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TRI β FUNCTION

J. CARLIN[1] AND M. MUTHUKUMARI

ABSTRACT. In this paper we study the properties of tri β continuous and define tri β open map, tri β closed map and tri β homeomorphism.

1. INTRODUCTION

M.E.Abd El-Monsef introduced a generalization of open set in topological space called β open set in topological space called β open set. The generalized open sets and generalized continuity is analyzed in [2] and [3], respectively. A subset A of a topological space is called β open if $A \subseteq cl(int(clA))$. Lellis Thivagar extended this concept to bitopological spaces. In [1], the authors considered α open sets and α continuous functions. Palaniammal in his PhD thesis, [4], extended this concept to tri topological spaces. Palaniammal and others defined the concept tri β continuous function.

2. TRI β FUNCTION

Definition 2.1. *Let X be a non empty set and T_1, T_2 and T_3 be three topologies on X. X together with three topologies is called a tri topological space. It is denoted by (X, T_1, T_2, T_3).*

[1]*corresponding author*

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Example 1. *Let* $X = \{a, b, c, d\}$*,* $T_1 = \{\emptyset, X\}$*,* $T_2 = P(X)$*,* $T_3 = \{\emptyset, \{a\}, X\}$ *Then* (X, T_1, T_2, T_3) *is a tri topological space.*

Definition 2.2. *Let* (X, T_1, T_2, T_3) *be a tri topological space. Let* $A \subset X$*.* A *is called a tri* β *open set if* $A \subset T_1 cl T_2 int T_3 cl A$*.*

Example 2. *Let* $X = \{1, 2, 3\}$*,* $T_1 = \{\emptyset, \{1\}, X\}$*,* $T_2 = \{\emptyset, \{2\}, X\}$*,* $T_3 = P(X)$ *Then tri* β *open sets are* $\{\emptyset, \{2\}, \{2, 3\}, X\}$*.*

Definition 2.3. *Let* (X, T_1, T_2, T_3) *be a tri topological space. Let* $A \subset X$*.* A *is called a tri* β *closed set if* A^c *is tri* β *open.*

Example 3. *Let* $X = \{1, 2, 3\}$*,* $T_1 = \{\emptyset, \{1\}, X\}$*,* $T_2 = \{\emptyset, \{2\}, X\}$*,* $T_3 = P(X)$ *Then tri* β *closed sets are* $\{\emptyset, \{1\}, \{1, 3\}, X\}$*.*

Result 1.
(1) *Arbitrary union of tri* β *open sets is tri* β *open.*
(2) *Arbitrary intersection of tri* β *closed sets is tri* β *closed.*
(3) $\emptyset$ *and X are always both tri* β *open and tri* β *closed.*

Definition 2.4. *Let* (X, T_1, T_2, T_3) *and* $(Y, T_1^{'}, T_2^{'}, T_3^{'})$ *be two tri topological spaces. A function* f *from* X *to* Y*, is called a tri* β *continuous function if* $f^{-1}(V)$ *is tri* β *open set in* X*, for every tri* β *open set* V *in* Y*.*

Example 4. *Let* (X, T_1, T_2, T_3) *and* $(Y, T_1^{'}, T_2^{'}, T_3^{'})$ *be two tri topological spaces, where* $X = \{1, 2, 3\}$*,* $Y = \{a, b, c\}$*,* $T_1 = \{\emptyset, \{1\}, X\}$*,* $T_1^{'} = \{\emptyset, \{a\}, Y\}$*,* $T_2 = \{\emptyset, \{1\}, \{1, 3\}, X\}$*,* $T_2^{'} = \{\emptyset, \{a\}, \{a, b\}, Y\}$*,* $T_3 = \{\emptyset, \{1\}, \{1, 2\}, X\}$*,*
$T_3^{'} = \{\emptyset, \{a\}, \{b\}, Y\}$*. Tri* β *open sets in* X *are* $\{\emptyset, \{1\}, \{1, 2\}, \{1, 3\}, X\}$*, tri* β *open sets in* Y *are* $\{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$*. Let* $f : X \to Y$ *be a function such that:* $f(1) = a, f(2) = b, f(3) = c$*. Therefore,* f *is a tri* β *continuous function.*

