ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.5, 2791–2800
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.5.40

FIXED POINTS THEOREMS OF (κ, μ) RATIONAL CONTRACTIVE MAPPINGS IN ORDERED COMPLEX VALUED QUASI METRIC SPACES

J. UMA MAHESWARI[1] AND A. ANBARASAN

ABSTRACT. In this article, we proved so many fixed point results with help of new notion (κ, μ) rational contractive mappings in ordered complex valued quasi metric spaces and show that the example exist as well as application on fixed point theorems.

1. INTRODUCTION

The Banach contraction principle is a basic tool for developing the fixed point results. Many authors contributed for proving fixed point results [1–5]. Doitchinov in [8], Adam et al. in [4], Dung in [10] have introduced fixed point theorems existence of complex valued quasi metric spaces. The concept of almost contraction initiated by Berinide. So many authors generalized that contraction, [6, 7].

Before entering into our main results we shall recall some basic definition and results which are needful.

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 54E35.
Key words and phrases. partially ordered metric spaces, complex valued quasi metric spaces, fixed point.

2. PRELIMINARIES

We recollect some basic definitions and notions which is useful for proving our main results.

Let $\mathbb{C}$ be the set of complex numbers and $p_1, p_2 \in \mathbb{C}$. Define a partial order $\leq$ on $\mathbb{C}$ as follows:

$p_1 \leq p_2$ if and only if $Re(p_1) \leq Re(p_2)$,$Im(p_1) \leq Im(p_2)$.

Consequently, one can infer that $p_1 \leq p_2$ if one of the following conditions is satisfied:

(i) $Re(p_1) = Re(p_2)$, $Im(p_1) < Im(p_2)$,
(ii) $Re(p_1) < Re(p_2)$, $Im(p_1) = Im(p_2)$,
(iii) $Re(p_1) < Re(p_2)$, $Im(p_1) < Im(p_2)$,
(iv) $Re(p_1) = Re(p_2)$, $Im(p_1) = Im(p_2)$.

In particular, we write $p_1 \lneqq p_2$ if $p_1 \neq p_2$ and one of $(i), (ii)$ and (iii) are satisfied and we write $p_1 < p_2$ if only (iii) is satisfied. Notice that

(a) If $0 \leq p_1 \lneqq p_2$, then $|p_1| < |p_2|$,
(b) If $p_1 \leq p_2$ and $p_2 < p_3$ then $p_1 < p_3$,
(c) If $a, b \in R$ and $a \leq b$ then $ap_1 \leq bp_1$ for all $p \in \mathbb{C}$.

Definition 2.1. *A complex quasi metric on a non-empty set X is a function $\psi_{cp} : X \times X \to \mathbb{C}$ such that for all $x, y, z \in X$:*

(1) $\psi_{cp}(x, y) = 0$ *if and only if* $x = y$*,*
(2) $\psi_{cp}(x, y) \leq \psi_{cp}(x, z) + \psi_{cp}(z, y)$.

Definition 2.2. *Let (X, ψ_{cp}) be a complex quasi metric space*

(1) *Let $\{x_n\}$ be a cauchy sequence if for every $0 \prec c \in \mathbb{C}$ find a integer N such that $\psi_{cp}(x_n, x_m) \prec c$ for every $m, n \succcurlyeq N$.*
(2) *Let $\{x_n\}$ converges to an element $x \in X$ if for every $0 \prec c \in \mathbb{C}$ find a integer N such that $\psi_{cp}(x_n, x) \prec c$ for all $n \succcurlyeq N$.*
(3) *Suppose that (X, ψ_{cp}) is complete if for every cauchy sequence in X converges to a point in X.*

Definition 2.3. *The function $\mu : [0, \infty) \to [0, \infty)$ is said to be an altering distance function if the following conditions are satisfied:*

(i) *μ is continuous and increasing;*
(ii) *$\mu(a) = 0$ iff $a = 0$.*

So many authors discussed alerting distance function. Khan et al. in [9] introduced the concept of altering distance function. Here we introduced new notion (κ,μ) rational contractive mappings in ordered complex valued quasi metric spaces where κ and μ are the altering distance function.

3. Main Results

In this section, we prove our main results.

