ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal **9** (2020), no.5, 2749–2759 ISSN: 1857-8365 (printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.9.5.37

HALL EFFECT UNSTEADY FLOW PAST AN ISOTHERMAL VERTICAL PLATE IN A ROTATING FLUID WITH VARIABLE MASS DIFFUSION IN THE PRESENCE OF CHEMICAL REACTION OF FIRST ORDER

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ABSTRACT. An exact analysis was performed to study the Hall Effect on unsteady flow past an infinite isothermal vertical plate in a rotating fluid with variable mass diffusion in the presence of homogeneous chemical reaction of first order. The mathematical model, derived from the Navier-Strokes equation was reduced to a system of coupled partial differential equation for velocity, temperature and concentration using Boussinesqs approximation. The dimensionless governing equations are tackled using the usual Laplace transform technique. Also, the effect of velocity, temperature and concentration fields were intercepted for various physical parameters like Hall parameter, Hartmann number, thermal Grashof number, mass Grashof number, Schmidt number, chemical reaction parameter and rotation parameter.

1. INTRODUCTION

Hall Effect on vertical plate is contemporarily undergoing a period of large enlargement and differentiated subject of matter. The mechanism of conduction in ionized gas in presence of magnetic field is different from that in a metallic substance. In ionized gases, the current is not proportional to the applied potential except when the field is very weak in an ionized gas where the density is low and

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²⁰¹⁰ Mathematics Subject Classification. 76U05.

Key words and phrases. isothermal, rotation parameter, Schmidt number, Hartmann number, Hall parameter.

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the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiralling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon is called the Hall Effect. Hall Effect, in the presence of heat and mass transfer, through chemical reaction has a diversified application in geophysical fluid dynamics, structure of stars and other cosmological applications.

The effect of Hall current on hydromagnetic flow near accelerated plate has been studied by Pop in [1]. Seth et al., [2] investigated the effects of Hall current, radiation and rotation on natural convection heat and mass transfer flow past a moving vertical plate. The Effect of Hall current on the magneto hydrodynamics free convective flow past semi infinite vertical plate with mass transfer has been obtained by Aboeldaheb et al., [3]. Pop and Watanabe in [4], deduced the effect of Hall current on MHD free convection about semi infinite vertical flat plate. Das et al. in [5] presented the Hall Effect on MHD free convection boundary layer flow past a Vertical flat plate. B.C. Sarkar et al. in [6], investigated Hall Effects on unsteady MHD free convective flow past an accelerated moving vertical plate with viscous and Joule dissipations. G.S. Seth et al., [7] obtained the Effects of Hall Current on Hydromagnetic Natural Convection Flow of a Heat Absorbing Fluid Past an Impulsively Moving Vertical Plate with Ramped Temperature. Muthucumaraswamy et al. in [8] presented the hall effect on MHD flow past an exponentially accelerated isothermal vertical plate with variable mass diffusion in the presence of rotating fluid.

Muthucumaraswamy and Jeyanthi in [9] studied the Hall effect on MHD flow past an infinite vertical plate in rotating fluid of varying temperature. The first order chemical reaction in the neighbourhood of a horizontal plate was analysed by Chambre and Young in [10]. In this problem, the Hall effect has been combined with an infinite vertical isothermal plate in a rotating fluid of varying mass diffusion following first order reaction kinetics and the effect of Hartmann number M, Schmidt number Sc, Rotation parameter Hall parameter m, thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Chemical reaction parameter K on the concentration and axial velocity of the fluid has been presented.

2. MATHEMATICAL FORMULATION

An unsteady hydromagnetic flow of fluid past an infinite isothermal vertical plate with varying mass diffusion exists. The fluid and the plate rotate in unison with a uniform angular velocity Ω' about the z' - axis normal to the plate. Initially the fluid is assumed to be at rest and surrounds an infinite vertical plate with temperature T'_{∞} and concentration C'_{∞} . A magnetic field of uniform strength B_0 is transversely applied to the plate. The x'-axis is taken along the plate in the vertically upward direction and the z'-axis is taken normal to the plate. At time $t^{'} > 0$, the plate and the fluid are at the same temperature $T^{'}_{\infty}$ in the stationary condition with concentration level C_∞^\prime at all the points. At time t' > 0, the plate is subjected to a uniform velocity $u = u_0$ in its own plane against the gravitational force. The plate temperature and concentration level near the plate are raised uniformly and are maintained constantly thereafter. All the physical properties of the fluid are considered to be constant except the influence of the body force term. Then under the usual Boussinesq's approximation the unsteady flow equations are momentum equation, energy equation, and mass equation respectively.

