

TRANSIENT HYDROMAGNETIC STUDIES OF ROTATING FLUID ON AN INFINITE VERTICAL PLATE WITH FIRST ORDER CHEMICAL REACTION THERMAL RADIATION AND HALL EFFECT

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ABSTRACT. An analysis of thermal radiation on the isothermal vertical plate with Hall effect and chemical reaction in the presence of rotating fluid is considered. The coupled of non dimensional partial differential equations are solved by Laplace transform technique. The effect of hall parameter, Hartman number, Grashof number, Prandtl number, Schmidt number, chemical and radiation parameter on axial velocity and concentration of the fluid are presented through the graph. An analysis of transient hydromagnetics fluid with hall effect and radiation is most important in modern electric and electronic plants, geophysics biomechanics and astro physics.

1. INTRODUCTION

Analysis of electrically conducting fluid like ironised gas and plasma with magnetic field refers to the magneto hydro dynamics. Reynolds transport theorem and Maxwell's theorem characterised flow of fluid in magnetic field. Hall effect phenomena induces potential which is normal to electric as well as magnetic field. Pop [4] analysis electro magnetic flow in hall effect. Hydromagnetics flow with hall effect on porous at plate was studied by Gupta [2]. Ram [5] has been analysed irrotating fluid with free convection affected by hall effect

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and iron slip current. Ghosh et al. [1] investigated Hall effect in magnetic flow with presence of rotating fluid. Muthucumaraswamy and Jayanti [3] studied chemical reaction and hall effect on hydromagnetic flow an infinite vertical plate with rotating fluid. Hall effect on isothermal vertical plate with uniform mass diffusion in the presence of rotating fluid and chemical reaction of first order has been analysed by Dhananjay Kumar and Muthucumaraswamy [6].

The magnetic field is very high, where the density is less in a gas which is ionised and the conductivity is normal to the magnetic field. This is produced due to the free spiralling of suffering collision in which also current is induced in direction normal to both electric and magnetic field. This phenomenon is called hall effect. The study of hydromagnetic viscous flow with Hall current has huge application in engineers and research community and many research articles published. Pop [4] studied Hall effect and magneto hydrodynamics free convection about semi infinite vertical plate. In this paper we have analysed the effect of hall current chemical reaction of rotating fluid with thermal radiation. The set of non dimensional equations are solved by technique of Laplace transform in terms of exponential function and complement error function. The effect of Hartmann number, velocity, concentration, Hall parameter, radiation magnetic effects, chemical reaction and time on a rotating field are discussed graphically.

2. FORMULATION OF THE PROBLEM

In this paper, hydromagnetics unsteady flow of vertical plate by constant mass diffusion is presented. Place the plate in x axis and normal to strong magnetic field along z axis. The plate and the fluid rotate with constant angular velocity simultaneously, Ω in same time. Assume the fluid kept at rest around the vertical plate with T'_∞ and C'_∞ , a uniform magnetic field, applied to the plate transversely. In stationary condition, the temperature and concentration of the plate and the fluid are same. The plate is accelerated with u_0 which refer motion of amplitude at $t' > 0$ and the accelerating parameter a' in the plate. At time $t' > 0$, the plate temperature T'_w and concentration C'_w are increased thereafter maintained constant. Except the influence body force consider remaining all the fluid physical properties are constant. Then the governing equation of flow are represented under the usual Boussinesqs approximation.

Equation of momentum is

$$(2.1) \quad \frac{\partial u}{\partial t'} - 2\Omega' v = v \frac{\partial u^2}{\partial z^2} - \frac{1}{\rho} \frac{\partial P}{\partial x} + g + \frac{B_0}{\rho} j_y$$

$$\frac{\partial v}{\partial t'} + 2\Omega' u = v \frac{\partial v^2}{\partial z^2} + \frac{B_0}{\rho} j_x.$$

Equation of energy is

$$\rho C_P \frac{\partial T'}{\partial t'} = k \frac{\partial T'^2}{\partial z^2} - \frac{q_r}{\partial z}.$$

