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NEIGHBORHOOD-PRIME LABELING OF GRID, TORUS AND SOME INFLATED GRAPHS

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ABSTRACT. Let $G = (V, E)$ be a graph with n vertices. A bijection $f : V(G) \to \{1, 2, 3, \ \dots \ , n\}$ is said to be a neighborhood-prime labeling if for every vertex $v \in V(G)$ with $deg(v) > 1$, $gcd\{f(u)|u \in N(v)\} = 1$. A graph which admits neighborhood-prime labeling is called a neighborhood-prime graph. In this paper, we investigate the neighborhood-prime labeling of grid, torus and some inflated graphs.

1. INTRODUCTION

The graphs we consider here are simple, finite, connected and undirected. The notion of prime labeling for graphs originated by Roger Entringer, was introduced in a paper by Tout et al., [8] in the early 1980s and since then it is an active field of research for many scholars. A triangular snake T_n, [1] is obtained from a path P_n by replacing each edge of P_n by a cycle C_3. Definitions of Ladder graph L_n, grid graph $P_m \times P_n$ and torus grid graph $C_m \times C_n$ are given in [3]. The helm H_n, [5], is the graph obtained from the wheel $W_n = C_n + K_1$ by attaching a pendent edge at each vertex of the cycle C_n. A closed helm CH_n, [5], is a graph obtained from a helm H_n by joining each pendent vertex to form a cycle. For the definition of inflated graphs we refer [2,6]. Inflated graph G_I of a graph

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G without isolated vertices it is obtained as follows: each vertex x_i of degree $d(x_i)$ of G is replaced by a clique $X_i \cong K_{d(x_i)}$ and each edge x_ix_j of G is replaced by an edge uv in such a way that $u \in X_i$, $v \in X_j$ and two different edges of G are replaced by non adjacent edges of G_I. Throughout this paper, we refer the clique corresponding to x_i in inflated graphs as $A(x_i)$. The neighborhood of v is the set of all vertices in G which are adjacent to v and is denoted by $N(v)$. Patel and Shrimali in [5], introduced one of the variation of prime labeling which is known as neighborhood-prime labeling of a graph. Let $G = (V, E)$ be a graph with n vertices. A bijective function $f : V(G) \to \{1, 2, 3, \ldots, n\}$ is said to be neighborhood-prime labeling, if for each vertex $v \in V(G)$, with $deg(v) > 1$, $gcd\{f(u)|u \in N(v)\} = 1$. A graph which admits neighborhood-prime labeling is called a neighborhood-prime graph. In this paper, we investigate the existence of neighborhood-prime labeling of grid, torus and some inflated graphs. The following facts are from [5]:

Remark 1.1. *A graph G in which every vertex is of degree atmost 1 is neighborhood-prime vacuously.*

Theorem 1.1. *The path P_n is a neighborhood-prime graph for every n.*

Theorem 1.2. *The cycle C_n is neighborhood-prime if $n \not\equiv 2(mod 4)$.*

Theorem 1.3. *The cycle C_n is not neighborhood-prime if $n \equiv 2(mod 4)$.*

Theorem 1.4. *[7]* **The Euclidean algorithm**
Given positive integers a and b, where b does not divide a. Let $r_0 = a$, $r_1 = b$, and apply the division algorithm repeatedly to obtain a set of remainders r_2 , r_3, $\ldots$, r_n , r_{n+1} defined successively by the relations

$$\begin{aligned}
r_0 &= r_1q_1 + r_2, 0 < r_2 < r_1,\\
r_1 &= r_2q_2 + r_3, 0 < r_3 < r_2,\\
&\vdots\\
r_{n-2} &= r_{n-1}q_{n-1} + r_n, 0 < r_n < r_{n-1},\\
r_{n-1} &= r_nq_n + r_{n+1}, r_{n+1} = 0.
\end{aligned}$$

Then r_n, the last nonzero remainder in this process, is (a, b), the gcd of a and b.

Remark 1.2. (1) *The torus grid graphs $C_m \times C_n$ are hamiltonian.*

(2) *A grid graph $P_m \times P_n$ is hamiltonian if either the number of rows or columns is even.*

Theorem 1.5. *[4], Let G be a graph of order n such that $n \not\equiv 2(mod\ 4)$. If G is Hamiltonian then G has a neighborhood-prime labeling.*

2. Main results

Remark motivated us to define the following definition.

