

## DECOMPOSITIONS OF $\pi g$ -CONTINUITY VIA IDEAL NANO TOPOLOGICAL SPACES

O. NETHAJI<sup>1</sup>, R. ASOKAN, AND I. RAJASEKARAN

ABSTRACT. In this paper, we introduce and discuss some notions of  $I_{n\pi g}$ -closed sets,  $I_{n\pi g}$ -continuity in ideal nano spaces.

### 1. INTRODUCTION AND PRELIMINARIES

According to [14], an ideal  $I$  on a topological space  $(X, \tau)$  is a non-empty collection of subsets of  $X$  which satisfies the following conditions.

- (i)  $A \in I$  and  $B \subseteq A$  imply  $B \in I$  and
- (ii)  $A \in I$  and  $B \in I$  imply  $A \cup B \in I$ .

Given a topological space  $(X, \tau)$  with an ideal  $I$  on  $X$ . If  $\wp(X)$  is the family of all subsets of  $X$ , a set operator  $(.)^* : \wp(X) \rightarrow \wp(X)$ , called a local function of  $A$  with respect to  $\tau$  and  $I$  is defined as follows: for  $A \subset X$ ,  $A^*(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$  where  $\tau(x) = \{U \in \tau : x \in U\}$ , [3].

The closure operator defined by  $cl^*(A) = A \cup A^*(I, \tau)$ , [13] is a Kuratowski closure operator which generates a topology  $\tau^*(I, \tau)$  called the  $\star$ -topology finer than  $\tau$ . The topological space together with an ideal on  $X$  is called an ideal topological space or an ideal space denoted by  $(X, \tau, I)$ . We will simply write  $A^*$  for  $A^*(I, \tau)$  and  $\tau^*$  for  $\tau^*(I, \tau)$ .

<sup>1</sup>corresponding author

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In this paper, we introduce and discuss some notions of  $I_{n\pi g}$ -closed sets,  $I_{n\pi g}$ -continuity in ideal nano spaces.

We denote a nano topological space by  $(U, \mathcal{N})$ , where  $\mathcal{N} = \tau_R(X)$ . The nano-interior, nano-closure and nano  $\alpha$ -closure of a subset  $A$  of  $U$  are denoted by  $I_n(A)$ ,  $C_n(A)$  and  $C_{n\alpha}(A)$ , respectively.

An ideal nanotopological space is denoted by  $(U, \mathcal{N}, I)$ . The nano-interior and nano-closure of a subset  $A$  of  $U$  are denoted by  $I_n^*(A)$  and  $C_n^*(A)$ , respectively.

**Definition 1.1.** A subset  $A$  of a space  $(U, \mathcal{N})$  is called

- (i) nano  $\alpha$ -open if  $A \subseteq I_n(C_n(I_n(A)))$ , [4];
- (ii) nano semi-open if  $A \subseteq C_n(I_n(A))$ , [4];
- (iii) nano pre-open if  $A \subseteq I_n(C_n(A))$ , [4];
- (iv) nano  $b$ -open if  $A \subseteq I_n(C_n(A)) \cup C_n(I_n(A))$ , [5];
- (v) nano  $\beta$ -open if  $A \subseteq C_n(I_n(C_n(A)))$ , [12].

The complements of the above mentioned sets are called their respective closed sets.

**Definition 1.2.** [4] A subset  $A$  of a nano space  $(U, \mathcal{N})$  is called nano regular-open (written in short as  $nr$ -open)  $A = I_n(C_n(A))$ .

The complement of  $nr$ -open set is said to be a  $nr$ -closed set.

**Definition 1.3.** [1] Let  $A$  be a subset of a space  $(U, \mathcal{N})$  is nano  $\pi$ -open (written in short as  $n\pi$ -open) if the finite union of  $nr$ -open sets.

The complement of  $n\pi$ -open set is said to be a  $n\pi$ -closed set.