Equation of diffusion is

$$\frac{\partial C'}{\partial t'} = v \frac{\partial^2 C}{\partial z^2} - K_l(C' - C'_\infty).$$

In the equation (2.1) viscous term vanishes in the absence of magnetic field and gradient along x -direction, so we have

$$0 = -\frac{\partial \rho}{\partial x} - g\rho_\infty.$$

Eliminating the pressure term we obtain

$$\frac{\partial u}{\partial t'} - 2\Omega' v = v \frac{\partial u^2}{\partial z^2} + (\rho_\infty - \rho)g + \frac{B_0}{\rho} j_y.$$

The Boussinesq approximation gives

$$\rho_\infty - \rho = \rho_\infty \beta(T' - T'_\infty) + \rho_\infty \beta^*(C' - C'_\infty).$$

Using the above equation in the momentum equation reduce to

$$\frac{\partial u}{\partial t'} - 2\Omega' v = v \frac{\partial^2 u}{\partial z^2} + \frac{B_0}{\rho} j_y + \rho_\infty \beta(T' - T'_\infty) + \rho_\infty \beta^*(C' - C'_\infty).$$

The generalized Ohm's law with Hall currents in to account and neglecting ion-slip give

$$j + \frac{\omega_e \tau_e}{B_0} (j \times B) = \sigma [q \times B + E].$$

From the above equation

$$J_x - m j_y = \sigma v B_0$$

$$J_y - m j_x = \sigma u B_0$$

where $m = \omega_e \tau_e$ is hall parameter.

Solving above equations, for J_x and J_y we have:

$$J_x = \frac{\sigma B}{(1 + m^2)}(v - mu),$$

$$J_y = \frac{\sigma B}{(1 + m^2)}(u + mv).$$

Using these equations in the moment equation, we obtain:

$$\frac{\partial u}{\partial t'} = v \frac{\partial^2 u}{\partial z^2} + 2\Omega_z v - \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)}(mv + u) + \rho_\infty \beta(T' - T'_\infty) + \rho_\infty \beta^*(C' - C'_\infty)$$

$$\frac{\partial v}{\partial t'} = v \frac{\partial^2 u}{\partial z^2} + 2\Omega_z v - \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)}(mu - v)$$

$$(2.2) \quad \rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} - \frac{qr}{\partial z}$$

$$\frac{\partial C'}{\partial t'} = v \frac{\partial^2 C'}{\partial z^2} - K_l(C' - C'_\infty).$$

The local radiant of thin rat gray gas is

$$(2.3) \quad \frac{\partial qr}{\partial z} = -4\sigma a^*(T'_\infty{}^4 - T'^4).$$

In the flow, the temperature difference is assumed to be sufficiently small so that T'^4 is first order differential function of temperature. The expansion of T'^4 in a Taylor series about T'_∞ and ignore higher order terms here, we obtain:

$$(2.4) \quad T'^4 \cong 4T'_\infty{}^3 T' - 3T'^4.$$

Using equation (2.3) and (2.4) in (2.2) we obtain:

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} + 16\sigma a^* T'_\infty{}^3 (T'_\infty - T').$$

The initial and boundary values are

$$\begin{aligned} u = 0, T = T_\infty, C' = C'_\infty, v = 0, \text{ for all } z, t' \leq 0. \\ v = 0, T = T_w, C' = C'_\infty, v = 0, \text{ at } Z = 0, \text{ for all } t' > 0. \\ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } z \rightarrow \infty \text{ for all } t' > 0. \end{aligned}$$

The non dimension boundary conditions are:

$$U = \frac{u}{u_0}, V = \frac{v}{v_0}, t = \frac{t'u_0^2}{v}, Z = \frac{zu_0^2}{v^2}, \Omega = \Omega_z \frac{v}{u_0^2}$$

$$M^2 = \frac{\sigma\mu_e^2 H_0^2 v}{2\rho u_0^2}, G_r = \frac{vg\beta(T_w - T_\infty)}{u_0^3}, G_c = \frac{vg\beta^*(C'_w - C'_\infty)}{u_0^3}$$

$$P_r = \frac{\mu C_p}{k}, S_c = \frac{v}{D}, K = K_l \frac{v}{u^2}, R = \frac{16\sigma a^* T_\infty'^3 V^2}{k P_r u_0^2}.$$

