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ON CONTRA-$I\pi G*\beta$-CONTINUOUS FUNCTIONS IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT. In this paper we have investigated the properties of contra-$I\pi g*\beta$-continuous functions in ideal topological spaces. Also, we have introduced the graph via $I\pi g*\beta$-closed sets. Relationships between the new classes and other classes of functions are established and some characterizations of their new classes of functions are studied.

1. INTRODUCTION

An ideal I on a topological space (X,τ) is a non-empty collection of subsets of X satisfying the following properties:

(1) $A\in I$ and $B\subseteq A$ imply $B\in I$ (heredity);
(2) $A\in I$ and $B\in I$ imply $A\cup B\in I$ (finite additivity).

A topological space (X,τ) with an ideal I on X is called an ideal topological space and is denoted by (X,τ,I). For a subset $A\subseteq X$, $A*(I)=\{x\in X : U\cap A/\in I$ for every $U\in\tau(x)\}$, is called the local function [9] of A with respect to I and τ. We simply write $A*$ in case there is no chance for confusion. A Kuratowski closure operator $cl*(.)$ for a topology $\tau*(I)$, called the $*$-topology finer than τ, is defined by $cl*(A)=A\cup A*$ [17]. Let (X,τ) denote a topological space on which no separation axioms are assumed unless explicitly stated. In a

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topological space (X, τ), the closure and the interior of any subset A of X will be denoted by $cl(A)$ and $int(A)$, respectively. The kernel [10] of A, denoted by $ker(A)$, is the intersection of all open supersets of A. A subset A of a topological space (X, τ) is said to be pre-open [11] if $A \subseteq cl(int(cl(A)))$.

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies the following conditions: $A \in I$ and $B \subset A$ implies $B \in I$; $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if $\mathcal{P}(X)$ is the set of all subsets of X, a set operator $(.)* : \mathcal{P}(X) \to \mathcal{P}(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subseteq X$, $A * (I, \tau) = \{x \in X : U \cap A \notin I$, for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau | x \in U\}$. A Kuratowski closure operator is $cl * (x) = A \cup A*$ for $\tau*$. When there is no chance for confusion, we will simply write A for $A(I, \tau)$. $X*$ is often a proper subset of X.

2. Preliminaries

Lemma 2.1. *[7] The following properties hold for subsets A, B of a topological space (X, τ) :*

(1) *$x \in ker(A)$ if and only if any closed set F of X contains x;*
(2) *$A \subset ker(A)$ and $A = ker(A)$ if A is open in X;*
(3) *If $A \subset B$ then $ker(A) \subset ker(B)$.*

Definition 2.1. *A topological space (X, τ) is said to be:*

(1) *extremally disconnected [15] if the closure of every open set of X is open in X;*
(2) *submaximal [14] if every dense set of X is open in X, equivalently if every preopen set is open.*

Definition 2.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(1) *almost pre-continuous [13] if for each $x \in X$ and each open neighbourhood V of $f(x)$, there exists $U \in PO(X)$ containing x such that $f(U) \subset cl(int(cl(V))$;*
(2) *$\pi g * \beta$-continuous if $f^{-1}(V)$ is $\pi g * \beta$-closed in (X, τ) for every closed set V of (Y, σ);*
(3) *contra-continuous [4] (briefly c-c) if $f^{-1}(V)$ is closed set in X for every open set V of Y;*

(4) *contra-pre-continuous [8] (briefly c-pre-c) if $f^{-1}(V)$ is a preclosed set in X for every open set V of Y;*
(5) *contra-β-continuous [3] (briefly c-β-c) if $f^{-1}(V)$ is β-closed set in X for every open set V of Y.*

Definition 2.3. *A subset A of an ideal topological space (X, τ, I) is said to be*

(1) *pre-I-open if $A \subseteq cl(int(cl * (A))$;*
(2) *semi-I-open [5] if $A \subseteq cl * (int(A))$;*
(3) *α-I-open [5] if $A \subseteq cl(int(cl * (int(A)))$;*
(4) *β-I-open [5] if $A \subseteq cl(cl(int(cl * (A)))$;*
(5) *strong β-I-open [6] if $A \subseteq cl * (cl(int(cl * (A)))$;*
(6) *δ-I-open [1] if $cl(int(cl * (A)) \subseteq cl * (int(A))$.*

Definition 2.4. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be pre-I-continuous [?] (resp. semi-I-continuous [5], α-I-continuous [5], β-I-continuous [5]) if for each open set V of (Y, σ), $f^{-1}(V)$ is a pre-I-open (resp. semi-I-open, α-I-open, β-I-open) set in (X, τ, I).*