Equation of Momentum:

(2.1)
$$\frac{\partial u}{\partial t'} - 2\Omega' v = v \frac{\partial^2 u}{\partial z^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} + g + \frac{B_0}{\rho} j_y + g \frac{\partial^2 u}{\partial z} + g \frac{\partial^2 u$$

(2.2)
$$\frac{\partial v}{\partial t'} + 2\Omega' u = v \frac{\partial^2 v}{\partial z^2} - \frac{B_0}{\rho} j_x.$$

Equation of Energy:

(2.3)
$$\rho C_P \frac{\partial T'}{\partial t'} = k \frac{\partial T'}{\partial z^2} - \frac{\partial q_r}{\partial z}$$

Equation of diffusion:

(2.4)
$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C}{\partial z^2}.$$

As, no large velocity gradient here, the viscous term in equation (2.1) vanishes for small and hence for the outer flow, beside there is no magnetic field along x-direction gradient, so this results in,

(2.5)
$$0 = -\frac{\partial \rho}{\partial x} - \rho_{\infty} g$$

By eliminating the pressure term from equation (2.1) and (2.5), we obtain

(2.6)
$$\frac{\partial u}{\partial t'} - 2\Omega' v = v \frac{\partial u^2}{\partial z^2} + (\rho_{\infty} - \rho)g + \frac{B_0}{\rho} j_y.$$

The Boussinesq approximation gives

(2.7)
$$\rho_{\infty} - \rho = \rho_{\infty}\beta(T' - T'_{\infty}) + \rho_{\infty}\beta^{-}(C' - C_{\infty}')$$

On using (2.7) in the equation (2.6) and noting that ρ_{∞} is approximately equal to 1, the momentum equation reduces to

(2.8)
$$\frac{\partial u}{\partial t'} - 2\Omega' v = v \frac{\partial^2 u}{\partial z^2} + \frac{B_0}{\rho} j_y + g\beta(T' - T'_\infty) + g\beta^*(C' - C_\infty').$$

The genralized Ohm's law with Hall currents is taken into account and ion - slip and thermo-electric

(2.9)
$$j + \frac{\omega_e \tau_e}{B_0} (j \times B) = \sigma [E + q \times B].$$

The equation (2.9) gives

$$(2.10) J_x - mj_y = \sigma v B_0$$

$$(2.11) J_y - mj_x = \sigma u B_0$$

where $m = \omega_e \tau_e$ is Hall parameter. Solving (2.10) and (2.11) for J_x and J_y , we have

(2.12)
$$J_x = \frac{\sigma B_0}{(1+m^2)} (v - mu) \,.$$

(2.13)
$$J_y = \frac{\sigma B_0}{(1+m^2)} (u+mv) \,.$$

On the use of (2.12) and (2.13), the momentum equations (2.8) and (2.2) become

(2.14)
$$\frac{\partial u}{\partial t'} = v \frac{\partial^2 u}{\partial z^2} + 2\Omega_z v - \frac{\sigma \mu_e^2 H_0^2}{\rho(1+m^2)} (u+mv) + g\beta(T-T_\infty) + g\beta^* (C'-C'_\infty)$$

(2.15)
$$\frac{\partial v}{\partial t'} = v \frac{\partial^2 u}{\partial z^2} + 2\Omega_z v - \frac{\sigma \mu_e^2 H_0^2}{\rho (1+m^2)} (v-mu)$$

(2.16)
$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial z^2}$$

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(2.17)
$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z^2} - K_l (C' - C'_{\infty}).$$

Due to small Coriolis force, the second term on the right side of the equation (2.14) and (2.15) comes into existence.

The boundary conditions are given by:

$$\begin{split} u &= 0, T = T_{\infty}, C' = C'_{\infty} & \text{for all } y, t' \leq 0 \\ t' &> 0 : u = u_0, T = T_w, C' = C'_{\infty} + (C'_w - C'_{\infty})At' & \text{at } y = 0 \\ u &\to 0, T \to T_{\infty}, C' \to C'_{\infty} & \text{as } y \to \infty \end{split}$$

(2.18)
$$u = 0, T = T_{\infty}, C' = C'_{\infty}, v = 0 \text{ for all } y, t' \le 0$$

(2.19)
$$u = u, T = T_w, C' = C'_w, v = 0 \ atz = 0 \ for \ all \ t' \le 0$$

 $u \to 0, T \to T_{\infty}, C' \to C'_{\infty}$ as $z \to \infty$ for all $t' \le 0$. The dimensionless quantities are introduced as follows:

$$U = \frac{u}{u_0}, V = \frac{v}{u_0}, t = \frac{t'u_0^2}{v}, Z = \frac{z(u_0^2}{v^2}), \Omega = \Omega_z \frac{v}{u_0^2}, M^2 = \frac{\sigma \mu_e^2 H_0^2 v}{2\rho u_0^2}$$