**Definition 1.4.** A subset  $A$  of a space  $(U, \mathcal{N})$  is called

- (i) nano  $g$ -closed (written in short as  $ng$ -closed) if  $C_n(A) \subseteq B$ , whenever  $A \subseteq B$  and  $B$  is  $n$ -open, [2];
- (ii) nano  $\pi g$ -closed (written in short as  $n\pi g$ -closed) if  $C_n(A) \subseteq B$ , whenever  $A \subseteq B$  and  $B$  is  $n\pi$ -open, [9];
- (iii) nano  $\alpha g$ -closed (written in short as  $n\alpha g$ -closed) if  $C_{n\alpha}(A) \subseteq B$ , whenever  $A \subseteq B$  and  $B$  is  $n$ -open, [9];
- (iv) nano  $\pi g\alpha$ -closed (written in short as  $n\pi g\alpha$ -closed) if  $C_{n\alpha}(A) \subseteq B$  whenever  $A \subseteq B$  and  $B$  is  $n\pi$ -open, [10].

The complements of the above mentioned sets are called their respective open sets.

**Definition 1.5.** [6] A subset  $A$  of a space  $(U, \mathcal{N}, I)$  is  $n\star$ -dense in itself (resp.  $n\star$ -perfect and  $n\star$ -closed) if  $A \subseteq A_n^*$  (resp.  $A = A_n^*$ ,  $A_n^* \subseteq A$ ).

The complement of a  $n\star$ -closed set is said to be a  $n\star$ -open set.

**Definition 1.6.** [7] An ideal  $I$  in a space  $(U, \mathcal{N}, I)$  is called  $\aleph$ -codense ideal if  $\aleph \cap I = \{\phi\}$ .

**Definition 1.7.** [11] A subset  $A$  of space  $(U, \mathcal{N}, I)$  is said to be

- (i) nano  $\alpha$ - $I$ -open (written in short as  $\alpha$ - $nI$ -open) if  $A \subseteq I_n(C_n^*(I_n(A)))$ ,
- (ii) nano semi- $I$ -open (written in short as semi- $nI$ -open) if  $A \subseteq C_n^*(I_n(A))$ ,
- (iii) nano pre- $I$ -open (written in short as pre- $nI$ -open) if  $A \subseteq I_n(C_n^*(A))$ ,
- (iv) nano  $b$ - $I$ -open (written in short as  $b$ - $nI$ -open) if  $A \subseteq I_n(C_n^*(A)) \cup C_n^*(I_n(A))$ ,
- (v) nano  $\beta$ - $I$ -open (written in short as  $\beta$ - $nI$ -open) if  $A \subseteq C_n^*(I_n(C_n^*(A)))$ .

The complements of the above mentioned sets are called their respective closed sets.

**Definition 1.8.** A subset  $A$  of a space  $(U, \mathcal{N}, I)$  is called a

- (i) nano  $I_g$ -closed (written in short as  $I_{ng}$ -closed) if  $A_n^* \subseteq B$  whenever  $A \subseteq B$  and  $B$  is  $n$ -open, [6];
- (ii) nano  $I_\omega$ -closed (or) nano  $I_{\hat{g}}$ -closed (written in short as  $I_{n\omega}$ -closed) if  $A_n^* \subseteq B$  whenever  $A \subseteq B$  and  $B$  is  $ns$ -open, [8].

The complements of the above mentioned sets are called their respective open sets.

## 2. $\pi g$ -CLOSED SETS IN IDEAL NANOTOPOLOGICAL SPACES

**Definition 2.1.** A subset  $A$  of an ideal nano space  $(U, \mathcal{N}, I)$  is called a nano  $I_{\pi g}$ -closed (written in short as  $I_{n\pi g}$ -closed) if  $A \subseteq H$ ,  $H \in n\pi$ -open  $\implies A_n^* \subseteq H$ .

Nano  $I_{\pi g}$ -open (written in short as  $I_{n\pi g}$ -open) if  $\mathcal{A} = H - A$  (where  $\mathcal{A}$  denotes the complement operator and  $A$  is  $I_{n\pi g}$ -closed).