Definition 2.1. *Any vertex is of degree atmost 1 is called a neighborhood-prime vertex of G.*

Remark 2.1. *By definition 2.1, a graph G in which every vertex is a neighborhood-prime vertex, then such a graph is neighborhood-prime vacuously.*

In [5] while proving theorem 1.3, the result when n is even $n \not\equiv 2(mod\ 4)$, gcd$\left(n, \frac{n}{2}+1\right) = 1$ is assumed directly. Here in this paper, we proved this assumption as the following lemma.

Lemma 2.1. *Let n be an even integer. Then, gcd$\left(n, \frac{n}{2}+1\right) = 1$ iff $n = 4k$.*

Proof. First to find gcd$\left(n, \frac{n}{2}+1\right)$. Applying Euclidean algorithm 1.4,
$n = 1\left(\frac{n}{2}+1\right) + \left(\frac{n}{2}-1\right)$, and $\frac{n}{2}+1 = 1\left(\frac{n}{2}-1\right)+2$.
Now $\frac{n}{2}-1 = k(2)+r$ where $r = 0$ or $r = 1$. If $r = 0$, then gcd$\left(n, \frac{n}{2}+1\right) = 2$. Also, $\frac{n}{2}-1 = 2k$ implies $\frac{n}{2} = 2k+1$. Therefore $n = 4k+2$. Therefore,

$$gcd\left(n, \frac{n}{2}+1\right) = 2\ if\ n = 4k+2\,. \tag{2.1}$$

Similarly, if $r = 1$, then gcd$\left(n, \frac{n}{2}+1\right) = 1$. Also, $\frac{n}{2}-1 = 2k+1$ implies $\frac{n}{2} = 2k+2$. Therefore $n = 4k+4 = 4(k+1) = 4k'$ where $k' = k+1$. Therefore,

$$gcd\left(n, \frac{n}{2}+1\right) = 1\ if\ n = 4k' \tag{2.2}$$

If n is even, then either

$$n = 4k \text{ or } n = 4k+2. \tag{2.3}$$

Hence, by (2.1), (2.2) and (2.3), gcd$\left(n, \frac{n}{2}+1\right) = 1$ iff $n = 4k$. □

Theorem 2.1. *Let G be a graph on n vertices. Suppose G has a $u-v$ hamiltonian path such that $d_G(u) = d_G(v) = 1$. Then G is neighborhood-prime.*

Proof. Let P_n denote the $u-v$ hamiltonian path in G such that $d_G\left(u\right) = d_G\left(v\right) = 1$. By theorem 1.2, P_n admits neighborhood-prime labeling.
That is, there exists a function $f : V(P_n) \to \{1, 2, 3, \ldots, n\}$ such that gcd $\{f(x)|x \in N(y)\} = 1$ for every $y \in V\left(P_n\right) - \{u, v\}$.
Further, $V(P_n) = V(G)$ and $d_{P_n}\left(w\right) = 2$ for all $w \in V\left(P_n\right) - \{u, v\}$.
Since gcd$(f\left(x\right), f\left(y\right)) = 1$ implies gcd$(f\left(x\right), f\left(y\right), f\left(x_1\right), f\left(x_n\right)) = 1$,
gcd$f(x)|x \in N(y) = 1$ for every $y \in V\left(G\right) - \{u, v\}$.
Also, $d_G\left(u\right) = d_G\left(v\right) = 1$ implies u and v are neighborhood-prime vertex of G.
Therefore, gcd$f(x)|x \in N(y) = 1$ for every $y \in V(G)$ with $d(y) > 1$.
Therefore f is a neighborhood-prime labeling of G.
Therefore G admits neighborhood-prime labeling. Hence G is neighborhood-prime. □

Observation 1. *The inflation of path $I(P_n)$ being isomorphic to P_1 or P_{2n-2} is neighborhood-prime.*

Theorem 2.2. *The inflation of cycle $I(C_n)$ is a neighborhood-prime graph iff n is even.*

Proof. Suppose $I(C_n)$ is a neighborhood-prime graph. Clearly, $I\left(C_n\right) = C_{2n}$. Therefore, by theorem 1.3, $2n \not\equiv 2(mod4)$. That is, $2n - 2 \not\equiv 0(mod4)$. That is, $2n - 2 \neq 4k$ for any integer k.
$\Rightarrow 2n \neq 4k + 2$.
$\Rightarrow n \neq 2k + 1$. $\Rightarrow n$ is not odd. Therefore n is even.
Conversely, let n be even. Suppose $n = 2k$. Then $2n = 2(2k) = 4k$. Therefore $2n \equiv 0(mod4)$. Therefore $2n \not\equiv 2(mod4)$. Therefore by theorem 1.3, $C_{2n} = I\left(C_n\right)$ is a neighborhood-prime graph. Hence the inflation of cycle $I(C_n)$ is a neighborhood-prime graph iff n is even. □

