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SOFT λ - CONTINUOUS FUNCTIONS

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ABSTRACT. In this paper, we present soft λ-open sets on soft topological spaces and examine some of their properties. We also look into the theory of soft λ-continuous functions and explain the relationships with soft continuous and other weaker forms of soft continuous functions.

1. INTRODUCTION

The theory of soft sets was initially introduced by Molodtsov [9] in 1999. He originated the basics of corresponding theory as a new idea to model the uncertainties. In [9, 10], Molodtsov successfully implemented the soft theory in smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and theory of measurement. In the recent past, number of papers has been emerged in soft sets theory and its applications in various fields [6, 12]. Shabir and Naz [11] bring out the notion of soft topological spaces which are defined to be over an initial universe with a fixed set of parameters. Moreover, Maji et al. [7] suggested different operations on soft sets, and some basic properties of these operations have been brought out so far. Later, Zorlutuna et al. [12], Aygunoglu and Aygun [3] carried on to examine the properties of soft topological space.

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Maki [8] investigated the notion of λ-sets in topological spaces. A λ-set is a set A which is equal to its kernel, that is, to the intersection of all open super sets of A. Arenas et.al. [2] introduced and investigated the notion of λ-closed sets by involving λ-sets and closed sets. Duraisamy et.al [4] examined some properties of λ-continuous functions.

In this paper, we are portraying soft-λ open sets on soft topological spaces and examine some of their properties. Besides, we investigate the concepts of soft λ-continuous functions and explain their relationships with soft continuous and other weaker forms of soft continuous functions.

2. Preliminaries

Definition 2.1. *[9] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a nonempty subset of E. A pair (F, A) is referred as soft set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 2.2. *[7] For two soft sets (F, A) and (G, B) over a common universe U, we state that (F, A) is a soft subset of (G, B), if*

(1) *$A \subseteq B$ and*
(2) *For all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A)\widetilde{\subset}(G, B)$.*

(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We refer it by $(F, A)\widetilde{\supset}(G, B)$.

Definition 2.3. *[7] Two soft sets (F, A) and (G, B) over a common universe U are referred to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).*

Definition 2.4. *[7] The complement of a soft sets (F, A) is denoted by $(F, A)^c$ and is outlined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\neg\alpha)$ for all $\alpha \in A$.*

Definition 2.5. *[7] A soft set (F, A) over U is said to be a null soft set, denoted by φ, if for all $e \in A$, $F(e) = \varphi$ (null set).*

Definition 2.6. *[7] A soft set (F, A) over U is said to be an absolute soft set, denoted by $\widetilde{A}$, if for all $e \in A$, $F(e) = U$. Clearly $\widetilde{A}^c = \varphi$ and $\varphi^c = \widetilde{A}$.*

Definition 2.7. *[7] The union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & if e \in A - B \\ G(e) & if e \in B - A \\ F(e) \cup G(e) & if e \in A \cap B \end{cases} (H, C) = (F, A) \cup (G, B).$$

Definition 2.8. *[7] The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted by $(F, A) \cap (G, B)$, is defined as $C = A \cap B$ and $(H, C) = (F, A) \cap (G, B)$ for all $e \in C$.*

Let X be an initial universe set and E be the non-empty set of parameters.

Definition 2.9. *[7] The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \backslash (G, E)$ is defined as $H(e) = F(e) \backslash G(e)$ for all $e \in E$.*

Definition 2.10. *[7] Consider Y as a non- empty subset of X, then $\widetilde{Y}$ denotes the soft set (Y, E) over X for which $Y(\alpha) = Y$ for all $\alpha \in E$. In particular (X, E) is denoted by $\widetilde{X}$.*

Definition 2.11. *[7] Let $x \in X$, then (x, E) denotes the soft set over X for which $x(\alpha) = \{x\}$ for all $\alpha \in E$.*

Definition 2.12. *[1] The relative complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha)$ for all $\alpha \in A$.*