**Definition 2.2.** A subset  $A$  of an ideal nano space  $(U, \mathcal{N}, I)$  is called a

- (i) nano  $\mathfrak{D}_I$ -set if  $A = H \cap V$ , where  $H$  is a  $n\pi$ -open set and  $V$  is a  $n\star$ -perfect set.
- (ii) nano  $\mathfrak{B}_I$ -set if  $A = H \cap V$ , where  $H$  is a  $n\pi$ -open set and  $V$  is a  $n\star$ -closed set.

**Theorem 2.1.** *Each  $n\pi g$ -closed set is  $I_{n\pi g}$ -closed.*

*Proof.* Let  $A$  be a every  $n\pi g$ -closed set. Then  $A \subseteq H$ ,  $H \in n\pi$ -open  $\implies C_n(A) \subseteq H$ . Since  $A_n^* \subseteq C_n(A) \subseteq H$ , we have  $A_n^* \subseteq H$  and hence  $A$  is  $I_{n\pi g}$ -closed.  $\square$

**Theorem 2.2.** *If  $(U, \mathcal{N}, I)$  is any ideal nano space and  $A \subseteq U$ , then the following hold.*

- (i) *If  $I = \phi$ , then  $A$  is  $I_{n\pi g}$ -closed  $\iff A$  is  $n\pi g$ -closed.*
- (ii) *If  $I = \aleph$ , then  $A$  is  $I_{n\pi g}$ -closed  $\iff A$  is  $n\pi g\alpha$ -closed.*

*Proof.* The proof follows from the fact that  $A_n^*(\{\phi\}) = C_n(A)$  and  $A_n^*(\aleph) = C_{n\alpha}(A)$ .  $\square$

**Theorem 2.3.** *If  $A$  and  $B$  is  $I_{n\pi g}$ -closed then  $A \cup B$  is  $I_{n\pi g}$ -closed.*

*Proof.* Suppose that  $A \cup B \subseteq H$  and  $H$  is  $n\pi$ -open, then  $A, B \subseteq H$ . Since  $A$  and  $B$  are  $I_{n\pi g}$ -closed,  $A_n^* \subseteq H$  and  $B_n^* \subseteq H$ .  $(A \cup B)_n^* = A_n^* \subseteq B_n^*$ ,  $(A \cup B)_n^* = A_n^* \cup B_n^* \subseteq H$ . Thus,  $A \cup B$  is also  $I_{n\pi g}$ -closed.  $\square$

**Theorem 2.4.** *If a subset  $A$  of  $(U, \mathcal{N}, I)$  is  $I_{n\pi g}$ -closed, then  $C_n^*(A) - A$  contains no nonempty  $n\pi$ -closed set.*

*Proof.* Suppose that  $A$  is  $I_{n\pi g}$ -closed and  $F$  be a  $n\pi$ -closed subset of  $C_n^*(A) - A$ . Then  $A \subseteq U - F$ . Since  $U - F$  is  $n\pi$ -open and  $A$  is  $I_{n\pi g}$ -closed,  $C_n^*(A) \subseteq U - F$ .

Consequently,  $F \subseteq U - C_n^*(A)$ . We have  $F \subseteq C_n^*(A)$ . Thus,  $F \subseteq C_n^*(A) \cap (U - C_n^*(A)) = \phi$  and so  $C_n^*(A) - A$  contains no nonempty  $n\pi$ -closed set.  $\square$

**Corollary 2.1.** *Let  $(U, \mathcal{N}, I)$  be an ideal nano space and  $A$  be an  $I_{n\pi g}$ -closed set. Then the following are equivalent.*

- (i)  *$A$  is a  $n\star$ -closed set.*
- (ii)  *$C_n^*(A) - A$  is a  $n\pi$ -closed set.*
- (iii)  *$A_n^* - A$  is a  $n\pi$ -closed set.*

*Proof.* (i)  $\implies$  (ii) : If  $A$  is  $n\star$ -closed set, then  $C_n^*(A) - A = \phi$  and so  $C_n^*(A) - A$  is  $n\pi$ -closed.

(ii)  $\implies$  (i) : suppose  $C_n^*(A) - A$  is  $n\pi$ -closed. Since  $A$  is  $I_{n\pi g}$ -closed, By Theorem 2.4  $C_n^*(A) - A = \phi$  and so  $A$  is  $n\star$ -closed.