Theorem 2.3. *The inflation of triangular snake $I(T_n)$ is a neighborhood-prime graph for all $n \neq 2$.*

Proof. Let $V(T_n) = \{v_i \,|1 \le i \le n\} \cup \{u_i \,|1 \le i \le n-1\}$ be the vertex set where $v_i's$ and $u_i's$ represent the vertices of the base path and top of the triangle.
Then $E(T_n) = \{v_iv_{i+1}|1 \le i \le n-1\} \cup \{v_iu_i\ ,\ u_iv_{i+1}|1 \le i \le n-1\}$ is the edge set.
In $I(T_n)$, corresponding to each vertex in T_n we get a clique. Suppose w is a vertex in T_n, adjacent to w_1, w_2, $\ldots\ldots$, w_k then label the vertices of the clique

corresponding to w as ww_1, ww_2, ..., ww_k. Correspondingly, every edge is either of the form $\{xyyx\}$ or $\{xy_ixz_j\}$. Here, the second pair represents the set of edges whose end vertices label start with a common vertex of T_n. Let $n = 1$. Then $I\,(T_1) \cong P_1$.
Therefore by remark 2.2, $I(T_1)$ is neighborhood-prime vacuously.
Let $n = 2$. Then $I(T_2) \cong C_6$ and $6 \equiv 2(mod\;4)$.
Therefore by theorem 1.4, $I(T_2)$ is not neighborhood-prime.
Let $n \geq 3$. Then the graph $I(T_n)$ looks as in figure 2.1. Here, $I(T_n)$ has $6n - 6$ vertices.

