

PERFECT INDEPENDENT DETOUR DOMINATION NUMBER OF SOME SPECIAL GRAPHS

J. VIJAYA XAVIER PARTHIPAN¹ AND S. MUTHU KUMAR

ABSTRACT. Let $G = (V, E)$ be a connected graph with at least two vertices. A detour dominating set S is called a perfect independent detour dominating set of G if the cardinality of minimum independent detour dominating set of G is equal to the maximum independent detour dominating set of G . The perfect independent detour domination number is denoted by $P\gamma_{Id}(G)$. The perfect independent detour dominating set is called $P\gamma_{Id}$ -set of G . We say that G has perfect independent detour domination number which is infinite if there is no independent detour dominating set in G . The perfect independent detour domination number of some special graphs are determined.

1. INTRODUCTION

For a graph $G = (V, E)$ we mean an undirected graph without loops or multiple edges. The order and size of G is denoted by p and q respectively. We consider connected graph G with at least two vertices. For basic definition and terminology we refer to [1] and [4].

For vertices u and v in a connected graph G , the detour distance $D(u, v)$ is the length of the largest u - v path in G . An u - v path of length $D(u, v)$ is called an u - v detour. It is known that the detour distance is a metric in the vertex set $V(G)$.

¹corresponding author

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The detour eccentricity $e_d(v)$ of a vertex v in G is the maximum detour distance from v to a vertex of G [2].

The detour radius, $rad_D(G)$ is the minimum detour eccentricity among the vertices of G while the detour diameter $diam_D(G)$ is the maximum detour eccentricity among the vertices of G .

This concept was studied by Chartrand et al. [2]. A vertex x is said to lie on an u - v detour path P including the vertices u and v .

A set $S \subseteq V(G)$ is called a detour set if every vertex v in G lies on a detour joining a pair of vertices of S . The detour number $dn(G)$ of a G is the minimum order of a detour set and any detour set of order $dn(G)$ is called a minimum detour set of G . This concept was studied by G. Chartrand et al. [3].

Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S \subseteq V(G)$ is called a dominating set of G if every vertices in $V(G)-S$ is adjacent to some vertices in S . The domination number $\gamma(G)$ of G is the minimum order of its dominating and any dominating set of order $\gamma(G)$ is called γ -set of G [4].

Let $G = (V, E)$ be a connected graph with at least two vertices. A set $S \subseteq V(G)$ is called a detour dominating set of G if S is both a detour and a dominating set of G . The detour number $\gamma_d(G)$ of G is the minimum order of its detour dominating set and any detour dominating set of order $\gamma_d(G)$ is called a γ_d -set of G [5].

A detour dominating set S is called an independent detour dominating set if no two vertices of S are adjacent in G . The independent detour domination number $P\gamma_{Id}(G)$ is the minimum cardinality taken over all independent detour dominating sets of G . The minimum independent detour dominating set is called $P\gamma_{Id}(G)$ -set of G . We say that G has independent detour domination number which is infinite if there is no independent detour dominating set in G .

Let $G = (V, E)$ be a connected graph with at least two vertices. A maximum independent detour dominating set is an independent detour dominating set containing largest possible number of vertices for a graph G . The maximum independent detour domination number is denoted by $P\gamma_{MId}(G)$. We say that the maximum independent detour domination number is infinite if there is no independent detour dominating set in G .

The next theorems are used in the sequence.

Theorem 1.1. *Every end vertex of a non-trivial connected graph G belongs to every detour set of G and if the set S of all end vertices of G is a detour set then S is the unique minimum detour set of G , [5–7].*

Theorem 1.2. *Every detour domination number in connected graph G satisfies $\gamma_{Id}(G) \geq 2$, [5–7].*

Theorem 1.3. *Every end vertices of G belongs to every detour dominating set of G , [5–7].*

2. PERFECT INDEPENDENT DETOUR DOMINATION NUMBER OF A GRAPH

Definition 2.1. *Let $G = (V, E)$ be a connected graph with at least two vertices. A detour dominating set S is called a perfect independent detour dominating set of G if the cardinality of minimum independent detour dominating set of G is equal to the maximum independent detour dominating set of G . We say that G has perfect independent detour domination number which is infinite if there is no independent detour dominating set in G .*

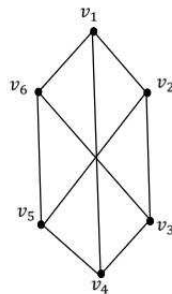


FIGURE 1

Example 1. *For the graph G given in the Figure 1, $S_1 = \{v_1, v_3, v_5\}$ and $S_2 = \{v_2, v_4, v_6\}$ are independent detour dominating set. Also $\gamma_{Id}(G) = \gamma_{MId}(G)$. Therefore, G has perfect independent detour dominating set.*

For the graph G in the Figure 2, $S_1 = \{v_1, v_6, v_4\}$ and $S_2 = \{v_1, v_3, v_7, v_5\}$ are independent detour dominating set of G . Here $\gamma_{Id}(G) \neq \gamma_{MId}(G)$. Therefore, G has no perfect independent detour domination number.

For the graph $G = P_2$ in the Figure 2, the perfect independent detour domination number is infinite.

