

## ANTI Q-FUZZY BI-IDEALS IN NEAR-SUBTRACTION SEMIGROUPS

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**ABSTRACT.** Our primary focus is to examine the notion of anti Q-fuzzy bi-ideals in near-subtraction semigroups. This is a continuation/furtherance of our earlier study regarding Q-fuzzy bi-ideals in near-subtraction semigroups. In this paper, we have attempted to define the notation of anti Q-fuzzy bi-ideals and investigated their properties in near-subtraction semigroups.

### 1. INTRODUCTION

The Concepts of fuzzy sets and fuzzy subsets, fuzzy logic finds roots in seminal work of L. A. Zadeh [4] in 1965. Fuzzy Logic and fuzzification is a transformative development in set theory, having been a ring in many latest scientific applications. Ideal of subtraction semigroup is thoroughly examined by K. H. Kim et.al. [3]. Anti Q-fuzzy bi-ideals of near-rings are researched and characterized by A. Balavickhneswari, V. Mahalakshmi [2]. Also the notation of Q-fuzzy bi-ideals of near-subtraction semigroups are researched and characterized by P. Annamalai Selvi and et.al [1].

Our present study is inspired by the above study and we have examined the concept of anti Q-fuzzy bi-ideals in near-subtraction semigroups and its characteristics.

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## 2. PRELIMINARIES

In this section, we collect all basic concepts of near-subtraction semigroups, which are used in this paper. Throughout this paper, by a near subtraction semigroup, we mean only a zero-symmetric right near-subtraction semigroup.

**Definition 2.1.** A family of a  $Q$ -fuzzy sets  $\{\mu_i/i \in \Omega\}$  in  $X$ , the union of  $\{\mu_i/i \in \Omega\}$  is defined by,  $\cup_{i \in \Omega} \mu_i(x, q) = \sup\{\mu_i(x, q)/i \in \Omega\}, \forall x \in X, q \in Q$  and the intersection of  $\{\mu_i/i \in \Omega\}$  is defined by,  $\cap_{i \in \Omega} \mu_i(x, q) = \inf\{\mu_i(x, q)/i \in \Omega\}, \forall x \in X, q \in Q$ .

**Definition 2.2.** Let  $f : X \rightarrow X'$ . Let  $\mu$  and  $\lambda$  be a  $Q$ -fuzzy sets of  $X$  and  $X'$  respectively. Then  $f(\mu)$ , the image of  $\mu$  under  $f$ , is a subset of  $X'$  defined by:

$$f(\mu)(b, q) = \begin{cases} (a, q) \in f^{-1} \inf_{b, q} \mu(a, q) & \text{if } f^{-1}(b, q) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

and pre-image of  $\lambda$  under  $f$  is a  $Q$ -fuzzy subset of  $X$ , defined by  $f^{-1}(\lambda(x, q)) = \lambda(f(x, q))$ , for all  $x \in X, q \in Q$  and  $f^{-1}(y, q) = \{(x, q)/x \in X, q \in Q, f(x, q) = (y, q)\}$  and also referred the notations of  $(\mu \cap \lambda)$ ,  $(\mu - \lambda)$ ,  $(\mu\lambda)$  &  $(\mu * \lambda)$  in [1].

In this paper,  $f_I$  is the characteristic function of the subsets  $I$  of  $X$ , and the characteristic function of  $X \times Q$  is denoted by  $\chi : X \times Q \rightarrow [0, 1]$  and it is mapping each element of  $X \times Q$  to 1.

**Definition 2.3.** For any  $Q$ -fuzzy set  $\mu$  in  $X$  and  $t \in [0, 1]$ , we define  $L(\mu; t) = \{(x, q)/x \in X, q \in Q, \mu(x, q) \leq t\}$ , which is called a lower  $t$ -level cut of  $\mu$ .

**Definition 2.4.** A mapping  $f : X \rightarrow X'$  is called a homomorphism if  $f(x - y) = f(x) - f(y)$  and  $f(xy) = f(x)f(y)$ , for all  $x, y \in X$ .

**Definition 2.5.** A mapping  $f : X \rightarrow X'$  is called an anti-homomorphism if  $f(x - y) = f(y) - f(x)$  and  $f(xy) = f(y)f(x)$  for all  $x, y \in X$ .