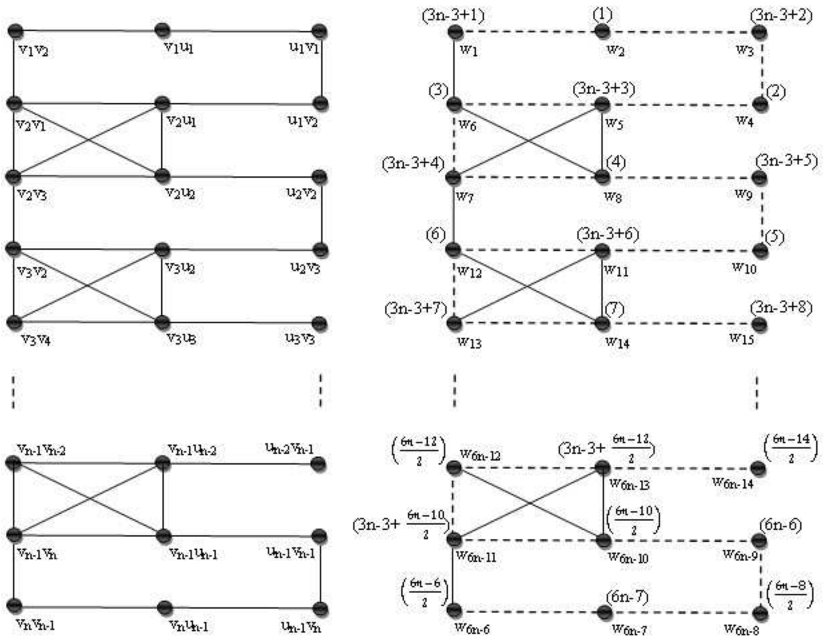

Figure 2.1: $\boldsymbol{I(T_n)}$

Consider the Hamiltonian path, v_1v_2, v_1u_1, u_1v_1, u_1v_2, v_2u_1, v_2v_1, v_2v_3, v_2u_2, u_2v_2, u_2v_3, v_3u_2, v_3v_2, v_3v_4, v_3u_3, u_3v_3, u_3v_4 ,......, $v_{n-2}u_{n-3}$, $v_{n-2}v_{n-3}$, $v_{n-2}v_{n-1}$, $v_{n-2}u_{n-2}$, $u_{n-2}v_{n-2}$, $u_{n-2}v_{n-1}$, $v_{n-1}u_{n-2}$, $v_{n-1}v_{n-2}$, $v_{n-1}v_n$, $v_{n-1}u_{n-1}$, $u_{n-1}v_{n-1}$, $u_{n-1}v_n$, v_nu_{n-1}, v_nv_{n-1}.
Rename the vertices in the Hamiltonian path as w_1, w_2, w_3, w_4, ..., w_{6n-8}, w_{6n-7}, w_{6n-6}. Define $f : V(I\,(T_n)) \rightarrow \{1, 2, 3, , 6n - 6\}$ as follows, $f\,(w_i) = \frac{i}{2}$ if i is even ; when i is odd & $1 \leq i \leq 6n - 11$, $f\,(w_i) = 3n - 3 + \left(\frac{i+1}{2}\right)$; $f\,(w_{6n-9}) = 6n - 6$ and $f\,(w_{6n-7}) = 6n - 7$. Since $w_{i-1}, w_{i+1} \subseteq N(w_i)$ and $f(w_{i-1})$, $f(w_{i+1})$ are

consecutive integers implies

(2.4) $$gcd\{f(x)|x \in N(w_i)\} = 1 \forall i \; such \; that \; 1 < i < 6n - 6 \& i \neq 6n - 10\,.$$

Further,

$$w_2 \subseteq N(w_1) \text{ and } f\,(w_2) = 1 \; implies \; gcd\{f(x)|x \in N(w_1)\} = 1\,,$$

and, $N(w_{6n-10})$ contains the vertices w_{6n-13} and w_{6n-11} with consecutive integers assigned to them under f. Therefore,

$$gcd\{f(x)|x \in N(w_{6n-10})\} = 1.$$

Similarly, $N\,(w_{6n-6}) = w_{6n-7}\,,\; w_{6n-11}$ and $f(w_{6n-7})$, $f(w_{6n-11})$ are consecutive integers implies

$$gcd\{f(x)|x \in N(w_{6n-6})\} = 1.$$

By (2.1),(2.2),(2.3) and (2.4), gcd$\{f(x)|x \in N(y)\} = 1$ for every $y \in \; V(I\,(T_n))$. Therefore f is a neighborhood-prime labeling. Hence $I(T_n)$ is a neighborhood-prime graph for all $n \neq 2$. □

Theorem 2.4. *The inflation of ladder graph $I(L_n)$ is a neighborhood prime graph for every n.*

Proof. Let $V(L_n) = \{u_i\,, v_i\,|1 \leq i \leq n\}$ be the vertex set.
Then $E(L_n) = \{u_i u_{i+1}\,, v_i v_{i+1}|1 \leq i \leq n-1\} \cup \{u_i v_i\,|1 \leq i \leq n\}$ is the edge set.
In $I(L_n)$, Corresponding to each vertex in L_n we get a clique. Suppose w is a vertex in L_n, adjacent to w_1 ,w_2 ,. . . . . ., w_k then label the vertices of the clique corresponding to w as ww_1, ww_2, ... ,ww_k. Correspondingly, every edge is either of the form $\{xyyx\}$ or $\{xy_i xz_j\}$. Here, the second pair represents the set of edges whose end vertices label start with a common vertex of L_n.
Let $n = 1$.
Then $I(L_1) \cong P_2$.
Therefore by remark 2.2, $I(L_1)$ is neighborhood-prime vacuously.
Let $n = 2$.
Then $I(L_2) \cong C_8$ and $8 \equiv 0(mod 4)$.
That is, $8 \not\equiv 2(mod 4)$.
Therefore by theorem 1.3, $I(L_2)$ is neighborhood-prime.
Let $n \geq 3$. Then the graph $I(L_n)$ looks as in figure 2.2(a) and figure 2.2(b).
Here, $I(L_n)$ has $6n - 4$ vertices.

Consider the Hamiltonian cycle, v_1u_1, u_1v_1, u_1u_2, u_2u_1, u_2v_2, u_2u_3, u_3u_2, u_3v_3, u_3u_4, $\ldots$, $u_{n-1}u_{n-2}$, $u_{n-1}v_{n-1}$, $u_{n-1}u_n$, u_nu_{n-1}, u_nv_n, v_nu_n, v_nv_{n-1}, $v_{n-1}v_n$, $v_{n-1}u_{n-1}$, $v_{n-1}v_{n-2}$, $v_{n-2}v_{n-1}$, $v_{n-2}u_{n-2}$, $v_{n-2}v_{n-3}$, $\ldots$, v_2v_3, v_2u_2, v_2v_1, v_1v_2, v_1u_1.
Rename the vertices in the Hamiltonian cycle as w_1, w_2, w_3, w_4, $\ldots$, w_{6n-6}, w_{6n-5}, w_{6n-4}, w_1.
Define $f : V(I(L_n)) \to \{1, 2, 3, , 6n-4\}$ as follows, $f(w_i) = \left(\frac{i+1}{2}\right)$ if i is odd; $f(w_{6n-6}) = 3n-1$; $f(w_{6n-4}) = 3n$; when i is even and $2 \le i \le 6n-8$, $f(w_i) = 3n + \left(\frac{i}{2}\right)$. Since w_{i-1} , $w_{i+1} \subseteq N(w_i)$ and $f(w_{i-1}), f(w_{i+1})$ are consecu-