Definition 3.1. *Let $(X,\leq,\psi_{cp})$be an ordered Quasi metric space. Let κ and μ be altering distance functions. Then the mapping $g : X \to X$ is an (κ,μ) rational contraction mapping if there exists $M \geq 0$ such that:*

$$\kappa(\psi_{cp}(gx,gy)) \leq \kappa(R(x,y)) - \mu(R(x,y)) + M\kappa(S(x,y)) \tag{3.1}$$

where $R(x,y) = max\{\psi_{cp}(x,y), \dfrac{\psi_{cp}(x,gx)\psi_{cp}(y,gx)}{1+(\psi_{cp}(x,y))^2}, \dfrac{\psi_{cp}(x,gy)\psi_{cp}(y,gy)}{1+\psi_{cp}(x,y)+\psi_{cp}(y,gy)}\}$
and $S(x,y) = min\{\dfrac{\psi_{cp}(x,gx)\psi_{cp}(y,gx)}{1+\psi_{cp}(x,y)}, \dfrac{\psi_{cp}(x,gy)\psi_{cp}(y,gx)}{1+\psi_{cp}(x,y)}\}$
for all comparable $x,y \in X$.

Theorem 3.1. *Let $(X,\leq,\psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing continuous mapping with respect to $\leq$. Suppose that g is an (κ,μ)- rational contractive mapping for all comparable $x,y \in X$ then g has a fixed point.*

Proof. It should be shown that g has a fixed point. Let us consider $x_0$be a point in X. We define a sequence $\{x_l\}$ in X such that $x_{l+1} = gx_l$.
Since g is a increasing sequence, $x_0 \leq gx_0 = x_1 = gx_0 \leq x_2 = gx_1$.
Again $x_1 \leq x_2$ and g is a increasing therefore by induction we show that:
$x_0 \leq x_1 \leq ..... \leq x_l \leq x_{l+1} \leq ....$ Consider $x_l \neq x_{l+1}$ for every $l \in N$. So from the equation (3.1) we have:

$$\begin{aligned} \kappa(\psi_{cp}(x_l,x_{l+1})) &= \kappa(\psi_{cp}(gx_{l-1},gx_l)) \leq \\ &\leq \kappa(R(x_{l-1},x_l)) - \mu(R(x_{l-1},x_l)) + M\kappa(S(x_{l-1},x_l)), \end{aligned} \tag{3.2}$$

where

$$R(x_{l-1},x_l) = max\{\psi_{cp}(x_{l-1},x_l), \frac{\psi_{cp}(x_{l-1},gx_{l-1})\psi_{cp}(x_l,gx_{l-1})}{1+(\psi_{cp}(x_{l-1},x_l))^2}, \frac{\psi_{cp}(x_{l-1},gx_l)\psi_{cp}(x_l,gx_l)}{1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,gx_l)}\} \leq$$
$$\leq max\{\psi_{cp}(x_{l-1},x_l), \frac{\psi_{cp}(x_{l-1},x_l)\psi_{cp}(x_l,x_l)}{1+(\psi_{cp}(x_{l-1},x_l))^2}, \frac{\psi_{cp}(x_{l-1},x_{l+1})\psi_{cp}(x_l,x_{l+1})}{1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})}\} \leq$$
$$\leq max\{\psi_{cp}(x_{l-1},x_l), \frac{\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})\psi_{cp}(x_l,x_{l+1})}{1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})}\}.$$

Therefore,

$$R(x_{l-1},x_l) \leq max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}\,. \tag{3.3}$$

Since $|1+\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})| > |\psi_{cp}(x_{l-1},x_l)+\psi_{cp}(x_l,x_{l+1})|\,.$

Now, let us take,

$$S(x_{l-1},x_l) = min\{\frac{\psi_{cp}(x_{l-1},gx_l)\psi_{cp}(x_l,gx_{l-1})}{1+\psi_{cp}(x_{l-1},x_l)}, \frac{\psi_{cp}(x_{l-1},gx_l)\psi_{cp}(x_l,gx_{l-1})}{1+\psi_{cp}(x_{l-1},x_l)}\}$$

$$\leq min\{\frac{\psi_{cp}(x_{l-1},x_{l+1})\psi_{cp}(x_l,x_l)}{1+\psi_{cp}(x_{l-1},x_l)}, \frac{\psi_{cp}(x_{l-1},x_{l+1})\psi_{cp}(x_l,x_l)}{1+\psi_{cp}(x_{l-1},x_l)}\} = 0\,. \tag{3.4}$$