Example 5. *Let* (X, T_1, T_2, T_3) *and* $(Y, T_1^{'}, T_2^{'}, T_3^{'})$ *be two tri topological spaces, where* $X = \{1, 2, 3\}$*,* $Y = \{a, b, c\}$*,* $T_1 = \{\emptyset, \{1\}, X\}$*,* $T_1^{'} = \{\emptyset, \{a\}, Y\}$*,* $T_2 = \{\emptyset, \{1\}, \{1, 3\}, X\}$*,* $T_2^{'} = \{\emptyset, \{a\}, \{a, b\}, Y\}$*,* $T_3 = \{\emptyset, \{1\}, \{1, 2\}, X\}$*,*
$T_3^{'} = \{\emptyset, \{a\}, \{b\}, Y\}$*. Tri* β *open sets in* X *are* $\{\emptyset, \{1\}, \{1, 2\}, \{1, 3\}, X\}$ *Tri* β *open sets in* Y *are* $\{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$*. Let* $f_1 : X \to Y$ *be a function such that:* $f_1(1) = b, f_1(2) = c, f_1(3) = a$*. Therefore,* f_1 *is not tri* β *continuous function.*

Definition 2.5. *Let* X *and* Y *be two tri topological spaces. A function* f *from* X *to* Y *is said to be tri* β *continuous at a point* $a \in X$ *if for every tri* β *open set* V *containing* $f(a)$*, there exists a tri* β *open set* U *containing a such that* $f(U) \subset V$*.*

Theorem 2.1. *f is a tri β continuous iff f is a tri β continuous at each point $x \in X$.*

Proof. Suppose f is tri β continuous. We prove that f is tri β continuous at each point $x \in X$.
Let $f : X \to Y$ be a tri β continuous function.
Take any $a \in X$. Let V be a tri β open set containing $f(a)$.
Since $f^{-1}(V)$ is tri β open set containing a.
Let $U = f^{-1}(V)$.
Then $f(U) \subset V \Rightarrow$ there exists a tri β open set U containing a and $f(U) \subset V$.
Hence, f is tri β continuous at a.
Conversely, suppose that f is tri β continuous at each point of X.
Let V be a tri β open set of Y. If $f^{-1}(V) = \emptyset$ then it is tri β open set.
Take any $a \in f^{-1}(V)$, f is tri β continuous at a. Hence there exists U_a, tri β open set containing a and $f(U_a) \subset V$.
Let $U = \cup\{U_a | a \in f^{-1}(V)$
We claim that: $U = f^{-1}(V)$
$a \in f^{-1}(V) \Rightarrow U_a \subset U \Rightarrow a \in U$ $x \in U \Rightarrow x \in U_a$ for some $a \Rightarrow f(x) \in V$ $\Rightarrow x \in f^{-1}(V)$ Hence, $U = f^{-1}(V)$.
Each U_a is tri β open. Hence U is tri β open.
$\Rightarrow f^{-1}(V)$ is tri β open in X.
Hence, f is tri β continuous function. □

Theorem 2.2. *Let (X, T_1, T_2, T_3) and (Y, T_1', T_2', T_3') be two tri topological spaces. Then f is a tri β continuous function iff $f^{-1}(V)$ is tri β closed in X for every tri β closed V in Y.*

Proof. Let $f : X \to Y$ is tri β continuous function.
Let V be any tri β closed in Y.
$\Rightarrow V^c$ is tri β open in Y
$\Rightarrow f^{-1}(V^c)$ is tri β open in X
$\Rightarrow [f^{-1}(V)]^c$ is tri β open in X
$\Rightarrow f^{-1}(V)$ is tri β closed in X.
Hence, $f^{-1}(V)$ is tri β closed in X. Whenever, V is tri β closed in Y.
Conversely,
suppose $f^{-1}(V)$ is tri β closed in X.
Whenever V is tri β closed in Y.