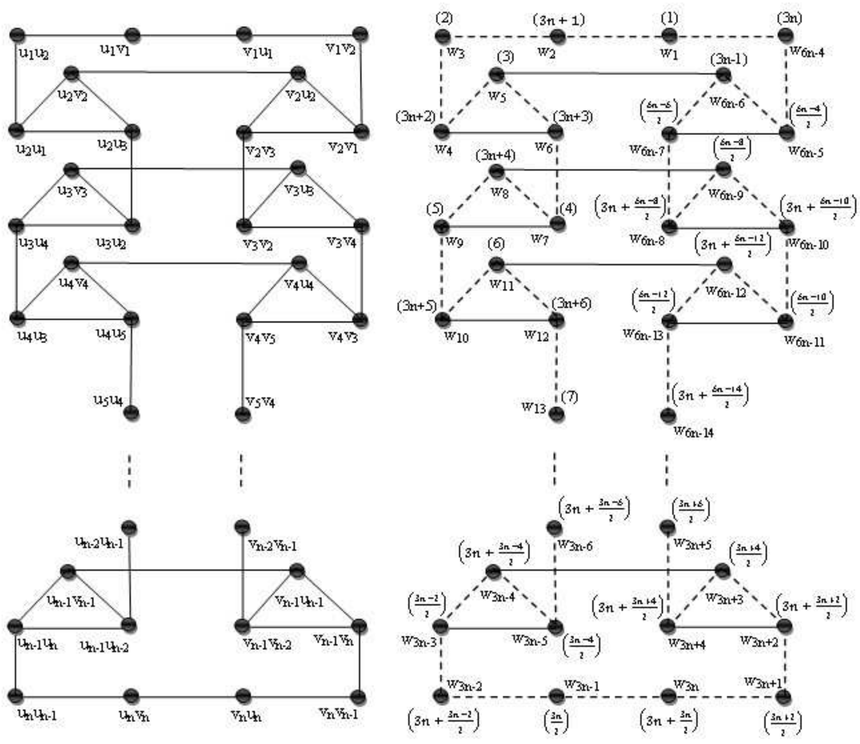

Figure 2.2(a): $I(L_n)$ when n is even

tive integers implies

$$gcd\{f(x)|x \in N(w_i)\} = 1 \forall i \; such \; that \; 1 < i < 6n-4 \; and \; i \neq 6n-7$$

Also, $N(w_1) = w_2$, w_{6n-4} and $f(w_2), f(w_{6n-4})$ are consecutive integers implies

$$gcd\{f(x)|x \in N(w_1)\} = 1.$$

Further,

$$w_1 \subseteq N(w_{6n-4}) \text{ and } f\,(w_1) = 1 \text{ implies } gcd\{f(x)|x \in N(w_{6n-4})\} = 1.$$

And, $N\,(w_{6n-7})$ contains the vertices w_{6n-5} and w_{6n-6} with consecutive integers assigned to them under f. Therefore,