From (3.2), (3.3), (3.4) and let κ and μ we obtain,

$$\kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) -$$
$$-\mu(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) \leq$$
$$\leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\})$$

$$\kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) \tag{3.5}$$

Suppose $max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\} = \psi_{cp}(x_l,x_{l+1})\,.$

Then (3.5) becomes,

$\kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\}) < \kappa(\psi_{cp}(x_l,x_{l+1}))$
which is the contradiction.
Therefore, $max\{\psi_{cp}(x_{l-1},x_l), \psi_{cp}(x_l,x_{l+1})\} = \psi_{cp}(x_{l-1},x_l)$. Now,

$$\kappa(\psi_{cp}(x_l,x_{l+1})) \leq \kappa(\psi_{cp}(x_{l-1},x_l)) - \mu(\psi_{cp}(x_{l-1},x_l)) < \kappa(\psi_{cp}(x_{l-1},x_l))\,. \tag{3.6}$$

Since κ is a increasing mapping, therefore$\{\psi_{cp}(x_l,x_{l+1}) : l \in N \cup \{0\}\}$is an increasing sequence of positive numbers, there exists $n \geq 0$ such that $\lim_{l\to\infty} \psi_{cp}(x_l,x_{l+1}) = n$. Let $l \to \infty$ in (3.6), we get $\kappa(n) \leq \kappa(n) - \mu(n) \leq \kappa(n)$.

Therefore, $\mu(n) = 0$. thus $n = 0$.
Hence we have

$$\lim_{l\to\infty} \psi_{cp}(x_l, x_{l+1}) = 0. \tag{3.7}$$

To show that $\{x_l\}$ is a Cauchy sequence in X, let suppose, $\{x_l\}$ is not a Cauchy sequence. Then there exists $\rho > 0$ and two subsequences $\{x_{k(i)}\}$ and $\{x_{l(i)}\}$ such that: $\psi_{cp}(x_{k(i)}, x_{l(i)}) \geq \rho, \quad l(i) > k(i) > i$. This shows that $\psi_{cp}(x_{k(i)}, x_{l(i)-1}) < \rho$.
Therefore we get,

$$\begin{aligned}
\rho &\leq \psi_{cp}(x_{k(i)}, x_{l(i)}) \\
&\leq \psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \psi_{cp}(x_{k(i)-1}, x_{l(i)}) \\
&\leq \psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \psi_{cp}(x_{k(i)-1}, x_{l(i)-1}) + \psi_{cp}(x_{l(i)-1}, x_{l(i)}) \\
&\leq 2\psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \psi_{cp}(x_{k(i)}, x_{l(i)-1}) + \psi_{cp}(x_{l(i)-1}, x_{l(i)}) \\
&< 2\psi_{cp}(x_{k(i)}, x_{k(i)-1}) + \rho + \psi_{cp}(x_{l(i)-1}, x_{l(i)}) \,.
\end{aligned}$$

Let $i \to \infty$ in the equation (3.7) and we obtain:

$$\begin{aligned}
\lim_{l\to\infty} \psi_{cp}(x_{k(i)}, x_{l(i)}) &= \lim_{l\to\infty} \psi_{cp}(x_{k(i)-1}, x_{l(i)}) \\
&= \psi_{cp}(x_{k(i)}, x_{l(i)-1}) \\
&= \psi_{cp}(x_{k(i)-1}, x_{l(i)-1}) \\
&= \rho\,.
\end{aligned}$$