Definition 2.5. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be:*

(1) *contra semi-I-continuous [12] (briefly c-semi-I-c) if $f^{-1}(V)$ is semi-I-open in (X, τ, I) for every closed set V of (Y, σ).*
(2) *contra α-I-continuous [16] (briefly c-α-I-c) if $f^{-1}(V)$ is α-I-open in (X, τ, I) for every closed set V of (Y, σ).*
(3) *contra β-I-continuous [2] (briefly c-β-I-c) if $f^{-1}(V)$ is β-I-open in (X, τ, I) for every closed set V of (Y, σ).*

Definition 2.6. *[?] A subset A of an ideal topological space (X, τ, I) is called $I\pi g * \beta$-closed if β-$Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is πg-open in X.*

Definition 2.7. *[?] A function $f : (X, \tau, I) \to (Y, \sigma)$ is called $I\pi g * \beta$-continuous if $f^{-1}(V)$ is $I\pi g * \beta$-closed in (X, τ, I) for every closed set V of (Y, σ).*

3. CONTRA $I\pi g^*\beta$-CONTINUOUS FUNCTIONS

Definition 3.1. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is called contra $I\pi$ generalized $^*\beta$-continuous (briefly c-$I\pi g^*\beta$-c) if $f^{-1}(V)$ is $I\pi g^*\beta$-open in (X, τ, I) for every closed set V of (Y, σ).*

Theorem 3.1. *The following statements hold for a function* $f : (X, \tau, I) \to (Y, \sigma)$

1. *Every contra-continuous function is contra* $I\pi g^*\beta$*-continuous.*
2. *Every contra* α*-I-continuous function is contra* $I\pi g^*\beta$*-continuous.*
3. *Every contra* $I\pi g^*\beta$*-continuous function is contra* $\pi g^*\beta$*-continuous.*

Remark 3.1. *We have the following implications:*

contra-continuous $\to$ *contra-*αI*-continuous* $\to$ *contra-semi-*I*-continuous*

$\downarrow$ $\downarrow$

contra-pre-I*-continuous* $\to$ *contra* $I\pi g^*\beta$*-continuous* $\to$ *contra* $\pi g^*\beta$*-cont.*

Example 1. *Let* $X = Y = \{a, b, c\}$*,* $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$*,* $\sigma = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ *and* $I = \{\emptyset, \{a\}, \{a, b\}\}$*. Let* $f(a) = b$*,* $f(b) = a$*,* $f(c) = c$*. Then* f *is* $I\pi g * \beta$*-continuous function but not contra-semi-I-continuous (resp. contra-pre-I-continuous) function.*

Theorem 3.2. *For a function* $f : (X, \tau, I) \to (Y, \sigma)$*, the following are equivalent:*

1. f *is contra* $I\pi g^*\beta$*-continuous.*
2. *For every closed subset* F *of* Y*,* $f^{-1}(F) \in I\pi g^*\beta O(X)$*.*
3. *For each* $x \in X$ *and each closed set* F *of* Y *containing* $f(x)$*, there exists* $U \in I\pi g^*\beta O(X)$ *such that* $f(U) \subset F$*.*
4. $f(I\pi g^*\beta cl(A)) \subset ker(f(A))$ *for every subset* A *of* X*.*
5. $I\pi g^*\beta cl(f^{-1}(B)) \subset f^{-1}(ker(B))$ *for every subset* B *of* Y*.*

Proof. The implications (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (2): Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in I\pi g^*\beta O(X)$ containing x such that $f(U_x) \subset F$. Therefore, we obtain $f^{-1}(F) = \cup\{U_x | x \in f^{-1}(F)\}$. Hence $f^{-1}(F) \in I\pi g^*\beta O(X)$.

(2) $\Rightarrow$ (4): Let A be any subset of X. Suppose that $y \notin ker(f(A))$. Then by Lemma 1.1, there exists a closed set F of Y containing y such that $f(A) \subset F = \emptyset$. Thus, we have $A \cap f^{-1}(F) = \emptyset$ and $I\pi g^*\beta cl(A) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(I\pi g^*\beta cl(A)) \cap F = \emptyset$ and $y \notin f(I\pi g^*\beta cl(A))$. This implies that $f(I\pi g^*\beta cl(A)) \subset ker(f(A))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4) and 2.1, we have: $f(I\pi g^*\beta cl(f^{-1}(B))) \subset ker(f(f^{-1}(B))) \subset ker(B)$ and $I\pi g^*\beta cl(f^{-1}(B)) \subset f^{-1}(ker(B))$.

