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ON GENERALIZED RIGHT PERMUTABLE Γ-NEAR SUBTRACTION SEMIGROUPS

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ABSTRACT. In analogy with the concept of generalized right permutable Γ-near-rings, we introduce the notion of generalized right permutable Γ-near subtraction semigroups. We show that any right permutable Γ-near subtraction semigroup is a generalized right permutable Γ-near subtraction semigroup. Also we study various properties of generalized right permutable Γ-near subtraction semigroups. Throughout this paper by X, we mean a Γ-near subtraction semigroup.

1. INTRODUCTION

Γ -near subtraction semigroup was introduced by Dr. S. J. Alandkar [1]. For basic terminology in near subtraction semigroup, we refer to Dheena [2] and for Γ-near subtraction semigroup, we refer to Dr. S. J. Alandkar [1].

M. Kaliselvi, N. Meenakumari and T. Tamizh Chelvam [3] introduced the notion of generalized right permutable Γ-near-ring. In this paper we introduce the notion of generalized right permutable in Γ-near subtraction semigroups by admiring the concepts of generalized right permutable Γ-near-ring.

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2. PRELIMINARIES

Definition 2.1. *A Γ-near subtraction semigroup is a triple $(X, -, \gamma)$, for all $\gamma \in \Gamma$, where Γ is a non-empty set of binary operators on X, such that $(X, -, \gamma)$ is a near-subtraction semigroup for all $\gamma \in \Gamma$. In practice, we called simply Γ-near-subtraction semigroup instead of right Γ-near- subtraction semigroup. Similarly we can define a Γ-near- subtraction semigroup (left).*

Definition 2.2. *An element $0 \neq a \in X$ is called nilpotent if there exists a positive integer $n \geq 1$ such that $(a\gamma)^n a = 0$ for each $\gamma \in \Gamma$.*

Definition 2.3. *X is called reduced if it has no nonzero nilpotent elements.*

Proposition 2.1. *X has no non-zero nilpotent elements if and only if $a\gamma a = 0$ implies $a = 0$ for all $\gamma \in \Gamma$.*

Definition 2.4. *An element $a \in X$ is called Boolean if $a\gamma a = a$ for all $\gamma \in \Gamma$.*

Definition 2.5. *X is said to have strong IFP if for all ideals I of X and for all $a, b, x \in X$, $a\gamma b \in I$ for all $\gamma \in \Gamma$ implies $a\gamma_1 x\gamma_2 b \in I$ for every pair of non-zero elements $\gamma_1, \gamma_2 \in \Gamma$.*

Definition 2.6. *X is said to fulfil the insertion-of-factors property(IFP) provided that for all $a, b, x \in X$, $a\gamma b = 0$ for all $\gamma \in \Gamma$ implies $a\gamma_1 x\gamma_2 b = 0$ for every pair of non-zero elements $\gamma_1, \gamma_2 \in \Gamma$.*

Definition 2.7. *X is said to have $*IFP$ if X has IFP and $x\gamma y = 0$ implies $y\gamma x = 0$ for every $x, y \in X$ and $\gamma \in \Gamma$.*

Definition 2.8. *Let P be an ideal of X. If P has strong IFP then the ideal P is called IFP-ideal of X.*

Definition 2.9. *An ideal P of X is called a completely semi prime if $a^2 (= a\Gamma a) \subseteq P$ implies $a \in P$.*

Definition 2.10. *A non-empty subset A of X is called a left $X - \Gamma$-subalgebra (or simply X-subalgebra) of X if A is a subalgebra of $(X, -)$ and $X\Gamma A \subseteq A$, i.e. $X\gamma A \subseteq A$ for all $\gamma \in \Gamma$.*

Definition 2.11. *A non-empty subset I of X is called:*

(i) *a left ideal of X if $y - y' \in I$ for every $y \in I$, $y' \in X$ and $x\gamma i - x\gamma(x' - i) \in I$, for all x, $x' \in X$ and $i \in I$;*

(ii) *a right ideal of X if $x - y \in I$ for every $x \in I$ and $y \in X$ and $I\Gamma X \subseteq I$;*
(iii) *an ideal of X if I is both left and right ideal of X.*

Definition 2.12. *Let X and Y be two Γ-Near Subtraction Semigroups. A map $f : X \to Y$ is said to be Γ-Near Subtraction Semigroup homomorphism if:*