(2.20)
$$G_r = \frac{vg\beta(T_w - T_\infty)}{u_0^3}, G_c = \frac{vg\beta^*(C'_w - C'_\infty)}{u_0^3}, P_r = \frac{\mu c_p}{k}, K = K_l(\frac{v}{u_0^2})$$

Together with the equation (2.1), (2.2), (2.3) and (2.4), boundary conditions (2.18), (2.19), using (2.20), we have

(2.21)
$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Z^2} + 2V(\Omega - \frac{M^2}{1+m^2}) - \frac{2m^2}{1+m^2}U + \theta G_r + CG_c$$

(2.22)
$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial Z^2} - 2U(\Omega + \frac{M^2 m}{1+m^2}) + \frac{2m^2 V}{1+m^2},$$

with the boundary conditions,

(2.23)
$$U = 0, \theta = 0, C = 0, v = 0 \text{ for all } Z, t \le 0,$$

$$U = 1, \theta = 1, C = t, V = 0$$
 at $Z = 0$, for all $t > 0$.

(2.24)
$$U \to 0, \theta \to 0, C \to 0, V \to 0 \text{ as } z \to \infty \text{ for all } t > 0$$

Now equations (2.21), (2.22) and the boundary conditions (2.23), (2.24) can be combined to give:

(2.25)
$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial Z^2} - Fa + \theta G_r + CG_c,$$

where F = U + iV and $a = 2\left[\left(\frac{M^2}{1+m^2} + i(\Omega - \frac{M^2m}{1+m^2})\right]$. In this study the value of Ω (rotation parameter) is taken a

In this study the value of Ω (rotation parameter) is taken to be $\Omega = \frac{M^2m}{1+m^2}$, as a result of this the transverse velocity vanishes.

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial Z^2} \,,$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2}{\partial Z^2} - KC \,,$$

with the boundary conditions,

 $F = 0, \theta = 0, C = 0$, for all $Z, t \le 0$ $F = 1, \theta = 1, C = t$ at Z = 0 and for all t > 0

(2.26)
$$F \to 0, \theta \to 0, C \to 0 \text{ as } z \to \infty \text{ for all } t > 0.$$

The exact solution for fluid temperature, concentration and velocity is expressed below using Laplace transform technique:

(2.27)
$$\theta(Z,t) = erfc(\eta\sqrt{P_r})$$

$$\begin{split} C(Z,t) &= t\{\frac{1}{2}[exp(-2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c}-\sqrt{Kt}) + \\ &+ exp(2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c}+\sqrt{Kt})]\} - \\ &- (\frac{\eta}{2}\sqrt{tS_c})\{\frac{1}{\sqrt{K}}[exp(-2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c}-\sqrt{Kt}) - \\ &- exp(2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c}+\sqrt{Kt})]\} \end{split}$$

$$\begin{split} F(Z,t) &= \frac{1}{2} [exp(-2\eta\sqrt{at}) erfc(\eta-\sqrt{at}) + exp(2\eta\sqrt{at}) erfc(\eta+\sqrt{at})] + \\ & \int \frac{\frac{-1}{2} [exp(-2\eta\sqrt{at}) erfc(\eta-\sqrt{at}) + exp(2\eta\sqrt{at}) erfc(\eta+\sqrt{at})] + \\ \frac{exp(\gamma t)}{2} [exp(-2\eta\sqrt{(\gamma+a)t} erfc(\eta-\sqrt{(\gamma+a)t}) +] \\ \end{split}$$

$$\frac{G_r}{a} \begin{cases} exp(2\eta\sqrt{(\gamma+a)terfc}(\eta+\sqrt{(\gamma+a)t})] + \\ erfc(\eta\sqrt{P_r} - \frac{exp(\gamma t)}{2}[exp(-2\eta\sqrt{P_r\gamma t})erfc(\eta\sqrt{P_r} - \sqrt{\gamma t}) + \\ exp(2\eta\sqrt{P_r\gamma t})erfc(\eta\sqrt{P_r} + \sqrt{\gamma t}] \end{cases}$$