**Definition 2.6.** A mapping  $\mu : X \times Q \rightarrow [0, 1]$ , where  $X$  is an arbitrary non-empty set is called  $Q$ -fuzzy set in  $X$ .

**Definition 2.7.** A  $Q$ -fuzzy subset  $\mu$  is called  $Q$ -fuzzy ideal of  $X$  if  $\forall x, y \in X$  and  $q \in Q$  it hold:

$$(i) \mu(x - y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$$

- (ii)  $\mu(xi - x(y - i), q) \geq \mu(i, q)$  and
- (iii)  $\mu(xy, q) \geq \mu(x, q)$ .

**Definition 2.8.** A  $Q$ -fuzzy set  $\mu$  in  $X$  is a  $Q$ -fuzzy bi-ideal of  $X$  if for  $\forall x, y, z \in X$  and  $q \in Q$  it hold:

- (i)  $\mu(x - y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$
- (ii)  $\mu(xyz, q) \geq \min\{\mu(x, q), \mu(z, q)\}$ .

**Definition 2.9.** A  $Q$ -fuzzy subset  $\mu$  is called an anti  $Q$ -fuzzy ideal of  $X$  if it satisfies:

- (i)  $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$
- (ii)  $\mu(xi - x(y - i), q) \leq \mu(i, q)$  and
- (iii)  $\mu(xy, q) \leq \mu(x, q)$ , for all  $x, y \in X$  and  $q \in Q$ .

### 3. ANTI Q-FUZZY BI-IDEAL IN NEAR-SUBTRACTION SEMIGROUPS

**Definition 3.1.** A  $Q$ -fuzzy set  $\mu$  in  $X$  is said to be an anti  $Q$ -fuzzy bi-ideal of  $X$  if for all  $x, y, z \in X$  and  $q \in Q$  it hold:

- (i)  $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$
- (ii)  $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$

**Example 1.** Let  $X = \{0, a, b, c\}$  with " $-$ " and " $\bullet$ " defined as

-	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

$\bullet$	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Define an anti  $Q$ -fuzzy subset  $\mu : X \times Q \rightarrow [0, 1]$  by  $\mu(0, q) = 0.5$ ,  $\mu(a, q) = 0.6$ ,  $\mu(b, q) = 0.7$ ,  $\mu(c, q) = 0.8$ . It is easy to verify that  $\mu$  is an anti  $Q$ -fuzzy bi-ideal of  $X$ .

**Theorem 3.1.** Let  $\{\mu_i / i \in \Omega\}$  be a family of an anti  $Q$ -fuzzy bi-ideal of  $X$ , then  $\cup_{i \in \Omega} \mu_i$  is also an anti  $Q$ -fuzzy bi-ideal of  $X$ , where  $\Omega$  is any index set.

*Proof.* Let  $\{\mu_i/i \in \Omega\}$  be a family of an anti Q-fuzzy bi-ideals of  $X$ . Let  $x, y, z \in X$ ,  $q \in Q$  and  $\mu = \cup_{i \in \Omega} \mu_i$ . Then

$$\begin{aligned}\mu(x, q) &= \cup_{i \in \Omega} \mu_i(x, q) = \sup_{i \in \Omega} \mu_i(x, q) \\ \mu(x - y, q) &= \sup_{i \in \Omega} \mu_i(x - y, q) \\ &\leq \sup_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(y, q)\} \\ &= \max\{\sup_{i \in \Omega} \mu_i(x, q), \sup_{i \in \Omega} \mu_i(y, q)\} \\ &= \max\{\cup_{i \in \Omega} \mu_i(x, q), \cup_{i \in \Omega} \mu_i(y, q)\} \\ &= \max\{\mu(x, q), \mu(y, q)\}.\end{aligned}$$

Therefore,  $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$ . Thus  $\mu$  is an anti Q-fuzzy subalgebra of  $X$ .