(ii)  $\iff$  (iii) : Follows from the fact that  $C_n^*(A) - A = A_n^* - A$ .  $\square$

**Theorem 2.5.** *In a space  $(U, \mathcal{N}, I)$ , every subset is  $I_{n\pi g}$ -closed  $\iff$  every  $n\pi$ -open set is  $n\star$ -closed.*

*Proof.* Suppose every subset of  $U$  is  $I_{n\pi g}$ -closed. If  $H$  is  $n\pi$ -open then by hypothesis,  $H$  is  $I_{n\pi g}$ -closed and so  $H_n^* \subseteq H$ . Hence,  $H$  is  $n\star$ -closed.

Conversely, suppose every  $n\pi$ -open set is  $n\star$ -closed. Let  $A$  be a subset of  $U$ . If  $H$  is a  $n\pi$ -open set such that  $A \subseteq H$  then  $A_n^* \subseteq H_n^* \subseteq H$  and so  $A$  is  $I_{n\pi g}$ -closed.  $\square$

**Remark 2.1.** If  $A$  is  $n\pi$ -open and  $I_{n\pi g}$ -closed, then  $A$  is  $n\star$ -closed.

**Theorem 2.6.** For each  $x \in (U, \mathcal{N}, I)$  either  $\{x\}$  is  $n\pi$ -closed or  $\{x\}^c$  is  $I_{n\pi g}$ -closed.

*Proof.* Suppose that  $\{x\}$  is not  $n\pi$ -closed, then  $\{x\}^c$  is not  $n\pi$ -open and the only  $n\pi$ -open set containing  $\{x\}^c$  is the space  $(U, \mathcal{N}, I)$  itself.

Therefore,  $C_n^*(\{x\}^c) \subseteq U$  and so  $\{x\}^c$  is  $I_{n\pi g}$ -closed.  $\square$

**Theorem 2.7.** If  $A$  is an  $I_{n\pi g}$ -closed set such that  $A \subseteq B \subseteq A_n^*$ , then  $B$  is also an  $I_{n\pi g}$ -closed set.

*Proof.* Let  $H$  be any  $n\pi$ -open set such that  $B \subseteq H$ , then  $A \subseteq H$ . Since  $A$  is  $I_{n\pi g}$ -closed, we have  $A_n^* \subseteq H$ . Now,  $B_n^* \subseteq (A_n^*)_n^* \subseteq A_n^* \subseteq H$ . Therefore,  $B$  is  $I_{n\pi g}$ -closed.  $\square$

**Theorem 2.8.** A subset  $A$  of an ideal nano space  $(U, \mathcal{N}, I)$  is  $I_{n\pi g}$ -open  $\iff F \subseteq I_n^*(A)$  whenever  $F$  is  $n\pi$ -closed and  $F \subseteq A$ .

*Proof.* Suppose that  $F \subseteq I_n^*(A)$  whenever  $F$  is  $n\pi$ -closed and  $F \subseteq A$ . Let  $A^c \subseteq H$ , whenever  $H$  is  $n\pi$ -open. Then  $H^c \subseteq A$  and  $H^c$  is  $n\pi$ -closed, therefore  $H^c \subseteq I_n^*(A)$ , which implies that  $C_n^*(A^c) \subseteq H$ . Hence,  $A^c$  is  $I_{n\pi g}$ -closed and so  $A$  is  $I_{n\pi g}$ -open. Conversely, suppose that  $A$  is  $I_{n\pi g}$ -open,  $F \subseteq A$  and  $F$  is  $n\pi$ -closed. Then  $F^c$  is  $n\pi$ -open and  $A^c \subseteq F^c$ . Therefore,  $C_n^*(A^c) \subseteq F^c$  and so  $F \subseteq I_n^*(A)$ .  $\square$

**Theorem 2.9.** A subset  $A$  of an ideal nano space  $(U, \mathcal{N}, I)$  is a nano  $\mathfrak{D}_I$ -set and a  $I_{n\pi g}$ -closed set, then  $A$  is a  $n\star$ -closed set.