Together with governing equation, boundary conditions and non dimension equation, we have:

$$\begin{aligned} \frac{\partial U}{\partial t} &= \frac{\partial^2 U}{\partial Z^2} + 2V(\Omega - \frac{M^2}{1+m^2}) - \frac{2m^2}{1+m^2}U + \theta G_r + CG_c \\ \frac{\partial V}{\partial t} &= \frac{\partial^2 V}{\partial Z^2} + 2U(\Omega - \frac{M^2}{1+m^2}) - \frac{2m^2}{1+m^2}V \\ \frac{\partial \theta}{\partial t} &= \frac{1}{P_r} \frac{\partial \theta^2}{\partial Z^2} - R\theta \\ (2.5) \quad \frac{\partial C}{\partial t} &= \frac{1}{S_c} \frac{\partial^2 C}{\partial Z^2} - KC, \end{aligned}$$

with the boundary conditions

$$\begin{aligned} U &= 0, V = 0, C = 0, \theta = 0 \text{ for all } Z, t \leq 0. \\ (2.6) \quad U &= 1, V = 0, \theta = 1, C = 1 \text{ at } Z = 0, \text{ for all } t > 0, \\ U &\rightarrow 0, \theta \rightarrow 0, C \rightarrow 0, V \rightarrow 0 \text{ as } z \rightarrow \infty \text{ for all } t > 0. \end{aligned}$$

Combining equations (2.5) and (2.6) we obtain:

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial Z^2} - Fa + \theta G_r + CG_c.$$

Hence $F = U + iV$ and $a = 2[(\frac{M^2}{1+m^2} + i(\Omega - \frac{M^2 m}{1+m^2}))]$.

The initial and boundary conditions are:

$$\begin{aligned} F &= 0, \theta = 0, C = 0, \text{ for all } Z, t \leq 0 \\ F &= 1, \theta = 1, C = 1 \text{ at } Z = 0, \text{ for all } t > 0 \\ F &\rightarrow 0, \theta \rightarrow 0, C \rightarrow 0, V \rightarrow 0 \text{ as } z \rightarrow \infty \text{ for all } t > 0 \end{aligned}$$

3. EXACT SOLUTION

By applying Laplace transform to the non dimensional governing equations, the results are:

$$\begin{aligned} \text{'ASXQ'1}\theta(Z, t) &= \frac{1}{2} \left\{ \exp(2\eta\sqrt{RP_r t}) \operatorname{erfc}(\eta\sqrt{P_r} + \sqrt{Rt}) + \right. \\ &\quad \left. \exp(-2\eta\sqrt{RP_r t}) \operatorname{erfc}(\eta\sqrt{P_r} - \sqrt{Rt}) \right\} \end{aligned}$$

$$\begin{aligned} C(Z, t) &= \frac{1}{2} \left\{ \exp(2\eta\sqrt{KS_c t}) \operatorname{erfc}(\eta\sqrt{S_c} + \sqrt{Kt}) + \right. \\ &\quad \left. \exp(-2\eta\sqrt{KS_c t}) \operatorname{erfc}(\eta\sqrt{S_c} - \sqrt{Kt}) \right\} \end{aligned}$$