(i) $f(a - b) = f(a) - f(b)$
(ii) $f(a\gamma b) = f\,(a)\,\gamma f(b)$ *for all a, $b \in X$, $\gamma \in \Gamma$.*

3. A study on regularities in Γ-near subtraction semigroups

Definition 3.1. *X is called regular if for each $a \in X$ and for every non-zero elements $\gamma \in \Gamma$, $a = a\gamma b\gamma a$ for some $b \in X$.*

Definition 3.2. *For $A \subseteq X$, we define the radical $\sqrt{A}$ of A to be $\{a \in X/a^k \in A$, for some $k > 0\}$. Obviously $A \subseteq \sqrt{A}$.*

Definition 3.3. *X is said to be right permutable if $a\,\gamma\,(b\gamma c) = a\gamma\,(c\gamma b)$ for all a,b,$c \in X$ and for all $\gamma \in \Gamma$.*

Proposition 3.1. *Let X be without non-zero nilpotent elements. Then X has IFP.*

Proof. If $x\gamma y = 0$ for $x, y \in X$ and $\gamma \in \Gamma$, then $(y\gamma x)^2 = y\gamma x\gamma y\gamma x = y\gamma 0 = 0$. This implies that $y\gamma x = 0$. For every pair of non-zero elements γ_1, $\gamma_2 \in \Gamma$ and $x \in X$, $(a\gamma_1 x\gamma_2 b)^2 = (a\gamma_1 x\gamma_2 b)\,\gamma\,(a\gamma_1 x\gamma_2 b) = a\gamma_1 x\gamma_2\,(b\gamma a)\,\gamma_1 x\gamma_2 b = a\gamma_1 x\gamma_2 0\gamma_1 x\gamma_2 b = a\gamma_1 x\gamma_2 0$ and hence a $\gamma_1 x\gamma_2 y = 0$. Therefore X has IFP. □

Lemma 3.1. *If X is a zero-symmetric, then for any ideal I of X, $X\Gamma I\Gamma X \subseteq I$.*

Proof. For any I, x, $x' \in X$ and $\gamma \in \Gamma$, $x\gamma i - x\gamma(x' - i) \in I$. Substituting $x' = 0$, we get $X\Gamma I \subseteq I$. Also, $I\Gamma X \subseteq I$. Hence $X\Gamma I\Gamma X \subseteq I$. □

Definition 3.4. *Let A and B be any two subsets of X. Let $\gamma \in \Gamma$, then $(A : B)_\gamma = \{x \in X/x\gamma B \subseteq A\}$.*

Proposition 3.2. *Let X be regular, $a \in X$ and $a = a\gamma_1 x\gamma_2 a$ for $x \in X$ and for every pair of non-zero elements γ_1, $\gamma_2 \in \Gamma$. Then for all $\gamma \in \Gamma$*

(i) *$a\gamma_1 x$, $x\gamma_2 a$ are idempotent elements in X for every pair of γ_1, $\gamma_2 \in \Gamma$;*
(ii) *$a\gamma_1 x\gamma_2 X = a\gamma_1 X$ and $X\gamma_1 x\gamma_2 a = X\gamma_2 a$ for every pair of non-zero elements γ_1, $\gamma_2 \in \Gamma$.*

Proof.

(i) $(x\gamma_2 a)^2 = (x\gamma_2 a)\,\gamma_1\,(x\gamma_2 a) = x\gamma_2\,(a\gamma_1 x\gamma_2 a) = x\gamma_2 a$. Similarly, $(a\gamma_1 x)^2 = (a\gamma_1 x)\gamma_2(a\gamma_1 x) = (a\gamma_1 x\gamma_2 a)\,\gamma_1 x = a\gamma_1 x$.

(ii) Trivially $X\gamma_2 a = X\gamma_2 a\gamma_1 x\gamma_2 a \subseteq X\gamma_1 x\gamma_2 a \subseteq X\gamma_2 a$. Similarly, $a\gamma_1 X = a\gamma_1 x\gamma_2 a\gamma_1 X \subseteq a\gamma_1 x\gamma_2 X \subseteq a\gamma_1 X$. Thereforea $\gamma_1 x\gamma_2 X = a\gamma_1 X$.