From (κ, μ) rational contraction mapping we have,

$$\begin{aligned}
\kappa(\psi_{cp}(x_{k(i)}, x_{l(i)})) &= \kappa(\psi_{cp}(gx_{k(i)-1}, gx_{l(i)} - 1)) \\
&\leq \kappa(R(x_{k(i)-1}, x_{l(i)-1})) - \mu(R(x_{k(i)-1}, x_{l(i)-1})) \\
&\quad + M\kappa(S(x_{k(i)-1}, x_{l(i)-1}))\,,
\end{aligned}$$

where

$$\begin{aligned}
R(x_{k(i)-1}, x_{l(i)-1}) = max\{(\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}),\ &\tfrac{\psi_{cp}(x_{k(i)-1}, gx_{k(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{k(i)-1})}{1+(\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}))^2}, \\
&\tfrac{\psi_{cp}(x_{k(i)-1}, gx_{l(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{l(i)-1})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})+\psi_{cp}(x_{l(i)-1}, gx_{l(i)-1})}\}
\end{aligned}$$

$$\begin{aligned}
= max\{(\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}), &\frac{\psi_{cp}(x_{k(i)-1}, x_{k(i)})\psi_{cp}(x_{l(i)-1}, x_{k(i)})}{1 + (\psi_{cp}(x_{k(i)-1}, x_{l(i)-1}))^2}, \\
&\frac{\psi_{cp}(x_{k(i)-1}, x_{l(i)})\psi_{cp}(x_{l(i)-1}, x_{l(i)})}{1 + \psi_{cp}(x_{k(i)-1}, x_{l(i)-1}) + \psi_{cp}(x_{l(i)-1}, x_{l(i)})}\}
\end{aligned} \tag{3.8}$$

$$S(x_{k(i)-1}, x_{l(i)-1}) = min\{\frac{\psi_{cp}(x_{k(i)-1}, gx_{k(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{k(i)-1})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})}, \frac{\psi_{cp}(x_{k(i)-1}, gx_{l(i)-1})\psi_{cp}(x_{l(i)-1}, gx_{k(i)-1})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})}\}$$

$$= min\{\frac{\psi_{cp}(x_{k(i)-1}, x_{k(i)})\psi_{cp}(x_{l(i)-1}, x_{k(i)})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})}, \frac{\psi_{cp}(x_{k(i)-1}, x_{l(i)})\psi_{cp}(x_{l(i)-1}, x_{k(i)})}{1+\psi_{cp}(x_{k(i)-1}, x_{l(i)-1})}\}. \tag{3.9}$$

let $i \to \infty$ in (3.9). Therefore

$$\lim_{i\to\infty} R(x_{k(i)-1}, x_{l(i)-1}) = \rho$$
$$\lim_{i\to\infty} S(x_{k(i)-1}, x_{l(i)-1}) = \rho.$$

Letting $i \to \infty$ in (3.8) then it becomes: $\kappa(\rho) \leq \kappa(\rho) - \mu(\rho) < \kappa(\rho)$, which is a contradiction. Hence $(x_{l+1} = gx_l)$ is a Cauchy sequence in X. Since X is a complete space find that $v \in X$ such that $\lim_{l\to\infty} x_{l+1} = \lim_{l\to\infty} gx_l = v.$
Let $gx_l \to gv$ since g is a continuous.
Therefore by limit uniqueness we find $fv = v$.
Hence, v is a fixed point of g. □

Without assuming the continuous the theorem 3.1 we have the following fixed point.

Theorem 3.2. *Let $(X, \leq, \psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing mapping with respect to $\leq$. Suppose that g is an (κ, μ)- rational contractive mapping for all comparable $x, y \in X$ then g has a fixed point.*

Proof. The same argument followed from the theorem 3.1, we construct an non-decreasing sequence $\{x_l\}$ in X such that $x_l \to v$ for some $v \in X$. It is enough to show that g has a fixed point. By (κ, μ) rational contraction mapping we have,

$$\kappa(\psi_{cp}(x_{l+1}, gv)) = \kappa(\psi_{cp}(gx_l, gv)) \leq \kappa(R(x_l, v)) - \mu(R(x_l, v)) + M\kappa(S(x_l, v)) \tag{3.10}$$

where

$$R(x_l, v) = max\{\psi_{cp}(x_l, v), \frac{\psi_{cp}(x_l,gx_l)\psi_{cp}(v,gx_l)}{1+(\psi_{cp}(x_l,v))^2}, \frac{\psi_{cp}(x_l,gv)\psi_{cp}(v,gv)}{1+\psi_{cp}(x_l,v)+\psi_{cp}(v,gv)}\}$$
$$= max\{\psi_{cp}(x_l, v), \frac{\psi_{cp}(x_l,x_{l+1})\psi_{cp}(v,x_{l+1})}{1+(\psi_{cp}(x_l,v))^2}, \frac{\psi_{cp}(x_l,gv)\psi_{cp}(v,gv)}{1+\psi_{cp}(x_l,v)+\psi_{cp}(v,gv)}\}$$