(5) $\Rightarrow$ (1): Let V be any open set of Y. Then by Lemma 2.1, we have $I\pi g^*\beta cl(f^{-1}(V)) \subset f^{-1}(ker(V)) = f^{-1}(V)$ and $I\pi g^*\beta cl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is $I\pi g^*\beta$-closed in (X,τ,I). □

Theorem 3.3. *If a function $f:(X,\tau,I)\to(Y,\sigma)$ is contra $I\pi g^*\beta$-continuous and Y is regular, then f is $I\pi g^*\beta$-continuous.*

Proof. Let x be an arbitrary point of X and V be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $cl(W)\subset V$. Since f is contra $I\pi g^*\beta$-continuous, by Theorem 3.2 there exists $UI\pi g^*\beta O(X)$ containing x such that $f(U)\subset cl(W)$. Then $f(U)\subset cl(W)\subset V$. Hence, f is $I\pi g^*\beta$-continuous. □

Theorem 3.4. *If a function $f:(X,\tau,I)\to(Y,\sigma)$ is contra $I\pi g^*\beta$-continuous, then f is almost weakly-I-continuous.*

Proof. Let V be any open set of Y. Since $cl(V)$ is closed in Y, $f^{-1}(cl(V))$ is $I\pi g^*\beta$-open in X and we have $f^{-1}(V)\subset f^{-1}(cl(V))\subset int(cl^*(f^{-1}(cl(V))))$. This shows that f is almost weakly-*I*-continuous. □

Definition 3.2. *A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to satisfy the $I\pi g^*\beta$-interiority condition if $I\pi g^*\beta Int(f^{-1}(cl(V)))\subset f^{-1}(V)$ for each open set V of (Y,σ).*

Theorem 3.5. *If $f:(X,\tau,I)\to(Y,\sigma)$ is a contra-$I\pi g^*\beta$-continuous function and satisfies the I-interiority condition, then f is $I\pi g^*\beta$-continuous.*

Proof. Let V be any open set of Y. Since f is contra-$I\pi g^*\beta$-continuous and $cl(V)$ is closed, $f^{-1}(cl(V))$ is $I\pi g^*\beta$-open in X. By hypothesis of f, $f^{-1}(V)\subset f^{-1}(cl(V))\subset I\pi g^*\beta Int(f^{-1}(cl(V)))\subset I\pi g^*\beta Int(f^{-1}(V))\subset f^{-1}(V)$. Therefore, we obtain $f^{-1}(V) = I\pi g^*\beta Int(f^{-1}(V))$ and consequently $f^{-1}(V)\in \beta IO(X)$. This shows that f is a $I\pi g^*\beta$-continuous function. □

4. GRAPHS VIA $I\pi g*\beta$-CLOSED SETS

Definition 4.1. *The graph $G(f)$ of a function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be contra $I\pi g*\beta$ -closed in $X\times Y$ if for each $(x,y)\in(X\times Y)G(f)$, there exists a $I\pi g*\beta$-open set U in X containing x and a closed set V in Y containing y such that $(U\times V)\cap G(f)=\emptyset$.*

Lemma 4.1. *The graph $G(f)$ of a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra $I\pi g * \beta$-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y)G(f)$, there exists an $I\pi g * \beta$-open set U in X containing x and a closed set V in Y containing y such that $f(U) \cap V = \emptyset$.*

Theorem 4.1. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a contra $I\pi g * \beta$ -continuous function and Y is a Urysohn space, then $G(f)$ is contra $I\pi g * \beta$ -closed in $X \times Y$.*

Theorem 4.2. *Let (X, τ, I) be any ideal topological space and let $f : (X, \tau, I) \to (Y, \sigma)$ be a function and $g : X \to X \times Y$ be the graph function, given by $g(x) = (x, f(x))$ for every $x \in X$. Then f is contra-$I\pi g^*\beta$-continuous if and only if g is contra-$I\pi g^*\beta$-continuous.*

Proof. Let $x \in X$ and let W be a closed subset of $X \times Y$ containing $g(x)$. Then $W \cap (\{x\} \times Y)$ is closed in $\{x\} \times Y$ containing $g(x)$. Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y | (x, y) \in W\}$ is a closed subset of Y. Since f is contra-$I\pi g^*\beta$-continuous, $\cup\{f^{-1}(y) \in Y | (x, y) \in W\}$ is a $I\pi g^*\beta$-open subset of (X, τ, I). Further, $x \in \cup\{f^{-1}(y) | (x, y) \in W\} \subset g^{-1}(W)$. Hence $g^{-1}(W)$ is $I\pi g^*\beta$-open. Then g is contra-$I\pi g^*\beta$-continuous.

Conversely, let F be a closed subset of Y. Then $X \times F$ is a closed subset of $X \times Y$. Since g is contra-$I\pi g^*\beta$-continuous, $g^{-1}(X \times F)$ is a $I\pi g^*\beta$-open subset of X. Also, $g^{-1}(X \times F) = f^{-1}(F)$. Hence f is contra-$I\pi g^*\beta$-continuous. □

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