□

Proposition 3.3. *Let X be reduced. For any $a, b \in X$, and e an idempotent, $a\gamma_1 x\gamma_2 e = a\gamma_2 e\gamma_1 b$ for every pair of non-zero elements γ_1, $\gamma_2 \in \Gamma$.*

Proof. Let e be an idempotent in X, a, $b \in X$ and γ_1, γ_2 a pair of non-zero elements in Γ. Since $(a - a\gamma_2 e)\gamma_2 e = 0$, we have $(a - a\gamma_2 e)\,\gamma_1 b\gamma_2 e = 0$ so that $a\gamma_1 b\gamma_2 e = a\gamma_2 e\gamma_1 b\gamma_2 e$. Since $(e\gamma_1 b - e\gamma_1 b\gamma_2 e)\gamma_2 e = 0$, we have $e\gamma_1 b\gamma_2(e\gamma_1 b - e\gamma_1 b\gamma_2 e)\gamma_2 e = 0$ so that $(e\gamma_1 b - e\gamma_1 b\gamma_2 e)^2 = 0$. Hence $e\gamma_1 b = e\gamma_1 b\gamma_2 e$. Thus, $a\gamma_1 x\gamma_2 e = a\gamma_2 e\gamma_1 b$. □

Proposition 3.4. *Let X be with IFP property. Then $(0 : S)_\Gamma = \{x \in X / x\Gamma S = \{0\}\}$ is an ideal where S is any non-empty subset of X.*

Proof. Let $I = (0 : S)_\Gamma = \{x \in X / x\Gamma S = \{0\}\}$. Let $y \in I$ and $y' \in X$, $(y - y')\,\Gamma S = y\Gamma S - y'\Gamma S = \{0\}$. For $i \in I$ and $x \in X$, since X has IFP, $i\gamma x\Gamma S = \{0\}$ implies $i\gamma x \in I$ implies $I\Gamma X \subseteq I$. For $x, x' \in X$ and $I \in I$, $(x\gamma i - x\gamma(x' - i))\Gamma S = x\gamma i\Gamma S - x\gamma(x' - i)\Gamma S = x\gamma i\Gamma S - x\gamma(x'\Gamma S - i\Gamma S) = x\gamma i\Gamma S - x\gamma(x'\Gamma S - 0) = x\gamma 0 - x\gamma 0 = \{0\}$. $x\gamma i - x\gamma(x' - i) \in I$. Hence I is a ideal. □

4. Generalized right permutable Γ-near subtraction semigroups

Definition 4.1. *X is said to be (GRP) generalized right permutable Γ-near subtraction semigroup if $X\Gamma a\Gamma b = X\Gamma b\Gamma a$ for all a, $b \in X$.*

Proposition 4.1. *Any homomorphic image of a GRP Γ-near subtraction semigroup is again a GRP Γ-near subtraction semigroup.*

Proof. Let $f : X \to X'$ be a homomorphism and X be GRP Γ-near subtraction semigroup. Let $X' = f(X)$. Then $X'\Gamma f(a)\,\Gamma f(b) = f(X)\,\Gamma f(a)\,\Gamma f(b) = f(X\Gamma a\Gamma b) = f(X\Gamma b\Gamma a) = f(X)\,\Gamma f(b)\,\Gamma f(a) = X'\Gamma f(b)\Gamma f(a)$. Thus every homomorphic image of a GRP Γ-near subtraction semigroup is again a GRP Γ-near subtraction semigroup. □

Proposition 4.2. *Any right permutable Γ-near subtraction semigroup X is a GRP Γ-near subtraction semigroup.*

Proof. Let a, $b \in X$. For $y \in X\Gamma a\Gamma b$, we have $y = x\gamma a\gamma b$ for some $x \in X$ and for all $\gamma \in \Gamma$. Since X is right permutable, $y = x\gamma b\gamma a \in X\Gamma b\Gamma a$. Therefore $X\Gamma a\Gamma b \subseteq X\Gamma b\Gamma a$. Similarly $X\Gamma b\Gamma a \subseteq X\Gamma a\Gamma b$. Hence $X\Gamma b\Gamma a = X\Gamma a\Gamma b$ for all a, $b \in X$. Thus X is a GRP Γ-near subtraction semigroup. $\square$

Theorem 4.1. *Let X be a GRP Γ-near subtraction semigroup. If X is regular, then we have the following:*

(1) *For every $a \in X$, there exists $x \in X$, such that $a = a^2\Gamma x$;*
(2) *X has no non-zero nilpotent elements;*
(3) *Any two principal $X - \Gamma$-subalgebra of X commute with each other;*
(4) *$a\Gamma X\Gamma a = a\Gamma X$ for every $a \in X$.*
(5) *$X\Gamma a = X\Gamma a^2$ for all $a \in X$.*

Proof.