*Proof.* Let  $A$  be a nano  $\mathfrak{D}_I$ -set and a  $I_{n\pi g}$ -closed set. Since  $A$  is a nano  $\mathfrak{D}_I$ -set,  $A = H \cap V$ , where  $H$  is a  $n\pi$ -open set and  $V$  is a  $n\star$ -perfect set. Now,  $A = H \cap V \subseteq H$  and  $A$  is a  $I_{n\pi g}$ -closed set implies that  $A_n^* \subseteq H$ . Also,  $A = H \cap V \subseteq V$  and  $V$  is  $n\star$ -perfect set implies that  $A_n^* \subseteq V$ . Thus,  $A_n^* \subseteq H \cap V = A$ . Hence,  $A$  is a  $n\star$ -closed set.  $\square$

**Theorem 2.10.** For a subset  $A$  of an ideal nano space  $(U, \mathcal{N}, I)$ ,  $A$  is a  $n\star$ -closed set  $\iff A$  is a nano  $\mathfrak{B}_I$ -set and a  $I_{n\pi g}$ -closed set.

*Proof.* Assuming that  $A$  is a  $n\star$ -closed set and  $A = U \cap V$ , where  $U$  is  $n\pi$ -open set and  $V$  is a  $n\star$ -closed set. Hence,  $A$  is a nano  $\mathfrak{B}_I$ -set. Suppose that  $A$  is a  $n\star$ -closed set and  $H$  is a  $n\pi$ -open set such that  $A \subseteq H$ . Then  $A_n^* \subseteq H$  and hence  $A$  is a  $I_{n\pi g}$ -closed set.

Conversely, let  $A$  be a nano  $\mathfrak{B}_I$ -set and a  $I_{n\pi g}$ -closed set. Since  $A$  is a nano  $\mathfrak{B}_I$ -set,  $A = H \cap V$ , where  $H$  is a  $n\pi$ -open set and  $V$  is a  $n\star$ -closed set. Now,  $A \subseteq H$  and  $A$  is a  $I_{n\pi g}$ -closed set implies that  $A_n^* \subseteq H$ . Also,  $A \subseteq V$  and  $V$  is a  $n\star$ -closed set implies that  $A_n^* \subseteq V$ . Thus,  $A_n^* \subseteq H \cap V = A$ . Hence,  $A$  is a  $n\star$ -closed set.  $\square$

### 3. ON NANO $I_{\pi g}$ -CONTINUOUS MAPS

**Definition 3.1.** A map  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X})$  is called nano  $I_{\pi g}$ -continuous (written in short as  $I_{n\pi g}$ -continuous) if  $f^{-1}(A)$  is  $I_{n\pi g}$ -closed in  $(U, \mathcal{N}, I)$  for every  $n$ -closed set  $A$  of  $F$ .

**Definition 3.2.** A map  $f : (U, \mathcal{N}) \rightarrow (F, \mathcal{X})$  is called a

- (i) a nano  $\pi$ -space (written in short as  $n\pi$ -space) if  $f(A)$  is  $n\pi$ -closed in  $(F, \mathcal{X})$  for every  $n\pi$ -closed set  $A$  in  $(U, \mathcal{N})$ .
- (ii) a nano regular map (written in short as  $nr$ -map) if  $f^{-1}(A)$  is  $nr$ -closed in  $(U, \mathcal{N})$  for every  $nr$ -closed set  $K$  of  $F$ .

**Theorem 3.1.** For a map  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X})$ , the following hold.

- (i)  $f$  is  $n\pi g$ -continuous  $\Rightarrow f$  is  $I_{n\pi g}$ -continuous.
- (ii)  $f$  is  $I_{ng}$ -continuous  $\Rightarrow f$  is  $I_{n\pi g}$ -continuous.