$$\begin{aligned} F(Z, t) &= \frac{1}{2} [\exp(2\eta\sqrt{at}) \operatorname{erfc}(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at}) \operatorname{erfc}(\eta - \sqrt{at})] \\ &+ \left(\frac{G_r}{(P_r - 1)\gamma} \right) \left(\frac{\exp(\gamma t)}{2} [[\exp(-2\eta\sqrt{(a + \gamma)t}) \operatorname{erfc}(\eta - \sqrt{(a + \gamma)t})] \right. \\ &+ \exp(2\eta\sqrt{(a + \gamma)t}) \operatorname{erfc}(\eta + \sqrt{(a + \gamma)t})] \\ &- [\exp(2\eta\sqrt{(R + \gamma)P_r t}) \operatorname{erfc}(\eta\sqrt{P_r} + \sqrt{(R + \gamma)t}) \\ &+ \exp(-2\eta\sqrt{(R + \gamma)P_r t}) \operatorname{erfc}(\eta\sqrt{P_r} - \sqrt{(R + \gamma)t})] \\ &- \frac{1}{2} [[\exp(2\eta\sqrt{at}) \operatorname{erfc}(\eta + \sqrt{at}) \\ &+ \exp(-2\eta\sqrt{at}) \operatorname{erfc}(\eta - \sqrt{at})] - [\exp(2\eta\sqrt{RP_r t}) \operatorname{erfc}(\eta\sqrt{P_r} + \sqrt{Rt}) \\ &+ \exp(-2\eta\sqrt{RP_r t}) \operatorname{erfc}(\eta\sqrt{P_r} - \sqrt{Rt})]] \\ &+ \left(\frac{G_r}{(S_c - 1)\alpha} \right) \left(\frac{\exp(\alpha t)}{2} [[\exp(-2\eta\sqrt{(a + \alpha)t}) \operatorname{erfc}(\eta - \sqrt{(a + \alpha)t})] \right. \\ &+ \exp(2\eta\sqrt{(a + \alpha)t}) \operatorname{erfc}(\eta + \sqrt{(a + \alpha)t})] \\ &- [\exp(2\eta\sqrt{(K + \alpha)S_c t}) \operatorname{erfc}(\eta\sqrt{S_c} + \sqrt{(K + \alpha)t}) \\ &+ \exp(-2\eta\sqrt{(K + \alpha)S_c t}) \operatorname{erfc}(\eta\sqrt{S_c} - \sqrt{(K + \alpha)t})] \\ &- \frac{1}{2} [[\exp(2\eta\sqrt{at}) \operatorname{erfc}(\eta + \sqrt{at}) \\ &+ \exp(-2\eta\sqrt{at}) \operatorname{erfc}(\eta - \sqrt{at})] - [\exp(2\eta\sqrt{KS_c t}) \operatorname{erfc}(\eta\sqrt{S_c} + \sqrt{Kt}) \\ &+ \exp(-2\eta\sqrt{KS_c t}) \operatorname{erfc}(\eta\sqrt{S_c} - \sqrt{Kt})]] \end{aligned}$$

where $\gamma = \frac{a - P_r R}{P_r - 1}$, $\alpha = \frac{a - S_c K}{S_c - 1}$, $\eta = \frac{z}{2\sqrt{t}}$.

The numerical values of $F(Z, t)$, $C(Z, t)$, $\theta(Z, t)$ have been computed and represented graphically.

4. RESULTS AND DISCUSSION

The exact solution of the problem with effect of parameter like M , Ω , mG_r , G_c , S_c , and K on the flow and concentration profile are presented graphically at time $t = 2$ and P_r is taken as 0.7 and 7. The effect of different physical parameters such as Hartman number, rotation parameter, radiation parameter, Prandtl number, Hall parameter, thermal Grashof number, mass Grashof number, reaction parameter, Schmidt number and time of the velocity, temperature and concentration fields are presented graphically.