$$\begin{aligned}\mu(xyz, q) &= \sup_{i \in \Omega} \mu_i(xyz, q) \\ &\leq \sup_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(z, q)\} \\ &= \max\{\sup_{i \in \Omega} \mu_i(x, q), \sup_{i \in \Omega} \mu_i(z, q)\} \\ &= \max\{\cup_{i \in \Omega} \mu_i(x, q), \cup_{i \in \Omega} \mu_i(z, q)\} \\ &= \max\{\mu(x, q), \mu(z, q)\}.\end{aligned}$$

Therefore,  $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$ . Hence  $\mu = \cup_{i \in \Omega} \mu_i$  is an anti Q-fuzzy bi-ideal of  $X$ , where  $\Omega$  is any index set.  $\square$

**Theorem 3.2.** Let  $f : X \rightarrow X'$  be an epimorphism of  $X$ . If  $\lambda$  is an anti Q-fuzzy bi-ideal of  $X'$ , then  $f^{-1}(\lambda)$  is an anti Q-fuzzy bi-ideal in  $X$ .

*Proof.* Let  $\lambda$  be an anti Q-fuzzy bi-ideal of  $X'$ . For  $x, y, z \in X$  and  $q \in Q$ ,

$$\begin{aligned}f^{-1}(\lambda)(x - y, q) &= \lambda(f(x - y, q)) \\ &= \lambda(f(x, q) - f(y, q)) \\ &\leq \max\{\lambda f(x, q), \lambda f(y, q)\}.\end{aligned}$$

Therefore  $f^{-1}(\lambda)(x - y, q) \leq \max\{f^{-1}(\lambda)(x, q), f^{-1}(\lambda)(y, q)\}$ .

$$\begin{aligned} f^{-1}(\lambda)(xyz, q) &= \lambda(f(xyz, q)) \\ &= \lambda\{f(x, q)f(y, q)f(z, q)\} \\ &\leq \max\{\lambda f(x, q), \lambda f(z, q)\}. \end{aligned}$$

Therefore,  $f^{-1}(\lambda)(xyz, q) \leq \max\{f^{-1}(\lambda)(x, q), f^{-1}(\lambda)(z, q)\}$ . Hence  $f^{-1}(\lambda)$  is an anti Q-fuzzy bi-ideal in  $X$ .  $\square$

**Theorem 3.3.** Let  $f : X \rightarrow X'$  be an epimorphism of  $X$ . If  $\mu$  is an anti Q-fuzzy bi-ideal in  $X$ , then  $f(\mu)$  is an anti Q-fuzzy bi-ideal in  $X'$ .

*Proof.*

- (i) Let  $\mu$  is an anti Q-fuzzy bi-ideal in  $X$  and  $y_1, y_2, y_3 \in X'$  and  $q \in Q$ . Then we have:

$$\begin{aligned} (x, q) \in f^{-1} \inf_{y_1 - y_2, q} (x, q) &\subseteq (x_1, q) \in f^{-1}(y_1, q) \\ (x_2, q) \in f^{-1} \inf_{y_2, q} (x_1 - x_2, q) \\ f(\mu)(y_1 - y_2, q) = (x, q) &\in f^{-1} \inf_{y_1 - y_2, q} \mu(x, q) \leq (x_1, q) \in f^{-1}(y_1, q) \\ (x_2, q) \in f^{-1} \inf_{(y_2, q)} \mu(x_1 - x_2, q) &\leq (x_1, q) \in f^{-1}(y_1, q) \\ (x_2, q) \in f^{-1} \inf_{(y_2, q)} \max\{\mu(x_1, q), \mu(x_2, q)\} &= \max\{(x_1, q) \in f^{-1} \inf_{(y_1, q)} \mu(x_1, q) \\ &\quad (x_2, q) \in f^{-1} \inf_{(y_2, q)} \mu(x_2, q)\}. \end{aligned}$$

Therefore,  $f(\mu)(y_1 - y_2, q) \leq \max\{f(\mu)(y_1, q), f(\mu)(y_2, q)\}$ . Thus  $f(\mu)$  is an anti Q-fuzzy subalgebra in  $X'$ .