V is tri β open set in Y.
$\Rightarrow V^c$ is tri β closed in Y
$\Rightarrow f^{-1}(V^c)$ is tri β closed in X
$\Rightarrow [f^{-1}(V)]^c$ is tri β closed in X
$\Rightarrow f^{-1}(V)$ is tri β open in X.
Hence, f is tri β continuous function. □

Definition 2.6. *Let (X, T_1, T_2, T_3) and $(Y, T_1^{'}, T_2^{'}, T_3^{'})$ be two tri topological spaces. A function f from X to Y is called tri β open map if $f(V)$ is tri β open in Y for every tri β open set V in X.*

Example 6. *In example 4, f is tri β open map also.*

Definition 2.7. *Let (X, T_1, T_2, T_3) and $(Y, T_1^{'}, T_2^{'}, T_3^{'})$ be two tri topological spaces. Let f from X to Y be a mapping. f is called tri β closed map if $f(V)$ is tri β closed in Y for every tri β closed set F in X.*

Example 7. *Let (X, T_1, T_2, T_3) and $(Y, T_1^{'}, T_2^{'}, T_3^{'})$ be two tri topological spaces, where $X = \{1, 2, 3\}$, $Y = \{a, b, c\}$, $T_1 = \{\emptyset, \{1\}, X\}$, $T_1^{'} = \{\emptyset, \{a\}, Y\}$, $T_2 = \{\emptyset, \{1\}, \{1, 3\}, X\}$, $T_2^{'} = \{\emptyset, \{a\}, \{a, b\}, Y\}$, $T_3 = \{\emptyset, \{1\}, \{1, 2\}, X\}$,
$T_3^{'} = \{\emptyset, \{a\}, \{b\}, Y\}$.
Tri β closed sets in X are $\{\emptyset, \{2\}, \{3\}, \{2, 3\}, X\}$
Tri β closed sets in Y are $\{\emptyset, \{b\}, \{c\}, \{b, c\}, Y\}$.
Let $f : X \to Y$ be a function such that
$f(1) = a, f(2) = b, f(3) = c$.
Therefore, f is a closed map.*

Result 2. *Let X & Y be a two tri topological spaces. Let $f : X \to Y$ be a mapping. Then f is tri β continuous function iff $f^{-1} : Y \to X$ is tri β open map.*

Definition 2.8. *Let (X, T_1, T_2, T_3) and $(Y, T_1^{'}, T_2^{'}, T_3^{'})$ be two topological spaces. Let f from X to Y be a mapping. f is called a tri β homeomorphism. If*

i) *f is a bijection;*
ii) *f is a tri β continuous;*
iii) *f^{-1} is tri β continuous.*

Example 8. *The function f defined in the example 4 is*

i) *f is a bijection;*

ii) *f is tri β continuous;*

iii) *f^{-1} is tri β continuous. Therefore, f is a tri β homeomorphism.*

REFERENCES

[1] A. S. MASSHHOUR, I. A. HASANEIN, S.N. EL-DEEB: *α continuous and α open mappings*, Acta Math. Hung. **41**(3-4) (1983), 213–218.

[2] A. CSASZAR: *Generalized open sets*, Acta Math. Hungar., **75**(1-2) (1997), 65–87.

[3] A. CSASZAR: *Generalized topology, generalized continuity*, Acta Math. Hungar., **96** (2002), 351–357.

[4] S. PALANIAMMAL: *A Study of tri topological spaces*, Ph.D Thesis Manonmaniam Sundaranar University, November 2011, 21–42.

A. P. C. MAHALAXMI COLLEGE FOR WOMEN
THOOTHUKUDI-2
TAMIL NADU, INDIA
E-mail address: carlinewilliams103@gmail.com

A. P. C. MAHALAXMI COLLEGE FOR WOMEN
THOOTHUKUDI-2
TAMIL NADU, INDIA
E-mail address: mmkumari27@gmail.com