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$$+ \frac{Gc}{(a-KSc)} \left\{ \begin{array}{l} \frac{1}{2\alpha} [exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at}) + exp(2\eta+\sqrt{at})] \\ \frac{-\frac{1}{2}}{2} [exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at}) + exp(2\eta+\sqrt{at})] \\ + \frac{\eta}{2}\sqrt{\frac{t}{a}} [exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at}) - exp(2\eta+\sqrt{at})] \\ + \frac{\eta}{2}\sqrt{\frac{t}{a}} [exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at}) - exp(2\eta+\sqrt{at})] \\ + \frac{exp(2\eta\sqrt{(\alpha+a)}terfc(\eta+\sqrt{(\alpha+a)t})] + \\ exp(2\eta\sqrt{(\alpha+a)}terfc(\eta+\sqrt{(\alpha+a)t})] + \\ + \frac{1}{2\alpha} [exp(-2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c} - \sqrt{Kt}) + \\ exp(2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c} - \sqrt{Kt}) + \\ exp(2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c} + \sqrt{Kt})] \\ + \frac{\eta}{2}\sqrt{\frac{Set}{k}} [exp(-2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c} - \sqrt{Kt}) + \\ exp(2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c} + \sqrt{Kt})] \\ + \frac{exp(2\eta\sqrt{S_cKt})erfc(\eta\sqrt{S_c} + \sqrt{Kt})] \\ + \frac{exp(2\eta\sqrt{S_ct}(K+\alpha))erfc(\eta\sqrt{S_c} + \sqrt{(K+\alpha)t})] \\ \end{array} \right\}$$
where $\gamma = \frac{a}{P_r - 1}, \alpha = \frac{a - KS_c}{S_c - 1}, a = \frac{2M^2m}{1 + m^2} \text{ and } \eta = \frac{z}{2\sqrt{t}}.$

3. RESULTS AND DISCUSSION

The problem has been formulated, analyzed and solved analytically using Laplace technique. The effect of parameters like thermal Grashof number **Gr**, mass Grashof number **Gc**, Schmidt number **Sc**, Prandtl number **Pr**, Reaction parameter **K**, Rotation parameter Ω , time **t**, Hartmann number **M**, Hall parameter **m**, on Axial Velocity **F**, Concentration **C** and Temperature **T** are computed and intercepted through graphs.



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Figure1 shows the Temperature profile for different values of Prandtl number. It is observed that temperature increases with decrease in values of Prandtl number and also heat transfer is predominant in air when compared to water. Figure2 represents the concentration profile for various values of Reaction parameter K at time t=2 and Schmidt number Sc=2.01. It is noticed that the concentration field decreases with increase in values of K.



In figure 3 the concentration profile for different values of Sc at t=2 and K=0.2 is presented and it is observed that the concentration field increases for decreasing value of Sc. In figure 4 the Axial Velocity profile for various values of Sc at t=2, K=0.2, Gr=5, Gc=10, Pr=7, M=1.0 and m=0.5 is obtained. Here, the axial velocity drops for high Sc values.



FIGURE 5: Axial velocity profile for Different values of Hartmann <u>number</u>. M

FIGURE 6: Axial velocity profile for Different values of Hall parameter, m

Figure 5 and Figure 6, depicts the Axial velocity profile at Sc=2.01, Pr=7.0, K=0.2, t=2, Gr=5, Gc=10 for varying Hartmann number, M and Hall parameter, m respectively. From figure 5 it is observed that the axial velocity falls as M is increased and in Figure 6 it is observed that axial velocity increases for increasing values of m.



4. CONCLUSION

An analysis was performed to inspect the combined Hall effect on an infinite vertical isothermal plate in a rotating fluid with varying mass diffusion under first order chemical reaction. The effects of thermo physical parameters on velocity, temperature and concentration were studied and the following conclusions were made. The temperature field increases for decreasing values of Prandtl number. Concentration profile decreases with increase in K and Sc. The Axial velocity drops when Sc is increased and it rises when Hall parameter m, is increased. Also, the axial velocity decreases for increasing values of Ω and M.

Nomenclature B_0 - Imposed magnetic field

m - Hall parameter

 ${\cal M}$ - Hartman number

v- Kinematic viscosity

- Ω_z -Component of angular viscosity
- Ω -Non-dimensional angular velocity
- J_z component of current density j
- Sc Schmidt number

- ρ Fluid density
- σ Electrical conductivity
- Gr Thermal GrashofâĂŹs number
- Gc Mass GrashofâĂŹs number
- Pr Thermal Prandtl number
- t' Time
- μ Coefficient of viscosity
- T Temperature of the fluid near the plate
- T_w Temperature of the plate
- $\boldsymbol{\theta}$ Dimensionless temperature
- $T\infty$ Temperature of the fluid far away from the plate
- C Dimensionless concentration
- k Thermal conductivity
- β Volumetric coefficient of thermal expansion
- $\beta*$ Volumetric coefficient of expansion with concentration
- C^\prime Species concentration in the fluid
- C_w Wall concentration
- $C\infty$ Concentration for away from the plate
- K Chemical reaction parameter
- t Non-dimensional time
- (u, v, w) Components of velocity field F
- (U, V, W) Non-dimensional velocity components
- (x, y, z) Cartesian co-ordinates

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