

COMPARISON OF WEIBULL AND BAYESIAN RELIABILITY MODELS FOR SOFTWARE RELIABILITY DATA

S. PARTHASARATHY¹ AND V. MADHU

ABSTRACT. In reliability analysis Weibull distribution plays a vital role, because it is terribly versatile model to check than the other distribution fit. Also Bayesian approach in reliability analysis is terribly necessary. When we tend to use the Bayesian model for any reliability knowledge, we will get correct parameter value. In this article we present the estimation parameter of Weibull model using Bayesian Paradigm and compare the results using classical Weibull model.

1. INTRODUCTION

Reliability testing is about exercising an application so that failures are discovered and removed before the system is deployed. The purpose of reliability testing is to determine product reliability, and to determine whether the software meets the customer's reliability requirements [8].

Nowadays industries are growing faster and faster. They are having competition in today's world. If they want to keep their place, they should manufacture quality products and goods. For that they need to go for reliability and quality control [10]. In every industry they have to keep up their own reliability and quality control unit. So reliability analysis is very important when we wish to study about a material or machine [7]. Suppose if we want to calculate reliability of a particular product, we need to get time to failure data for that particular

¹corresponding author

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product, like machine or material. Statistics plays an important role in the reliability analysis. In statistics many type of analysis approach are there, to model the reliability. Bayesian approach is one of the most important method in reliability [9]. Bayesian method has been used in reliability and research area in statistics. Generally in Bayesian approach, we need to estimate prior distribution based on some time to failure data. In reliability analysis probability plays an important role.

Using some statistical model and time to failure data, we can predict the parameter value. Weibull distribution is one of the main distribution in reliability analysis, because we can calculate both scale and shape parameter for this distribution [13]. In this article we are going to compare Weibull distribution for classical model and Weibull distribution through Bayesian point of view.

2. CLASSICAL WEIBULL MODEL

The Weibull distribution is particularly useful in reliability work since it is a general distribution by which adjustment of the distribution parameters can be made to model a wide range of life distribution characteristics of different classes of engineered items. The parameters like shape, scale, location are denoted as β, η [1].

Generally we won't use location parameter for Weibull distribution. The value for this parameter can be set by zero (failure assumed to start at $t = 0$). In this case, the probability density function equation for three parameter Weibull distribution reduced to two parameter Weibull distribution.

$$f(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta-1} e^{-\left(\frac{t}{\eta} \right)^{\beta}}.$$

The probability density function is:

$$f(x) = \left(\frac{b}{a} \right) \left(\frac{x}{a} \right)^{b-1} e^{-\left(\frac{x}{a} \right)^b}.$$

The Cumulative distribution:

$$\begin{aligned}
 F(t) &= \int_0^t \left(\frac{b}{a}\right) \left(\frac{1}{a}\right)^{b-1} (x)^{b-1} e^{-\left(\frac{x}{a}\right)^b} dx \\
 &= \left(\frac{b}{a}\right) \left(\frac{1}{a}\right)^{b-1} \left[\frac{a^b e^{\frac{e^b \ln(x)}{a^b}}}{b} \right]^{0 \text{ to } t} \\
 &= \left(\frac{b}{a}\right) \left(\frac{1}{a}\right)^{b-1} \left[\frac{a^b (e^{\frac{t^b}{a^b}} - 1)}{b} \right] = e^{\frac{t^b}{a^b}} - 1.
 \end{aligned}$$

The Reliability function is:

$$R(t) = 1 - F(t) = -e^{\frac{t^b}{a^b}}.$$

Mean time to failure is:

$$MTTF = \int_0^\infty R(t) dt.$$

The Maximum Likelihood estimation is:

$$L = \frac{a^n}{b^n} \sum_{i=1}^n \frac{t_i^{a-1}}{b} e^{-\sum_{i=1}^n \left(\frac{t_i}{b}\right)^a}.$$

$$\begin{aligned}
 l = \log L &= \log \left(\frac{a^n}{b^n}\right) + (a-1) \log \sum_{i=1}^n \frac{t_i}{b} - \sum_{i=1}^n \left(\frac{t_i}{b}\right)^a \\
 \frac{\partial l}{\partial a} &= \frac{n \cdot \ln \frac{a}{b}}{a} + \sum_{i=1}^n \ln \left(\frac{t_i}{b}\right) - \sum_{i=1}^n \ln \left(\frac{t_i}{b}\right) \left(\frac{t_i}{b}\right) \\
 \frac{\partial l}{\partial b} &= \frac{-n \cdot \ln \frac{a}{b}}{b} - \frac{(a-1) \sum_{i=1}^n a \ln \frac{a}{b} \left(\frac{t_i}{b}\right)}{b} - \sum_{i=1}^n \frac{a \left(\frac{t_i}{b}\right)^a}{b}.
 \end{aligned}$$

3. BAYESIAN WEIBULL RELIABILITY MODEL

Weibull model is commonly used in reliability analysis. The density function for Weibull distributed is as follows

$$p(t_i | \alpha, \lambda_i) = \alpha t_i^{\alpha-1} \exp(\lambda_i - \exp(\lambda_i) t_i^\alpha).$$

This can be re-written as

$$p(t_i | \alpha, \gamma_i) = \exp\left(-\left(\frac{t_i}{\gamma_i}\right)^\alpha\right) \frac{\alpha}{\gamma_i} \left(\frac{t_i}{\gamma_i}\right)^{\alpha-1}.$$