(1) Since X is a regular GRP Γ-near subtraction semigroup, $a = a\gamma x\gamma a \in X\Gamma x\Gamma a = X\Gamma a\Gamma x$ for all $\gamma \in \Gamma$. Therefore $a = x'\Gamma a\Gamma x$ for some $x' \in X$. This gives $x'\Gamma a = x'\Gamma\left(a\Gamma x\Gamma a\right) = \left(x'\Gamma a\Gamma x\right)\Gamma a = a\Gamma a = a^2$. In conclusion, we have $a = x'\Gamma a\Gamma x = a^2\Gamma x$.
(2) Let $a \in X$. Suppose $a\Gamma a (= a^2) = 0$. By (1) there exists $x \in X$ such that $a = a^2\Gamma x$ and therefore $a = 0$. By Proposition 2.1, X has no non-zero nilpotent elements.
(3) Let $y \in X\Gamma a\Gamma X$. Then $y = x\Gamma a\Gamma x'$ for some $x, x' \in X$. Since X is a GRP, $x\Gamma a\Gamma x' = x\Gamma x'\Gamma a$ for some $x \in X$. Hence $y = x\Gamma x'\Gamma a = \left(x\Gamma x'\right)\Gamma a \in X\Gamma a$. Also $X\Gamma a = X\Gamma a\Gamma x\Gamma a = X\Gamma a\Gamma(x\Gamma a) \subseteq X\Gamma a\Gamma X$. Therefore $X\Gamma a\Gamma X = X\Gamma a$. Let b, $c \in X$. Now $X\Gamma b\Gamma X\Gamma c = (X\Gamma b\Gamma X)\Gamma c = (X\Gamma b)\Gamma c = X\Gamma c\Gamma b = (X\Gamma c)\Gamma b = (X\Gamma c\Gamma X)\Gamma b = X\Gamma c\Gamma X\Gamma b$. That is, $X\Gamma b\Gamma X\Gamma c = X\Gamma c\Gamma X\Gamma b$ and (3) follows.
(4) Let $a \in X$. For any $y \in a\Gamma X$, there exists $x \in X$, such that $y = a\Gamma x = (a\Gamma x'\Gamma a)\Gamma x = a\Gamma(x'\Gamma a\Gamma x)$. Since X is a GRP, $x'\Gamma a\Gamma x = x'\Gamma x\Gamma a$. Hence $y = a\Gamma(x'\Gamma x\Gamma a) = a\Gamma(x'\Gamma x)\Gamma a \in a\Gamma X\Gamma a$ which implies that $a\Gamma X \subseteq a\Gamma X\Gamma a$. Clearly, $a\Gamma X\Gamma a \subseteq a\Gamma X$ and hence $a\Gamma X\Gamma a = a\Gamma X$.

(5) $X\Gamma a = X\Gamma(a\Gamma x\Gamma a) = (X\Gamma a\Gamma x)\Gamma a = (X\Gamma x\Gamma a)\Gamma a = X\Gamma x\Gamma(a\Gamma a) = X\Gamma x\Gamma a^2 \subseteq X\Gamma a^2$. Therefore, $X\Gamma a \subseteq X\Gamma a^2$ and consequently $X\Gamma a = X\Gamma a^2$.

□

Theorem 4.2. *Let X be a GRP. If X is Boolean, then the following are true.*

(1) $X\Gamma a\Gamma X\Gamma b = X\Gamma a\Gamma b$ *for all a, $b \in X$.*
(2) *All principal X-Γ-subalgebras of X commute with one another.*
(3) *Every ideal of X is a GRP.*
(4) *Every X-Γ- subalgebra of X is a GRP.*
(5) *Every X-Γ-subalgebra of X is an invariant X- Γ- subalgebra of X.*

Proof.