**Definition 3.3.** A map  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X}, I)$  is called nano  $I_{\pi g}$ -irresolute (written in short as  $I_{n\pi g}$ -irresolute) if  $f^{-1}(A)$  is  $I_{n\pi g}$ -closed in  $(U, \mathcal{N}, I)$  for every  $I_{n\pi g}$ -closed set  $A$  of  $(F, \mathcal{X}, I)$ .

**Theorem 3.2.** If  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X}, I)$  is  $I_{n\pi g}$ -continuous and  $n\pi$ -space, then  $f$  is  $I_{n\pi g}$ -irresolute.

*Proof.* Assume that  $A$  is  $I_{n\pi g}$ -closed in  $F$ . Let  $f^{-1}(A) \subseteq H$ , where  $H$  is  $n\pi$ -open in  $U$ . Then  $(U - H) \subseteq f^{-1}(F - A)$  and hence  $f(U - H) \subseteq F - A$ . Since  $f$  is  $n\pi$ -space,  $f(U - H)$  is  $n\pi$ -closed. Then, since  $F - A$  is  $I_{n\pi g}$ -open. By Theorem 2.8,  $f(U - H) \subseteq I_n^*(F - A) = F - C_n^*(A)$ . Thus,  $f^{-1}(C_n^*(A)) \subseteq H$ . Since  $f$  is  $I_{n\pi g}$ -continuous,  $f^{-1}(C_n^*(A))$  is  $I_{n\pi g}$ -closed. Therefore,  $C_n^*(f^{-1}(C_n^*(A))) \subseteq H$

and hence  $C_n^*(f^{-1}(A)) \subseteq C_n^*(f^{-1}(C_n^*(A))) \subseteq H$  which proves that  $f^{-1}(A)$  is  $I_{n\pi g}$ -closed and therefore  $f$  is  $I_{n\pi g}$ -irresolute.  $\square$

**Definition 3.4.** A map  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X})$  is called almost nano  $I_{\pi g}$ -continuous (written in sort as almost  $I_{n\pi g}$ -continuous) if  $f^{-1}(A)$  is  $I_{n\pi g}$ -closed in  $(U, \mathcal{N}, I)$  for every  $A$  is  $n$ -regular closed in  $F$ .

**Theorem 3.3.** For a map  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X})$ , the following are equivalent.

- (i)  $f$  is almost  $I_{n\pi g}$ -continuous.
- (ii)  $f^{-1}(A) \in I_{n\pi g}$ -open for every  $A$  is  $nr$ -open in  $F$ .
- (iii)  $f^{-1}(I_n^*(C_n^*(A))) \in I_{n\pi g}$ -open for every  $A \in \mathcal{X}$ .
- (iv)  $f^{-1}(C_n^*(I_n^*(A))) \in I_{n\pi g}$ -closed for every  $n$ -closed set  $A$  of  $F$ .

*Proof.* (i)  $\iff$  (ii) : Obvious.

(ii)  $\iff$  (iii) : Assuming that  $A$  is  $n$ -regular open in  $F$ , we have  $A = I_n(C_n(A))$  and  $f^{-1}(I_n(C_n(A))) \in I_{n\pi g}$ -open. Conversely, suppose  $A \in \mathcal{X}$ , we have  $I_n(C_n(A)) \in n$ -regular open  $(F)$  and  $f^{-1}(I_n(C_n(A))) \in I_{n\pi g}$ -open.

(iii)  $\iff$  (iv) : Let  $A$  be a  $n$ -closed set in  $F$ . Then  $F - A \in \mathcal{X}$ . We have  $f^{-1}(I_n(C_n(F - A))) = f^{-1}(F - (C_n(I_n(A)))) = U - f^{-1}(C_n(I_n(A))) \in I_{n\pi g}$ -open. Hence,  $f^{-1}(I_n(C_n(A))) \in I_{n\pi g}$ -closed. Converse can be obtained similarly.  $\square$

**Theorem 3.4.** The following hold for the maps  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X}, J)$  and  $g : (F, \mathcal{X}, J) \rightarrow (G, \mathcal{M})$ ,