The relationship between λ and γ in these two parameterizations is as follows:

$$\lambda_i = -\alpha \log \gamma_i.$$

The corresponding reliability function, using the λ_i formulation, is as follows:

$$S(t_i | \alpha, \lambda_i) = \exp(-\exp(\lambda_i) t_i^\alpha).$$

Suppose that we have sample t_i of n independent failure times, with parameters α , and λ_i , then the likelihood function in terms of α and λ is as follows:

$$\begin{aligned} L(\alpha, \lambda | t) &= \prod_{i=1}^n p(t_i | \alpha, \lambda_i)^{v_i} S(t_i | \alpha, \lambda_i)^{1-v_i} \\ &= \prod_{i=1}^n (\alpha t_i^{\alpha-1} \exp(\lambda_i - \exp(\lambda_i) t_i^\alpha))^{V_i} (\exp(-\exp(\lambda_i) t_i^\alpha))^{1-V_i} \\ &= \prod_{i=1}^n (\alpha t_i^{\alpha-1} \exp(\lambda_i))^{V_i} (\exp(-\exp(\lambda_i) t_i^\alpha)). \end{aligned}$$

If we link the covariances to λ with $\lambda_{i=x_i', \beta}$ where x_i is the vector of covariates corresponding to the i^{th} observation and β is a vector of regression coefficients, the log-likelihood function becomes this:

$$\begin{aligned} l(\alpha, \beta | t, x) &= \\ &\sum_{i=1}^n V_i (\log(\alpha) + (\alpha - 1) \log(t_i) + (\alpha - 1) \log(t_i) + x_i' \beta) - \exp(x_i' \beta) t_i^\alpha. \end{aligned}$$

As with the exponential model, in the absence of prior information about the parameters in this model, we can use diffuse normal priors on β [5]. We might wish to choose a diffuse gamma distribution for α . Note that when $\alpha = 1$, the Weibull reliability likelihood reduces to the exponential reliability likelihood. Equivalently, by looking at the posterior distribution of α , we can conclude whether fitting an exponential reliability model would be more appropriate than the Weibull model [2, 4]. PROC MCMC also permits to build logical thinking on any functions of the parameters. Quantities of interest in reliability analysis include the worth of the reliability perform at specific times for specific operation and also the relationship between the reliability curves for various operations. With PROC MCMC, we will calculate a sample from the posterior distribution of the interested reliability functions at any variety of points.

4. BAYESIAN ANALYSIS

The LIFEREG procedure fits parametric models to failure-time data that can be uncensored, right-censored, left-censored, or interval-censored [3, 6]. The models for the response variable consist of a linear effect (which is composed of the covariates) and a random disturbance term. The distribution of the random disturbance can be taken from a class of distributions that includes the extreme value, normal, logistic, and, by using a log transformation, the exponential, Weibull, lognormal, log-logistic, and three-parameter gamma distributions. PROC LIFEREG uses either frequentist or Bayesian methods to fit models [12].

To perform Bayesian analyses with PROC LIFEREG, you specify a model essentially the same way you do for a frequentist approach, but you add a Bayes statement to request Bayesian estimation methods for fitting the model [11]. The Bayes statement requests that the parameters of the model be estimated by Markov chain Monte Carlo sampling techniques and provides options that enable you to specify prior information, control the sampling, and obtain posterior summary statistics and convergence diagnostics. You can also save the posterior samples to a SAS data set for further analysis.

We have analyzed software reliability failure data by Meeker 1987. There are 183 processor failure data collected from distributed system which is connected by a local network with all the five processors [9]. We have calculated failure count and mean time between failures for each processor.

Failure Time In Months	Recovery Time In Months
11.42	11.43
11.46	11.47
11.48	11.48
11.54	11.54
11.55	11.56
11.56	11.57
-	-
-	-

TABLE 1. Data format description

Model Information

Parameter	DF	Estimate	Standard Error	95% Confidence Limits	
Intercept	1	1.658	0.0076	1.45	1.76
Scale	1	0.0091	0.00081	0.0080	0.0142
Weibull Shape	1	107.725	94.09	94.09	123.33

TABLE 2. Analysis of Maximum Likelihood Parameter Estimates
Weibull Model

Information

Parameter	DF	Estimate	Standard Error	95% Confidence Limits	
Intercept	1	1.670	0.0077	1.59	1.72
Scale	1	0.0092	0.00082	0.0081	0.0131
Weibull Shape	1	108.075	95.09	94.12	121.33

TABLE 3. Analysis of Maximum Likelihood Parameter Estimates
Bayesian Weibull Model

Model	DIC
Weibull	793
Baysian Weibull	786

TABLE 4. Goodness of Fits (DIC)

5. DISCUSSION AND CONCLUSION

From the Table 2 and Table 3 when you compare the Bayesian Weibull and Weibull Model, Bayesian Model has narrow credible interval for this data.

From the Table 4 results reveals that Bayesian Weibull Model shows a smaller DIC value (786) comparing with Weibull (793). It shows Bayesian Weibull is better fit than the Weibull model for this data.

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DEPARTMENT OF MATHEMATICS
SRM IST, RAMAPURAMCHENNAI-89
CHENNAI-89
E-mail address: parthass@srmist.edu.in

DEPARTMENT OF MATHEMATICS
VIT
VELLORE CAMPUS