- (ii) Let  $y_1, y_2, y_3 \in X'$  and  $q \in Q$ . Then we have:

$$\begin{aligned} f(\mu)(y_1 y_2 y_3, q) = (x, q) &\in f^{-1} \inf_{(y_1 y_2 y_3, q)} \mu(x, q) \leq (x_1, q) \in f^{-1}(y_1, q) \\ (x_3, q) \in f^{-1} \inf_{(y_3, q)} \mu(x_1 x_2 x_3, q) &\leq (x_1, q) \in f^{-1}(y_1, q) \\ (x_3, q) \in f^{-1} \inf_{(y_3, q)} \max\{\mu(x_1, q), \mu(x_3, q)\} &= \max\{(x_1, q) \in f^{-1} \inf_{(y_1, q)} \mu(x_1, q) \\ &\quad (x_3, q) \in f^{-1} \inf_{(y_3, q)} \mu(x_3, q)\}. \end{aligned}$$

Therefore,  $f(\mu)(y_1 y_2 y_3, q) \leq \max\{f(\mu)(y_1, q), f(\mu)(y_3, q)\}$ . Hence  $f(\mu)$  is an anti Q-fuzzy bi-ideal in  $X'$ .

□

**Theorem 3.4.** *Let  $\mu$  be an anti Q-fuzzy subalgebra of  $X$ , then  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$  iff  $\mu X \mu \supseteq \mu$ .*

*Proof.* Assume that  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ . To prove:  $\mu X \mu \supseteq \mu$ , let  $x', x, y, x_1, x_2 \in X$  and  $q \in Q$  such that  $x' = xy$  and  $x = x_1x_2$ . Now,

$$\begin{aligned} (\mu X \mu)(x', q) &= \inf_{x'=xy} \max\{(\mu X)(x, q), \mu(y, q)\} \\ &= \inf_{x'=xy} \max\left\{\inf_{x=x_1x_2} \max\{\mu(x_1, q), X(x_2, q), \mu(y, q)\}\right\} \\ &= \inf_{x'=xy} \max\left\{\inf_{x=x_1x_2} \max\{\mu(x_1, q), 1, \mu(y, q)\}\right\} \\ &= \inf_{x'=x_1x_2y} \max\{\mu(x, q), \mu(y, q)\} \\ &\geq \inf_{x'=x_1x_2y} \mu(x_1x_2y, q). \end{aligned}$$

Therefore,  $(\mu X \mu)(x', q) \geq \mu(x', q)$ . Hence  $\mu X \mu \supseteq \mu$ .

Conversely, assume that  $\mu X \mu \supseteq \mu$ . To prove:  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ , let  $x, y, z \in X$  and  $q \in Q$ . Now,

$$\begin{aligned} \mu(xyz, q) &\leq \mu X \mu(xyz, q) \\ &= \inf_{xyz=ab} \max\{\mu X(a, q), \mu(b, q)\} \\ &\leq \max\{(\mu X)(xy, q), \mu(z, q)\} \\ &\leq \max\{\mu(x, q), X(y, q), \mu(z, q)\} \\ &= \max\{\mu(x, q), 1, \mu(z, q)\} \\ &= \max\{\mu(x, q), \mu(z, q)\}. \end{aligned}$$

Therefore,  $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$ . Hence  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ . □

**Theorem 3.5.** *Let  $X$  and  $X'$  be two near-subtraction semigroups. Let a mapping  $f : X \rightarrow X'$  be a homomorphism. If  $\mu$  is an anti Q-fuzzy bi-ideal of  $X'$  and  $L(\mu; t)$  is a bi-ideal of  $X'$ , then  $L(f^{-1}(\mu); t)$  is a bi-ideal of  $X$ .*

*Proof.* Let  $f : X \rightarrow X'$  be a homomorphism,  $\mu$  is an anti Q-fuzzy bi-ideal of  $X'$  and  $L(\mu; t)$  is a bi-ideal of  $X'$ . Let  $x, y \in L(f^{-1}(\mu); t)$  and  $q \in Q$ . Then we have:

$$f^{-1}(\mu)(x, q) \leq t$$

$$f^{-1}(\mu)(y, q) \leq t \Rightarrow \mu(f(x, q)) \leq t$$

$$\mu(f(y, q)) \leq t.$$

Now,

$$\begin{aligned} f^{-1}(\mu)(x - y, q) &= \mu(f(x - y, q)) \\ &= \mu(f(x, q) - f(y, q)) \\ &\leq \max\{\mu(f(x, q)), \mu(f(y, q))\} \\ &= \max\{t, t\} = t. \end{aligned}$$

Therefore,  $f^{-1}(\mu)(x - y, q) \leq t$ . We get  $x - y \in L(f^{-1}(\mu); t)$ . Hence  $L(f^{-1}(\mu); t)$  is a subalgebra of  $X$ .