- (i)  $g \circ f$  is  $I_{n\pi g}$ -continuous, if  $f$  is almost  $I_{n\pi g}$ -continuous and  $g$  is completely nano continuous.
- (ii)  $g \circ f$  is  $I_{n\pi g}$ -continuous, if  $f$  is  $I_{n\pi g}$ -continuous and  $g$  is nano continuous.
- (iii)  $g \circ f$  is  $I_{n\pi g}$ -continuous, if  $f$  is  $I_{n\pi g}$ -irresolute and  $g$  is  $I_{n\pi g}$ -continuous.
- (iv)  $g \circ f$  is almost  $I_{n\pi g}$ -continuous, if  $f$  is almost  $I_{n\pi g}$ -continuous and  $g$  is nano  $nr$ -map.
- (v)  $g \circ f$  is almost  $I_{n\pi g}$ -continuous, if  $f$  is  $I_{n\pi g}$ -irresolute and  $g$  is almost  $I_{n\pi g}$ -continuous.
- (vi)  $g \circ f$  is almost  $I_{n\pi g}$ -continuous, if  $f$  is  $I_{n\pi g}$ -continuous and  $g$  is almost  $I_{n\pi g}$ -continuous.

**Definition 3.5.** A map  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X})$  is called nano  $\mathfrak{B}_I$ -continuous (written in sort as  $\mathfrak{B}_{nI}$ -continuous) if  $f^{-1}(A)$  is nano  $\mathfrak{B}_I$ -set in  $(U, \mathcal{N}, I)$  for every  $n$ -closed set  $A$  of  $F$ .

**Theorem 3.5.** *A map  $f : (U, \mathcal{N}, I) \rightarrow (F, \mathcal{X})$  is  $n\star$ -continuous  $\iff \mathfrak{B}_{nI}$ -continuous and  $I_{n\pi g}$ -continuous.*

*Proof.* This is an immediate consequence of Theorem 2.10.  $\square$

**Remark 3.1.** *The concepts of  $\mathfrak{B}_{nI}$ -continuity and the concepts of  $I_{n\pi g}$ -continuity are independent of each other as shown in the following Example.*

**Example 1.** *Let  $U = \{a, b, c\}$  be a non empty finite set with*

- (i)  $U/R = \{\{a\}, \{b\}, \{c\}\}$  and  $X = \{a, b\}$  then  $\mathcal{N} = \{\phi, U, \{a\}, \{b\}, \{a, b\}\}$ .
- (ii)  $U/R = \{\{a, b\}, \{c\}\}$  and  $X = \{b, c\}$  then  $\mathcal{X} = \{\phi, U, \{c\}, \{a, b\}\}$ .
- (iii)  $U/R = \{\{b, c\}, \{a\}\}$  and  $X = \{b, c\}$  then  $\mathcal{M} = \{\phi, U, \{b, c\}\}$ .

*And let ideal be  $I = \{\phi, \{c\}\}$ .*

*In the ideal nano space  $(U, \mathcal{N}, I)$ , then*

- (i) *the identity function  $F : (U, \mathcal{N}, I) \rightarrow (U, \mathcal{M})$  is  $\mathfrak{B}_{nI}$ -continuous but not -continuous.*
- (ii) *the identity function  $G : (U, \mathcal{X}, I) \rightarrow (U, \mathcal{M})$  is  $I_{n\pi g}$ -continuous but not  $\mathfrak{B}_{nI}$ -continuous.*

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SCHOOL OF MATHEMATICS, MADURAI KAMARAJ UNIVERSITY,  
MADURAI-21, TAMIL NADU, INDIA  
*E-mail address:* jionetha@yahoo.com

DEPARTMENT OF MATHEMATICS  
SCHOOL OF MATHEMATICS, MADURAI KAMARAJ UNIVERSITY,  
MADURAI-21, TAMIL NADU, INDIA  
*E-mail address:* rasoka\_mku@yahoo.co.in

DEPARTMENT OF MATHEMATICS  
TIRUNELVELI DAKSHINA MARA NADAR SANGAM COLLEGE, T. KALLIKULAM - 627 113  
TIRUNELVELI DISTRICT, TAMIL NADU, INDIA  
*E-mail address:* sekarmelakkal@gmail.com