(1) Let a, $b \in X$. Since X is Boolean, $a = a\Gamma a \in a\Gamma X$, which implies $X\Gamma a \subseteq X\Gamma a\Gamma X$. Hence $X\Gamma a\Gamma b \subseteq X\Gamma a\Gamma X\Gamma b$. Then $y \in X\Gamma a\Gamma X\Gamma b$. Then $y = x\Gamma a\Gamma x'\Gamma b$ for some x, $x' \in X$. Since X is a GRP, we get that $x\Gamma a\Gamma x' = x\Gamma x'\Gamma a$. Thus $y = (x\Gamma x'\Gamma a)\Gamma b = (x\Gamma x')\Gamma a\Gamma b \in X\Gamma a\Gamma b$.
(2) For a, $b \in X$, we have $X\Gamma a\Gamma X\Gamma b = X\Gamma a\Gamma b = X\Gamma b\Gamma a = X\Gamma b\Gamma X\Gamma a$.
(3) Let I be any ideal of X and $a, b \in I$. Now $I\Gamma a\Gamma b = I\Gamma a^2\Gamma b = (I\Gamma a)\left((I\Gamma b) \subseteq I\Gamma\left(X\Gamma a\Gamma b\right) = I\Gamma\left(X\Gamma b\Gamma a\right) \subseteq I\Gamma b\Gamma a\right.$.That is $I\Gamma a\Gamma b \subseteq I\Gamma b\Gamma a$. Similarly, we get $I\Gamma b\Gamma a \subseteq I\Gamma a\Gamma b$ and therefore I is a GRP.
(4) For any X-Γ-subalgebra A of X, we have $X\Gamma A \subseteq A$. Let x, $y \in A$. Now, $A\Gamma x\Gamma y \subseteq X\Gamma x\Gamma y = X\Gamma y\Gamma x = X\Gamma y^2\Gamma x = (X\Gamma y)\,\Gamma y\Gamma x \subseteq (X\Gamma A)\,\Gamma y\Gamma x \subseteq A\Gamma y\Gamma x$. Similarly we get $A\Gamma y\Gamma x \subseteq A\Gamma x\Gamma y$. Consequently, A is a GRP.
(5) Let A be an X-Γ-Subalgebra of X. For $z \in A(\Gamma X)$, $z = a\Gamma x$ for some $a \in A$, $x \in X$ which implies that $z = a^2(\Gamma x) = a\Gamma a\Gamma x = a\Gamma x\Gamma a$. Therefore $z = a\Gamma x\Gamma a \in X\Gamma A \subseteq A$ and hence $A\Gamma X \subseteq A$, i.e. A is an invariant X-Γ-subalgebra.

□

Proposition 4.3. *Let X be a Boolean regular GRP and A an X-Γ- subalgebra of X. Then $A = \sqrt{A}$ where $\sqrt{A}$ is radical of A.*

Proof. Let $a \in \sqrt{A}$. There exists some positive integer k such that $a^k \in A$. By (1) of Theorem 4.1, $a = a^2\Gamma x$ for every $x \in X$. Now $a = a\Gamma a\Gamma x = a\Gamma(a^2\Gamma x)\Gamma x = a^3\Gamma x^2 = ... = a^k\Gamma x^{k-1} \in A\Gamma X \subset A$, by (5) of Theorem 4.1. Therefore $\sqrt{A} \subset A$. Obviously $A \subseteq \sqrt{A}$. □

Proposition 4.4. *Let X be a zero symmetric regular GRP. Then the following are true.*

(1) *X has IFP.*
(2) *Any ideal of X is semi completely prime.*
(3) *For any ideal I of X, X has the property that for a, $b \in X$, if $a\Gamma b \subseteq I$, then $b\Gamma a \subseteq I$.*
(4) *For every ideal I of X, and x_1,x_2,..$x_n \in X$ if $x_1\Gamma x_2\Gamma \ldots \Gamma x_n \subseteq I$, then $< x_1 > \Gamma < x_2 > \ldots \Gamma < x_{n-1} > \Gamma < x_n > \subseteq I$.*

Proof.