Let  $x, z \in L(f^{-1}(\mu); t)$  and  $y \in X$ . Then we have:

$$f^{-1}(\mu)(x, q) \leq t$$

$$f^{-1}(\mu)(z, q) \leq t \Rightarrow \mu(f(x, q)) \leq t$$

$$\mu(f(z, q)) \leq t.$$

Now,

$$\begin{aligned} f^{-1}(\mu)(xyz, q) &= \mu(f(xyz, q)) \\ &= \mu(f(x, q)f(y, q)f(z, q)) \\ &\leq \max\{\mu(f(x, q)), \mu(f(z, q))\} \\ &= \max\{t, t\} = t. \end{aligned}$$

Therefore,  $f^{-1}(\mu)(xyz, q) \leq t$ . We get  $xyz \in L(f^{-1}(\mu); t)$ . Hence  $L(f^{-1}(\mu); t)$  is a bi-ideal of  $X$ .  $\square$

**Theorem 3.6.** Let  $\mu$  be a Q-fuzzy set of  $X$ . Then  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$  iff  $\mu^c$  is a Q-fuzzy bi-ideal of  $X$ .

*Proof.* Assume that  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ . To prove that  $\mu^c$  is a Q-fuzzy bi-ideal of  $X$ , let  $x, y, z \in X$  and  $q \in Q$ . Now,

$$\begin{aligned}\mu^c(x - y, q) &= 1 - \mu(x - y, q) \\ &\geq 1 - \max\{\mu(x, q), \mu(y, q)\} \\ &= \min\{1 - \mu(x, q), 1 - \mu(y, q)\}.\end{aligned}$$

Therefore,  $\mu^c(x - y, q) \geq \min\{\mu^c(x, q), \mu^c(y, q)\}$ .

Now,

$$\begin{aligned}\mu^c(xyz, q) &= 1 - \mu(xyz, q) \\ &\geq 1 - \max\{\mu(x, q), \mu(z, q)\} \\ &= \min\{1 - \mu(x, q), 1 - \mu(z, q)\} \\ &= \min\{\mu^c(x, q), \mu^c(z, q)\}.\end{aligned}$$

Therefore,  $\mu^c(xyz, q) \geq \min\{\mu^c(x, q), \mu^c(z, q)\}$ . Hence  $\mu^c$  is a Q-fuzzy bi-ideal of  $X$ .

Conversely, assume that  $\mu^c$  is a Q-fuzzy bi-ideal of  $X$ . To prove:  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ , let  $x, y, z \in X$  and  $q \in Q$ . Now,

$$\begin{aligned}\mu(x - y, q) &= 1 - \mu^c(x - y, q) \\ &\leq 1 - \min\{\mu^c(x, q), \mu^c(y, q)\} \\ &= \max\{1 - \mu^c(x, q), 1 - \mu^c(y, q)\}.\end{aligned}$$

Therefore  $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$ . Now,

$$\begin{aligned}\mu(xyz, q) &= 1 - \mu^c(xyz, q) \\ &\leq 1 - \min\{\mu^c(x, q), \mu^c(z, q)\} \\ &= \max\{1 - \mu^c(x, q), 1 - \mu^c(z, q)\}.\end{aligned}$$

Therefore  $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$ . Hence  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ .  $\square$

**Theorem 3.7.** Let  $\mu$  be a Q-fuzzy set of  $X$ . Then  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$  iff the lower level cut  $L(\mu; t)$  of  $X$  is a bi-ideal of  $X$  for each  $t \in [\mu(0), 1]$ .