Definition 2.13. *[11] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if:*

(1) *φ, $\widetilde{X}$ belongs to τ;*
(2) *the union of any number of soft sets in τ belongs to τ;*
(3) *the intersection of any number of soft sets in τ belongs to τ.*

The triplet (X, τ, E) is named as soft topological space over X. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition 2.14. *[11] Let us consider the soft space over X as (X, τ, E). A soft set (F, E) over X is referred to be soft closed in X, if its relative complement $(F, E)^c$ belongs to τ.*

Definition 2.15. *[2] A subset A of topological spaces (X, τ) is said to be λ-closed if $A = B \cap C$, where B is a Λ set and C is a closed set. The complement of λ-closed is λ-open.*

3. Soft λ-open sets

In this part, we examine soft λ-open sets in soft topological spaces and study some of their properties.

Definition 3.1. *Let (X, τ, E) be a soft topological space over X and let (A, E) be a soft set over X. Then (A, E) is said to be soft λ-closed if $(A, E) = (B, E) \cap (D, E)$, where (B, E) is soft λ-open and (D, E) is soft λ-closed. The complement of soft λ-closed is soft λ-open.*

Definition 3.2. *Let (X, τ, E) be a soft topological space over X and let (A, E) be a soft set over X.*

(1) *The soft λ-interior of (A, E) is denoted by $SInt_\lambda(A, E)$ and is defined as $SInt_\lambda(A, E) = \cup\{(O, E) : (O, E)$ is soft λ-open and $(O, E)\widetilde{\subset}(A, E)\}$.*
(2) *The soft λ-closure of (A, E) is denoted by $SCl_\lambda(A, E)$ and is defined as $SCl_\lambda(A, E) = \cap\{(F, E) : (F, E)$ is soft λ-closed and $(A, E)\widetilde{\subset}(F, E)\}$.*

Clearly $SCl_\lambda(A, E)$ is the smallest soft λ-closed set over X which contains (A, E) and $SInt_\lambda(A, E)$ is the biggest soft λ-open set over X which is contained in (A, E). The collection of all soft λ-open sets of X is referred by $S\lambda O(X)$ and the accumulation of all soft λ-closed set of X is denoted by $S\lambda C(X)$.

Proposition 3.1.

(1) *The arbitrary union of soft λ-open sets is soft λ-open in X.*
(2) *The arbitrary intersection of any number of soft λ-closed sets is a soft λ-closed set over X.*

Remark 3.1.

(1) *Every soft open set is soft λ-closed.*
(2) *Union of soft closed sets and soft λ-open sets is soft λ-open.*
(3) *Every soft open set is soft λ-open.*

Proposition 3.2. *Let (X, τ, E) be a soft topological space over X and let (A, E) be a soft set over X. Then:*

(1) (A, E) *belongs to soft* λ*-closed set in* (X, τ, E) *if and only if* $(A, E) = SCl_\lambda(A, E)$;

(2) (A, E) *belongs to soft*λ*-open set in* (X, τ, E) *if and only if* $(A, E) = SInt_\lambda(A, E)$.

Proof. (1) Let $(A, E) = SCl_\lambda(A, E) = \tilde{\cap}\{(F, E) : (F, E)$ is a soft λ-closed set and $(A, E)\tilde{\subset}(F, E)\ \}$. This shows that $(A, E) \in \{(F, E) : (F, E)$ is a soft λ-closed set and $(A, E)\tilde{\subset}(F, E)\}$. Hence (A, E) is softλ-closed.

Conversely, let (A, E) be soft λ-closed set. Since $(A, E)\tilde{\subset}(A, E)$ and (A, E) is soft λ-closed, $(A, E) \in \{(F, E) : (F, E)$ is a soft λ-closed set and $(A, E)\tilde{\subset}(F, E)\}$. Further $(A, E)\tilde{\subset}(F, E)$ for all such $(F, E)'s$. $(A, E) = \tilde{\cap}\{(F, E) : (F, E)$ is a soft λ-closed set and $(A, E)\tilde{\subset}(F, E)\}$.

(2) Similar to (1).