$$S(x_l, v) = min\{\frac{\psi_{cp}(x_l, gx_l)\psi_{cp}(v, gx_l)}{1+\psi_{cp}(x_l, v)}, \frac{\psi_{cp}(x_l, gv)\psi_{cp}(v, gx_l)}{1+\psi_{cp}(x_l, v)}\}$$
$$= min\{\frac{\psi_{cp}(x_l, x_{l+1})\psi_{cp}(v, x_{l+1})}{1+\psi_{cp}(x_l, v)}, \frac{\psi_{cp}(x_l, gv)\psi_{cp}(v, x_{l+1})}{1+\psi_{cp}(x_l, v)}\} \tag{3.11}$$

As $l \to \infty$ in (3.10) we obtain $R(x_l, v) \to \psi_{cp}(v, gv)$ and $S(x_l, v) \to 0$. When $l \to \infty$ in (3.11) we obtain $\kappa(\psi_{cp}(v, gv)) \leq \kappa(\psi_{cp}(v, gv)) - \mu(\psi_{cp}(v, gv))$ so, $(\psi_{cp}(v, gv)) = 0$. Therefore $v = gv$. Thus v is a fixed point of g. □

Corollary 3.1. *Let $(X, \leq, \psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing continuous mapping with respect to $\leq$. Suppose that $b \in [0, 1)$ and $M \geq 0$ such that*

$$\psi(gx, gy) \leq bmax\{\psi_{cp}(x, y), \frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1+(\psi_{cp}(x, y))^2}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gy)}{1+\psi_{cp}(x, y)+\psi_{cp}(y, gy)}\}$$
$$+Mmin\{\frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1+\psi_{cp}(x, y)}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gx)}{1+\psi_{cp}(x, y)}\}$$

for all comparable $x, y \in X$ then g has a fixed point.

Proof. From the theorem 3.1 let us consider $\kappa(a) = a$ and $\mu(a) = (1 - b)a$ for every $a \in [0, \infty]$. Hence it shows that g has a fixed point. □

Without assuming continuity of g in the corollary 3.1.

Corollary 3.2. *Let $(X, \leq, \psi_{cp})$ be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let $g : X \to X$ be a increasing mapping with respect to $\leq$. Suppose that $b \in [0, 1)$ and $M \geq 0$ such that*

$$\psi(gx, gy) \leq bmax\{\psi_{cp}(x, y), \frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1+(\psi_{cp}(x, y))^2}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gy)}{1+\psi_{cp}(x, y)+\psi_{cp}(y, gy)}\}$$
$$+Mmin\{\frac{\psi_{cp}(x, gx)\psi_{cp}(y, gx)}{1+\psi_{cp}(x, y)}, \frac{\psi_{cp}(x, gy)\psi_{cp}(y, gx)}{1+\psi_{cp}(x, y)}\}$$

for all comparable $x, y \in X$ then g has a fixed point.

Proof. It follows from the theorem 3.2. Let us consider $\kappa(a) = a$ and $\mu(a) = (1-b)a$ for every $a \in [0, \infty]$. Hence it shows that g has a fixed point. □

Example 1. *Consider* $X = \{0, 1, 2, 3.....\}$ *Define the mapping* $g : X \to X$ *defined by:*

$$gx = \begin{cases} 0, & x = 0. \\ x - 3, & x \neq 0. \end{cases}$$

$$gy = \begin{cases} 0, & x \in \{0, 1, 2\}. \\ x - 5, & x \geq 3. \end{cases}$$

Define $\psi_{cp} : X \times X \to C$ *such that*

$$\psi_{cp} = \begin{cases} 0, & x = y. \\ x + 2y, & x \neq y. \end{cases}$$

Then (κ, μ) *rational contraction mapping has a fixed point.*

4. Applications

Let ζ be the set of mapping $\mu : [0, \infty) \to [0, \infty)$ satisfying the hypotheses