*Proof.* Let  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ . Let  $x, y \in L(\mu; t)$  and  $q \in Q$ . Then  $\mu(x, q) \leq t$  and  $\mu(y, q) \leq t$ . Now,

$$\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\} = \max\{t, t\} = t.$$

Therefore,  $\mu(x - y, q) \leq t$ , we get  $x - y \in L(\mu; t)$ . Hence  $L(\mu; t)$  is a subalgebra of  $X$ .

Let  $x, z \in L(\mu; t)$  and  $y \in X, q \in Q$ . Then  $\mu(x, q) \leq t$  and  $\mu(z, q) \leq t$ . Now,

$$\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\} = \max\{t, t\} = t.$$

Therefore,  $\mu(xyz, q) \leq t$ . We get  $xyz \in L(\mu; t)$ . Hence  $L(\mu; t)$  is a bi-ideal of  $X$ .

Conversely, assume that  $L(\mu; t)$  is a bi-ideal of  $X$ . To prove that  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ , suppose  $\mu$  is not an anti Q-fuzzy bi-ideal of  $X$ . Let  $x, y \in X$  and  $q \in Q$   $\mu(x - y, q) > \max\{\mu(x, q), \mu(y, q)\}$ . Choose  $t$  such that  $\mu(x - y, q) > t > \max\{\mu(x, q), \mu(y, q)\}$ . Then we get  $x, y \in L(\mu; t)$ , but  $x - y \notin L(\mu; t)$ , which is a contradiction. Hence  $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$ .

Let  $x, y, z \in X$  and  $q \in Q$   $\mu(xyz, q) > \max\{\mu(x, q), \mu(z, q)\}$ . Choose  $t$  such that  $\mu(xyz, q) > t > \max\{\mu(x, q), \mu(z, q)\}$ . Then we get  $xyz \in L(\mu; t)$ , but  $xyz \notin L(\mu; t)$ , which is a contradiction. Hence  $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$ . Hence  $\mu$  is an anti Q-fuzzy bi-ideal of  $X$ .  $\square$

**Theorem 3.8.** Let  $\{\mu_i/i \in \Omega\}$  be a family of an anti Q-fuzzy bi-ideal of a near-subtraction semigroup  $X$ . Then  $\cap_{i \in \Omega} \mu_i$  is also an anti Q-fuzzy bi-ideal of  $X$ , where  $\Omega$  is any index set.

*Proof.* Let  $\{\mu_i/i \in \Omega\}$  be a family of an anti Q-fuzzy bi-ideals of  $X$ . Let  $x, y, z \in X$  and  $q \in Q$  and  $\mu = \cap_{i \in \Omega} \mu_i$ . Then:

$$\begin{aligned} \mu(x, q) &= \cap_{i \in \Omega} \mu_i(x, q) = \inf_{i \in \Omega} \mu_i(x, q) \\ \mu(x - y, q) &= \inf_{i \in \Omega} \mu_i(x - y, q) \\ &\leq \inf_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(y, q)\} \\ &= \max\{\inf_{i \in \Omega} \mu_i(x, q), \inf_{i \in \Omega} \mu_i(y, q)\} \\ &= \max\{\cap_{i \in \Omega} \mu_i(x, q), \cap_{i \in \Omega} \mu_i(y, q)\} \\ &= \max\{\mu(x, q), \mu(y, q)\}. \end{aligned}$$

Therefore,  $\mu(x - y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$ . Thus  $\mu$  is an anti Q-fuzzy subalgebra of  $X$ .

$$\begin{aligned}\mu(xyz, q) &= \inf_{i \in \Omega} \mu_i(xyz, q) \\ &\leq \inf_{i \in \Omega} \max\{\mu_i(x, q), \mu_i(z, q)\} \\ &= \max\{\inf_{i \in \Omega} \mu_i(x, q), \inf_{i \in \Omega} \mu_i(z, q)\} \\ &= \max\{\cap_{i \in \Omega} \mu_i(x, q), \cap_{i \in \Omega} \mu_i(z, q)\} \\ &= \max\{\mu(x, q), \mu(z, q)\}.\end{aligned}$$

Therefore,  $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$ . Hence  $\mu = \cap_{i \in \Omega} \mu_i$  is an anti Q-fuzzy bi-ideal of  $X$ , where  $\Omega$  is any index set.  $\square$

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