(1) It follows from Theorem 4.1 (2) and Proposition 3.4.
(2) Let I be an ideal of X and $a^2 \in I$ for $a \in X$. By Theorem 4.1 (1), for $a \in X$, there exists $x \in X$ such that $a = a^2\Gamma x$. Here we have $a = a^2\Gamma x \in I\Gamma X \subseteq I$. Therefore I is semi completely prime.
(3) Let I be an ideal of X and $a\Gamma b \subseteq I$ for a, $b \in X$. We have $(b\Gamma a)^2 = (b\Gamma a)\,\Gamma\,(b\Gamma a) = b\Gamma\,(a\Gamma b)\,\Gamma a \subseteq X\Gamma I\Gamma X$. By Lemma 3.1, $X\Gamma I\Gamma X \subseteq I$ and hence by (2) $(b\Gamma a) \subseteq I$.
(4) Let $x_1\Gamma x_2\Gamma \ldots \Gamma x_n \subseteq I$. It can be easily verified that $(I:S)_\Gamma$ is an ideal for any subset S of X. Since $x_1 \in (I : x_2\Gamma \ldots . \Gamma x_n)_\Gamma$ we have $< x_1 > \subseteq (I : x_2\Gamma \ldots \Gamma x_n)_\Gamma$ so that $< x_1 > \Gamma x_2\Gamma \ldots \Gamma x_n \subseteq I$. By (3) we have $x_2\Gamma \ldots \Gamma x_n\Gamma < x_1 > \subseteq I$. Now $x_2 \in (I : x_3\Gamma \ldots \Gamma x_n)_\Gamma$ which implies that $< x_2 > \Gamma(I : x_3\Gamma \ldots \Gamma x_n)_\Gamma$ and hence $< x_2 > \Gamma x_3 \ldots \Gamma x_n\Gamma < x_1 > \subseteq I$. This implies that $x_3 \ldots \Gamma x_n\Gamma < x_1 > \Gamma < x_2 > \subseteq I$. Continuing this process, we get $< x_1 > \Gamma < x_2 > \ldots \Gamma < x_{n-1} > \Gamma < x_n > \subseteq I$.

□

Lemma 4.1. *Let X be a zero-symmetric reduced. If X is regular, then every X-Γ-subalgebra of X is an ideal.*

Proof. Let $a \in X$. Since X is regular, for each $a \in X$ and for every pair of non-zero elements γ_1,$\gamma_2 \in \Gamma$, $a = a\gamma_1 b\gamma_2 a$ for some $b \in X$. By (i) of Proposition 3.2, $b\gamma_2 a$ is an idempotent. Let $b\gamma_2 a = e$. By (ii) of Proposition 3.2, $X\gamma_1 e = X\gamma_1 b\gamma_2 a = X\gamma_2 a$ for all γ_1, $\gamma_2 \in \Gamma$. Therefore, $X\Gamma e = X\Gamma a$. Let $S = \{x - x\gamma e / x \in X, \gamma \in \Gamma\}$. Since $(x - x\gamma_1 e)\gamma_1 e = 0$ for all $x \in X$. $(x - x\gamma_1 e)\,\gamma_1 X\gamma_2 e = 0$. This implies that $X\gamma_2 e \subseteq (0 : S)$ for all $\gamma_2 \in \Gamma$ and so $X\Gamma e \subseteq (0 : S)$. Now let $y \in (0 : S)$. Since X is regular, $y = y\gamma_1 x\gamma_2 y$ for some $x \in X$, γ_1, $\gamma_2 \in \Gamma$

and so $y\gamma_1 x - (y\gamma_1 x)\,\gamma_2 e \in S$. So, we have $(y\gamma_1 x - (y\gamma_1 x)\,\gamma_2 e)\gamma_2 y = 0$. Hence $y\gamma_1 x\gamma_2 y - y\gamma_1\,(x\gamma_2 e\gamma_2 y) = 0$, i.e. $y - y\gamma_1\,(x\gamma_2 e\gamma_2 y) = 0$ and by Proposition 3.3, $y - y\gamma_1\,(x\gamma_2 y\gamma_2 e) = 0$ implies $y - (y\gamma_1 x\gamma_2 y)\,\gamma_2 e = 0$ implies $y - y\gamma_2 e = 0$. Hence $y = y\gamma_2 e \in X\Gamma e$. It follows that $(0 : S) = X\Gamma e = X\Gamma a$. By Proposition 3.4, we get $X\Gamma a$ is an ideal of X. Now if Y is X-Γ-subalgebra of X, then $X = \sum_{a\in Y} X\Gamma a$. Thus Y is an ideal of X. $\square$

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