(i) Every $\mu \in \zeta$ is a Lebesgue integrable on each compact subset of $[0, \infty)$

(ii) For all $\mu \in \zeta$ and $\rho > 0$

$$\int_0^\rho \mu(e)de > 0.$$

Let the function $\kappa : [0, \infty) \to [0, \infty)$ be defined by

$$\kappa(w) = \int_0^w \mu(e)de > 0,$$

is an altering distance function. It is obvious to check the function. Now the results follows

Corollary 4.1. *Let* $(X, \leq, \psi_{cp})$ *be a partially ordered complex quasi metric spaces such that the quasi metric is complete. Let* $g : X \to X$ *be a increasing continuous* *mapping with respect to $\leq$. Suppose that $b \in [0,1)$ and $M \geq 0$ such that*

$$\int_0^{\psi_{cp}(gx,gy)} \mu(e)de \leq b\int_0^{max\{\psi_{cp}(x,y), \frac{\psi_{cp}(x,gx)\psi_{cp}(y,gx)}{1+(\psi_{cp}(x,y))^2}, \frac{\psi_{cp}(x,gy)\psi_{cp}(y,gy)}{1+\psi_{cp}(x,y)+\psi_{cp}(y,gy)}\}} \mu(e)de$$
$$+M\int_0^{min\{\frac{\psi_{cp}(x,gx)\psi_{cp}(y,gx)}{1+\psi_{cp}(x,y)}, \frac{\psi_{cp}(x,gy)\psi_{cp}(y,gx)}{1+\psi_{cp}(x,y)}\}} \mu(e)de$$

for all comparable $x, y \in X$ then g has a fixed point.

Proof. It follows from the corollary 3.1 by taking

$$\kappa(w) = \int_0^w \mu(e)de.$$

□

REFERENCES

[1] F. ROUZKARD, M. IMDAD: *Some common fixed point theorems on complex valued metric spaces*, Computers and Mathematics with Applications, **64**(6) (2012), 1866–1874.

[2] V. W. BRYANT: *A Remark on a fixed point theorem for iterated mappings*, Amer. Math. Monthly, **75** (1968), 399–400.

[3] W. SINTUNAVARAT, P. KUMAM: *Generalized common fixed point theorems in complex valued metric spaces and applications*, Journal of Inequalities and Applications, 84 (2012).

[4] A. AZAM, B. FISHER, M.KHAN: *Common fixed point theorems in complex valued metric spaces*, Num. Func. Anal. Opt., **32** (2011), 243–253.

[5] M. IMDAD, T. I. KHAN: *On Common fixed point of pairwise coincidently commuting non-continuous mappings satisfying a rational inequality*, Bull. Calcutta Math. Soc., **93**(4) (2001), 263–268.

[6] M. ABBAS, V. C. RAJIC , T. NAZIR, S. RADENOVIC: *Common fixed point of mappings satisfying rational inequalities in ordered complex valued generalized metric spaces*, Afrika Matematika, **26** (2015), 17–30.

[7] P. DHIVYA, M. MARUDAI: *Common fixed point theorems for mappings satisfying a contractive condition of rational expression on a ordered complex partial metric spaces*, Cogent Mathematics, **4**(1) (2017), 1–11.

[8] D. DOITCHINOV: *On completeness in Quasi metric spaces*, Topology and its Applications **30** (1988), 127–148.

[9] M. S. KHAN, M. SWALEH, S. SESSA: *Fixed point theorems by altering distances between the points*, Bull Aust math. Soc., **30** (1984), 1–9.

[10] N. V. DUNG: *Remarks on Quasi metric spaces*, Miskolc Mathematical Notes, **15** (2) (2014), 401–422.

DEPARTMENT OF MATHEMATICS
ST.JOSEPH'S COLLEGE(AUTONOMOUS)
TIRUCHIRAPPALLI-620 002, INDIA ADDRESS
E-mail address: umasjc@gmail.com

DEPARTMENT OF MATHEMATICS
ST.CHARLES COLLEGE OF ARTS AND SCIENCE
ERAIYUR, ULUNDURPET-607 201, INDIA
E-mail address: anbumaths23@gmail.com