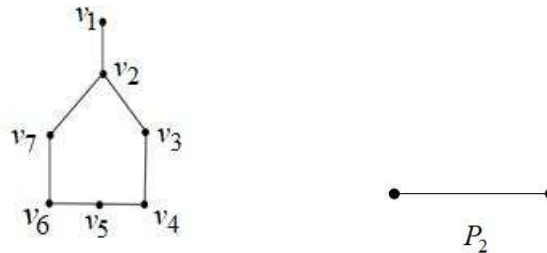


FIGURE 2

Remark 2.1. If G has a perfect independent detour domination number, all the independent detour domination number has same cardinality. Also it cannot necessarily have unique independent detour dominating set. For the graph in the Figure 1, G has two independent detour dominating sets.

Proposition 2.1. Every perfect independent detour dominating set is a minimum independent detour dominating set and also maximum independent detour dominating set, but converse does not need to be true. For the graph in the Figure 2, S_1 is the minimum independent detour dominating set and S_2 is the maximum independent detour dominating set, but G has no independent detour dominating set.

Definition 2.2. A gear graph G_n is a wheel graph with a graph vertex added between each pair of adjacent graph vertices of the outer cycle.

Proposition 2.2. In a gear graph G_n , $P\gamma_{Id}(G_n) = n + 1$.

Proof. Let $\{v, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ be the vertices of gear graph G_n , where $\{u_1, u_2, \dots, u_n\}$ is added between the each pair of adjacent to vertices of the outer cycle.

Now, consider $S = \{v_1, v_2, \dots, v_n\}$ and $S' = \{u, u_1, u_2, \dots, u_n\}$ are the independent detour set and clearly S' is the only independent detour dominating set of G_n .

Therefore, $\gamma_{Id}(G_n) = \gamma_{MId}(G_n) = n + 1$. Hence $P\gamma_{Id}(G_n) = n + 1$. $\square$

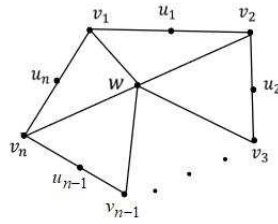


FIGURE 3

Definition 2.3. The windmill graph $Wd(n, k)$ is constructed for $k \geq 2$ and $n \geq 2$ by joining k copies of the complete graph K_n at a shared universal vertex.

Proposition 2.3. For a windmill graph $Wd(n, k)$, $P\gamma_{Id}(Wd(n, k)) = k$.

Proof. Consider the vertices from each component which forms the maximum independent detour dominating set. Therefore, $\gamma_{Id}(Wd(n, k)) = \gamma_{MId}(Wd(n, k)) = k$. Hence $P\gamma_{Id}(Wd(n, k)) = k$. $\square$

Definition 2.4. The friendship graph F_n can be constructed by joining n copies of the cycle C_3 with a common vertex.

Corollary 2.1. For the friendship graph F_n , $P\gamma_{Id}(F_n) = n$.

Proof. Using the above proposition, we get $P\gamma_{Id}(F_n) = n$. $\square$

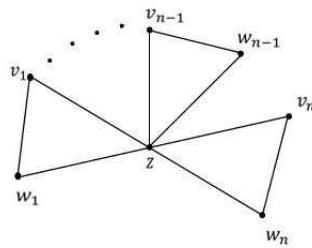


FIGURE 4

Definition 2.5. The butterfly graph F_2 is constructed by joining 2 copies of the cycle C_3 with a common vertex.

Corollary 2.2. For Butterfly graph F_2 , $P\gamma_{Id}(F_2) = 2$.

Proof. Follows by the above proposition, $P\gamma_{Id}(F_2) = 2$. $\square$

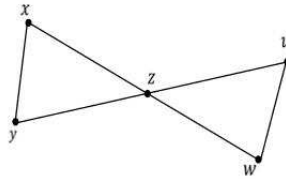


FIGURE 5

Definition 2.6. The n -book graph is defined as the graph cartesian product $B_n = S_{n+1} \times P_2$ where S_{n+1} is a star graph and P_2 is the path graph on two vertices.

Proposition 2.4. For a book graph B_n , $P\gamma_{Id}(B_n) = n+1$.

Proof. Consider the set of a vertex from every page of book B_n , we get $P\gamma_{Id}(B_n) = n+1$. $\square$

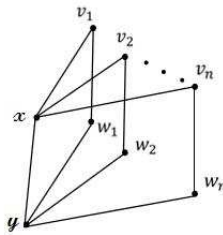


FIGURE 6

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DEPARTMENT OF MATHEMATICS
ST.JOHN'S COLLEGE, PALAYAMKOTTAI
MANONMANIAM SUNDARANAR UNIVERSITY
ABISHEKAPETTI, TIRUNELVELI, TAMIL NADU, INDI.
E-mail address: parthi68@rediffmail.com

DEPARTMENT OF MATHEMATICS
ST.JOHN'S COLLEGE, PALAYAMKOTTAI
MANONMANIAM SUNDARANAR UNIVERSITY
ABISHEKAPETTI, TIRUNELVELI, TAMIL NADU, INDIA
E-mail address: muthukumar.smk1996@gmail.com