$$gcd\{f(x)|x \in N(w_{6n-7})\} = 1\,.$$

By (2.1), (2.2), (2.3) and (2.4), $gcd\{f(x)|x \in N(y)\} = 1$ for every

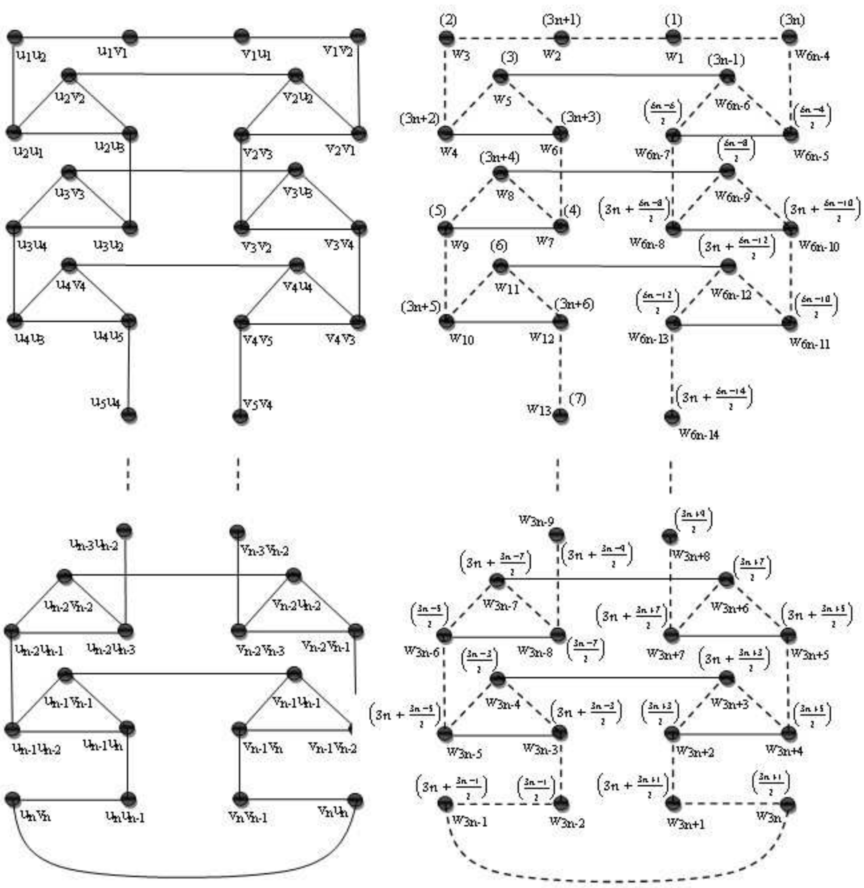

Figure 2.2(b): $I(L_n)$ when n is odd

$y \in V(I\,(L_n))$.

Therefore f is a neighborhood-prime labeling.

Hence $I(L_n)$ is a neighborhood-prime graph for every n. $\square$

Theorem 2.5. *The inflation of star $I(K_{1,n})$ is a neighborhood-prime graph for every n.*

Proof. Let $V(K_{1,n}) = \{v\ , u_i\ |1 \leq i \leq n\}$ be the vertex set where v represent the root vertex and u_i's represent the set of end vertices. Then $E(K_{1,n}) = \{vu_i|1 \leq i \leq n\}$ is the edge set. Let $n = 1$. Then $I(K_{1,1}) \cong P_2$.
Therefore by remark 2.2, $I(K_{1,1})$ is neighborhood-prime vacuously.
Let $n = 2$. Then $I(K_{1,2}) \cong P_4$.
Therefore by theorem 1.2, $I(K_{1,2})$ is neighborhood-prime.
Let $n \geq 3$. Then the graph $I(K_{1,n})$ looks as in figure 2.3.
In $I(K_{1,n})$, v is represented by K_n ; each u_i by K_1.

(1) Label the vertices of $A(v)$ as $v_1, v_2, v_3, , v_n$.
(2) Label the vertices of $A(u_i)$ as w_i for $1 \leq i \leq n$.

Here, $I(K_{1,n})$ has $2n$ vertices. Define $f : V\left(I\left(K_{1,n}\right)\right) \rightarrow \{1, 2, 3, , 2n\}$ by $f\left(v_i\right) = i$

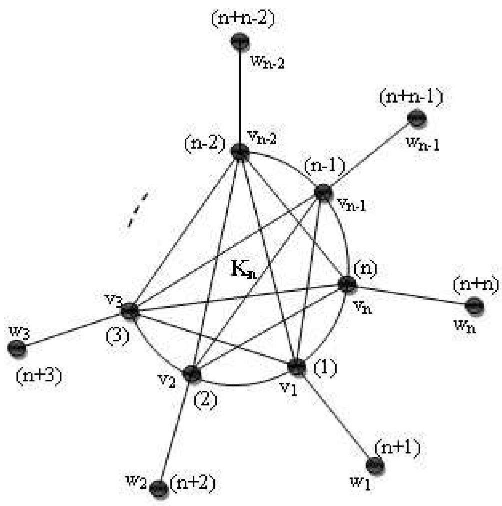

Figure 2.3: $I(K_{1,n})$

and $f\left(w_i\right) = n + i$ for $1 \leq i \leq n$.
Now for every $i = 1\ to\ n, N(v_i)$ contains atleast two vertices whose f values are two consecutive integers or two consecutive odd integers.
Therefore,

$$gcd\{f(x)|x \in N(v_i)\} = 1\ for 1 \leq i \leq n.$$

Further,

$$N(w_i) \text{ is a singleton set for every } i = 1 \text{ } to \text{ } n$$

By (2.1) and (2.2), $gcd\{f(x)|x \in N(y)\} = 1$ for every $y \in V(I(K_{1,n}))$ with $d(y) > 1$.
Therefore f is a neighborhood-prime labeling.
Hence $I(K_{1,n})$ is a neighborhood-prime graph for every n. □

