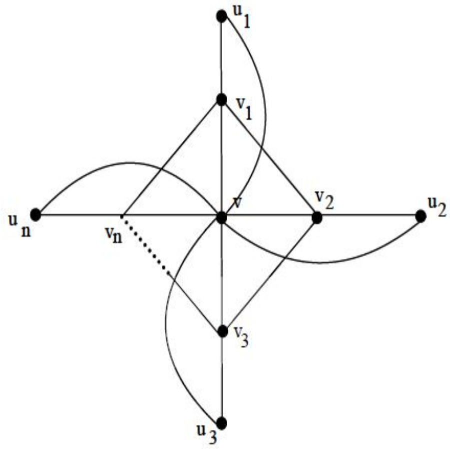

FIGURE 7. Flower graph Fl_n

Definition 2.7. *A Bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets U and V such that every edge connects a vertex in U to one in V.*

Proposition 2.7. *In a Bipartite graph, $\gamma_{ied}(B_{m,n}) = 2$.*

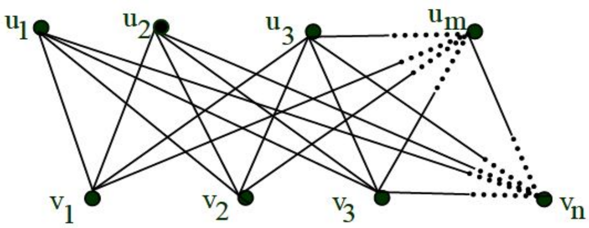

FIGURE 8. Bipartite graph $B_{m,n}$

Proof. Let $U = \{u_1, u_2, u_3, ..., u_m\}$ and $V = \{v, v_1, v_2, ..., v_n\}$be the vertices of a bipartite graph. Consider any two vertices from the same set. Clearly, this would be an isolated edge detour dominating set. Hence, $\gamma_{ied}(B_{m,n}) = 2$. □

Definition 2.8. *A Gear graph, G_n is graph obtained by inserting an extra vertex between each pair of adjacent vertices on the perimeter of a wheel graph W_n.*

Proposition 2.8. *For a Gear graph G_n, $\gamma_{ied}(G_n) = n + 1$.*

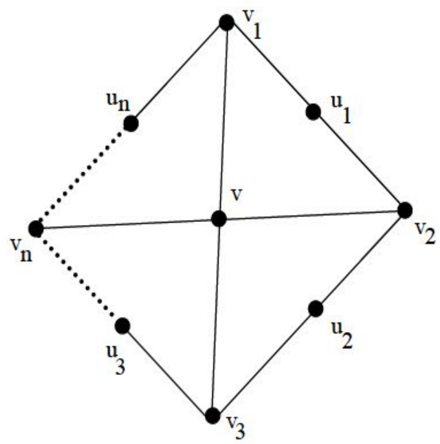

FIGURE 9. Gear graph G_n

Proof. Let $V(G_n) = \{v, v_1, v_2, ..., v_n, u_1, u_2, u_3, ..., u_n\}$, where v is a central vertex. Consider the set $S = \{v, u_1, u_2, u_3, ..., u_n\}$.
The set S is both edge detour dominating and isolated edge detour dominating set. Hence, $\gamma_{ied}(G_n) = n + 1$. □

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