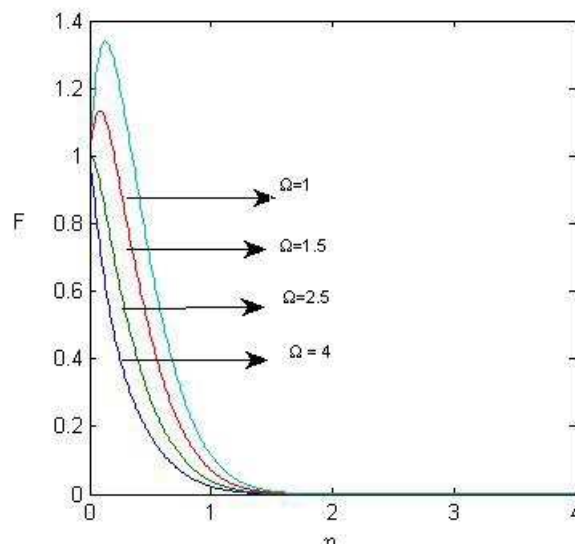
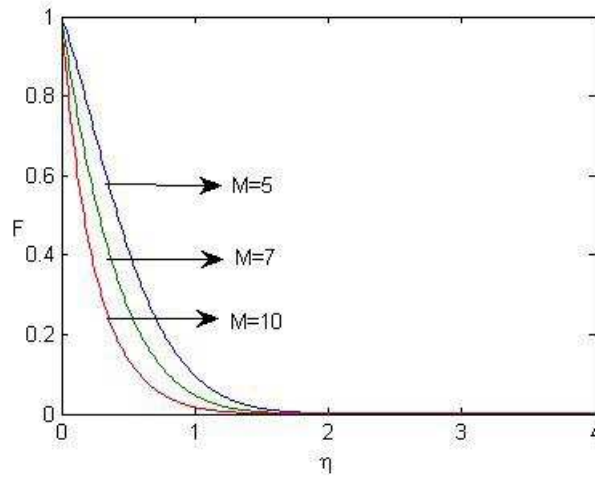


FIGURE 1. Axial velocity for various values of Ω

For various values of the rotation parameter ($\Omega = 1, 1.5, 2.5, 4$) Figure 1 shows the axial velocity profile at $S_c = 2.01$, $K = 0.2$, $m = 5$, $P_r = 7$, $Gr = 5$, $G_c = 10t = 2$, $Gr = 5$ in the presence of water. From Figure 1 we can notice that as Ω is increasing, the axial velocity F is decreasing. Also the axial velocity gets highest value at $\Omega = \frac{M^2 m}{1 + m^2}$ and the transverse velocity then becomes zero.

FIGURE 2. Axial velocity for different values of m

From Figure 2 we can see that the axial velocity decreases when the Hartmann number M increases. In the presence of water we can see the effect when $M = 5, 7$, and 10 with rotation parameter $\Omega = 10$, $Sc = 2.01$, $K = 0.2$, $Pr = 7$, $m = 2$, $Gc = 10$, $t = 2$, $Gr = 5$ on axial velocity (it is represented in Figure 2).

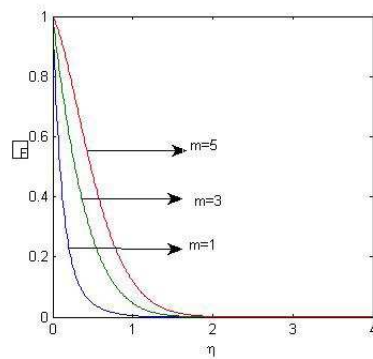
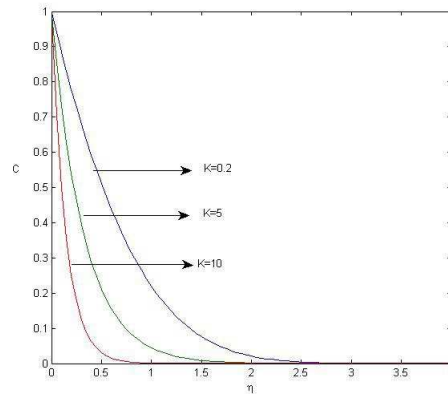
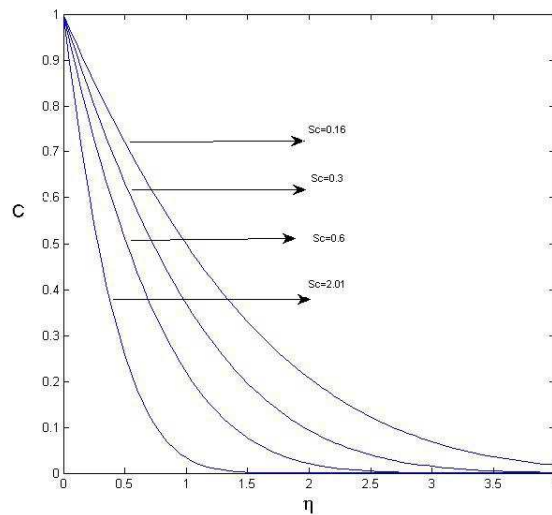
FIGURE 3. Axial velocity for different values of m