Theorem 2.6. *The inflation of complete graph $I(K_n)$ is a neighborhood-prime graph for all $n \neq 3$.*

Proof. Let $V(K_n) = \{v_i \mid 1 \leq i \leq n\}$ be the vertex set.
Let $n \leq 2$. Then $I(K_i) \cong P_i$.
Therefore by remark 2.2, $I(K_n)$ is neighborhood-prime vacuously if $n \leq 2$.
Let $n = 3$. Then $I(K_3) \cong C_6$ and $6 \equiv 2(mod\ 4)$.
Therefore by theorem 1.4, $I(K_3)$ is not neighborhood-prime.
Let $n \geq 4$. In $I(K_n)$, each v_i is represented by K_{n-1}.
Label the vertices of $A(v_i)$ as v_{ij} for $1 \leq j \leq n-1$; Here, $I(K_n)$ has $n(n-1)$ vertices.
Define $f : V(I(K_n)) \to \{1, 2, 3, , n(n-1)\}$ by $f(v_{ij}) = (i-1)n + j - (i-1)$ for $1 \leq i \leq n, 1 \leq j \leq n-1$.
Correspondingly, for every $v \in V(I(K_n))$, $N(v)$ contains atleast two vertices u and w such that $f(u)$ and $f(w)$ are consecutive integers or $N(v)$ contains atleast one vertex say $'u'$ with $f(u) = 1$.
Therefore, $gcd\{f(x)|x \in N(v)\} = 1$ for every $v \in V(I(K_n))$. Therefore f is a neighborhood-prime labeling. Hence $I(K_n)$ is a neighborhood-prime graph for all $n \neq 3$. □

Theorem 2.7. *G is Hamiltonian iff $I(G)$ is Hamiltonian.*

Proof. Suppose G is Hamiltonian.
Let $C = (v_1, e_1, v_2, e_2, v_3, \ldots, v_n, e_n, v_1)$ be a Hamiltonian cycle in G.
In $I(G)$, each vertex $'v'$ of G is replaced by a clique $K_{d(v)}$ of order $d(v)$.
Since the complete graph is hamiltonian, it contains a hamiltonian path also.
Let the spanning path in $K_{d(v)}$ be $P_{d(v)}$ for $v \in V$.
Now, $P_{d(v_1)}, e'_1, P_{d(v_2)}, e'_2, P_{d(v_3)}, \ldots, P_{d(v_n)}, e'_n, v_{11}$ is a Hamiltonian cycle in $I(G)$ where v_{11} is the initial vertex of $P_{d(v_1)}$ and e'_i is the edge replacing e_i in $I(G)$. Hence $I(G)$ is Hamiltonian.

Conversely, Suppose $I\,(G)$ is Hamiltonian and let $P_{d(v_1)}$, e'_1, $P_{d(v_2)}$, e'_2, $P_{d(v_3)}$, $\ldots$, $P_{d(v_n)}$, e'_n, v_{11} is a hamiltonian cycle in $I(G)$ formed where $P_{d(v)}$ represents the spanning path of $k_{d(v)}$ and v_{11} is the initial vertex of $P_{d(v_1)}$.
Replacing each $P_{d(v_i)}$ by v_i and e'_i by e_i we get $v_1\ ,\ \ e_1\ ,v_2\ ,e_2\ ,v_3\ ,\ldots,v_n\ ,e_n\ ,v_1$ as a Hamiltonian cycle in G. Hence G is Hamiltonian. □

Theorem 2.8. *The inflation of grid graph $I(P_m \times P_n)$ is neighborhood-prime if both m and n are even.*

Proof. Suppose $G \cong P_m \times P_n$ is a grid graph with both $m\&n$ are even and let $V\,(G) = \{v_{ij}|1 \le i \le m\ ,\ 1 \le j \le n\}$ be the vertex set.
In $I(G)$, the vertices $\{v_{11}\ ,\ v_{m1}\ ,\ v_{1n}\ v_{mn}\}$ are represented by K_2; the vertices $\{v_{i1}\ ,\ v_{in}\ ,\ v_{1j}\ ,\ v_{mj}|2 \le i \le m-1\ ,\ 2 \le j \le n-1\}$ are represented by K_3 and the vertices $\{v_{ij}|2 \le i \le m-1\ ,\ 2 \le j \le n-1\}$ are represented by K_4.
Therefore,

$$|V(I(G)| = 2\,(4)+3\,(2m-4)+3\,(2n-4)+4\,((m-2)\,(n-2)) = 4mn-2(m+n)\,.$$

Since both m and n are even, $|V(I(G)| \equiv 0\,(mod\ 4) \not\equiv 2(mod\ 4)$
Also by remark 1.6 (2), G is Hamiltonian.
Therefore by theorem 2.7, $I(G)$ is Hamiltonian.
Therefore, by theorem 1.4, $I(G)$ is neighborhood-prime. □

Theorem 2.9. *The torus graph $C_m \times C_n$ is neighborhood-prime if $mn \not\equiv 2\,(mod\ 4)$.*

Proof. Suppose $G \cong C_m \times C_n$ is a torus graph with $mn \not\equiv 2(mod\ 4)$.
By remark 1.6 (1), G is Hamiltonian. Hence, by theorem 1.4, G is neighborhood-prime. □

Theorem 2.10. *The inflation of torus graph $I(C_m \times C_n)$ is neighborhood-prime for every $m\&n$.*

Proof. Suppose $G \cong C_m \times C_n$ is a torus graph and let $V\,(G) = \{v_{ij}|1 \le i \le m\ ,\ 1 \le j \le n\}$ be the vertex set.
In $I(G)$, the vertices $\{v_{ij}|1 \le i \le m\ ,\ 1 \le j \le n\}$ are represented by K_4.
Therefore, $|V(I(G)| = 4mn \equiv 0\,(mod\ 4) \not\equiv 2(mod\ 4)$.
By remark 1.6 (2), G is Hamiltonian.
Therefore By theorem 2.7, $I(G)$ is Hamiltonian.
Therefore, by theorem 1.4, $I(G)$ is neighborhood-prime. □

Theorem 2.11. *The inflation of closed helm $I(CH_n)$ is a neighborhood-prime graph for every n.*

Proof. Let $V(CH_n) = \{v\ , u_i\ ,\ w_i\ |1 \leq i \leq n\}$ be the vertex set where v represent the central vertex, $u_i's$ represent the vertices of inner cycle and $w_i's$ represent the vertices of the outer cycle. The graph CH_n looks as in figure 2.4. In $I(CH_n)$, v

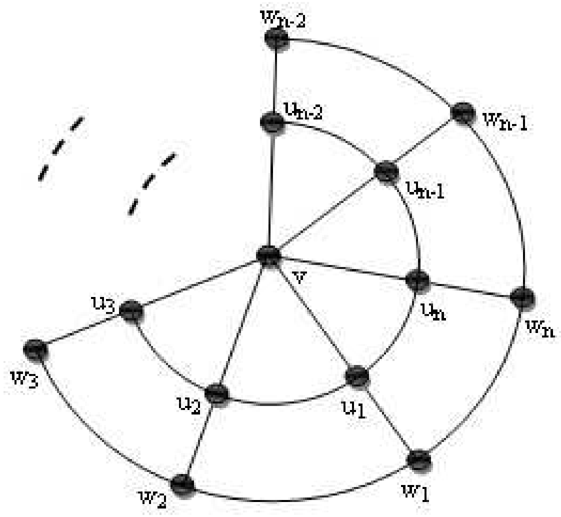

Figure 2.4: CH_n

is represented by K_n; each u_i by K_4 and each w_i by K_3.
Therefore, $|V\,(I(CH_n))| = n + 4n + 3n = 8n \equiv 0\,(mod\,4) \not\equiv 2(mod\,4)$.
Further, v, u_1, u_2, $\ldots$, u_{n-1}, w_{n-1}, w_{n-2}, $\ldots$, w_1, w_n, u_n, v is a hamiltonian cycle in CH_n. Therefore, CH_n is Hamiltonian.
Therefore by theorem 2.7, $I(CH_n)$ is Hamiltonian.
Hence, by theorem 1.4, $I(CH_n)$ is a neighborhood-prime graph for every n. □

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