Figure 3 shows for various values of Hall parameter, ($m = 5, 3, 1$), effect on Axial velocity profile in the presence of water at $\Omega = 10$, $Sc = 2.01$, $Pr = 7$, $Gc = 10$, $M = 10$, $Gr = 5$, $K = 0.2$, $t = 2$. Note that when Hall parameter is increasing, the the axial velocity is also increasing.

FIGURE 4. Concentration profile for different values of K

For different values of chemical reaction parameter $K = 0.2, 2$ and 5 the concentration profile with $Sc = 0.6$ and $t = 2$ is shown in Figure 4. The concentration field decreases with an increase of K .

Figure 5 shows that with the decreasing value of $Sc = 2.01, 0.6, 0.3$ and 0.16 , concentration profile is increasing at $K = 0.2$ and $t = 2$. The temperature profiler is shown in Figure 6 for different value at Prandtl number at $K = 0.2$ and time $t = 2$.

FIGURE 5. Concentration profile for distinct value of Sc

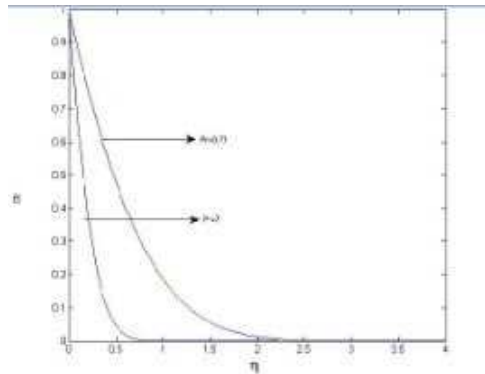


FIGURE 6. Temperature profile for different value of Pr

5. CONCLUSION

An analytic solution of hydro magnetic flow with hall effect and radiation with various parameters in the presence of rotating fluid has been presented graphically. The graphical result concluded that:

- (i) Axial velocity decreases with increasing of the rotational parameter and Hartman number. It increases with increase of Hall parameter.
- (ii) Concentration of the fluid is increasing by decreasing of Schmidt number and it is decreasing with increase of chemical reaction.

Nomenclature

- C - dimensionless concentration
- T - fluid temperature near the plate
- B_0 - magnetic field
- C' - concentration of the fluid
- C_w - concentration near the plate
- G_r - Mass Grashofs number
- G_c - Thermal Grashofs number
- J_z - current density along z axis
- R - Radiation parameter
- K - Chemical reaction parameter
- k - Thermal conductivity
- M - Hartman number
- m - Hall parameter

- P_r - Thermal Prantal number
 ω_z - angular viscosity along z axis
 S_c - Schmidt number
 C_1 - concentration far away from the plate
 T_w - plate temperature near wall
 T_1 - Fluid temperature far away from the plate
 t - dimensionless time
 t' - time
 (x, y, z) - Cartesian co-ordinates
 (u, v, w) - components of velocity field F
 β - volumetric coefficient of thermal expansion
 β^* - volumetric coefficient of expansion with concentration
 θ - dimensionless temperature
 μ - coefficient of viscosity
 ν - viscosity of kinematic
 ρ - density of fluid
 σ - conductivity of electrical charge
 ω - dimensionless angular velocity
 (U, V, W) - dimensionless velocity
 S_c - Schmidt number

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