□

Theorem 3.1. *For a soft set* (F, E) *in a soft topological space* X*, it holds:*

(1) (F, E) *is a soft* λ*-open set if and only if* $(F, E)^c$ *is a soft* λ*-closed set in* X.

(2) (F, E) *is a soft* λ*-closed set if and only if* $(F, E)^c$ *is a soft* λ*-open set in* X.

4. SOFT λ-CONTINUOUS FUNCTIONS

Definition 4.1. *A mapping* $f : (X, \tau, E) \to (Y, \sigma, K)$ *is said to be soft mapping if* (X, τ, E) *and* (Y, σ, K) *are soft topological spaces and* $u : X \to Y$ *and* $p : E \to K$ *are mappings.*

Definition 4.2. *[5] Let* (X, τ, E) *and* (Y, σ, K) *be soft classes. Let* $u : X \to Y$ *and* $p : E \to K$ *be mappings. Then a mapping* $f : (X, \tau, E) \to (Y, \sigma, K)$ *is defined as for a soft set* (F, A) *in* (X, E)*,* $f((F, A), B)$*,* $B = p(A) \subseteq K$ *is a soft set in* (Y, K) *given by* $f(F, A)(\beta) = u(\bigcup_{\alpha \in p^{-1}(\beta) \cap A} F(\alpha))$ *for* $\beta \in K$. $f((F, A), B)$ *is called a soft image of a soft set* (F, A). *If* $B = K$*, then we shall write* $f((F, A), K)$ *as* $f(F, A)$.

Definition 4.3. *[5] Let* $f : (X, \tau, E) \to (Y, \sigma, K)$ *be a soft mapping from* (X, τ, E) *to* (Y, σ, K) *and* (G, C) *a soft set in* (Y, σ, K). *Then* $f^{-1}((G, C), D)$*,* $D = p^{-1}(C)$ *is a soft set in* (X, τ, E)*, defined as:* $f^{-1}(G, C)(\alpha) = u^{-1}(G(p(\alpha)))$ *for* $\alpha \in D \subseteq E$. $(f^{-1}(G, C), D)$ *is referred as soft inverse image of* (G, C).

Definition 4.4. *A soft mapping* $f : X \to Y$ *is said to be soft* λ*-continuous if the inverse image of each soft open subset of* Y *is a soft* λ*-open in* X.

Theorem 4.1. *Every soft continuous function is soft λ-continuous function.*

Proof. Let $f : X \to Y$ is soft continuous function. Let (G, A) be a soft open set in Y. Since f is soft continuous, $f^{-1}((G, A))$ is soft open in X. And so $f^{-1}((G, A))$ is soft λ-open in X. Hence, f is soft λ-continuous. □

Remark 4.1. *Converse of the above theorem is false as it can be verified from the following example.*

Example 1. *Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, $E = \{e_1, e_2\}$ and $K = \{k_1, k_2, k_3\}$ and let (X, τ, E) and (Y, σ, K) be soft topological spaces. Define $f_{pu} : X \to Y$ such that $u : X \to Y$ and $p : E \to K$ as $u(x_1) = \{y_1\}$, $u(x_2) = \{y_2\}$, $u(x_3) = \{y_3\}$; $p(e_1) = \{k_1\}$, $p(e_2) = \{k_3\}$. Let $\tau = \{\widetilde{\varphi}, \widetilde{X}, (F_1, E), (F_2, E)\}$ where (F_1, E) and (F_2, E) are soft sets over X defined as $(F_1, E) = \{(e_1, \{x_2, x_3\}), (e_2, \{x_1\})\}$ and $(F_2, E) = \{(e_1, \{X\}), (e_2\{x_1, x_3\}), (e_1, \{X\}), (e_2, \{x_1, x_3\})\}$. Let $\sigma = \{\varphi, \widetilde{Y}, (G, K)\}$ where$(G, K) = \{(k_1, \{y_1\}), (k_2, \{y_1, y_3\}), (k_3, \{y_2, y_3\})\}$, then ${f^{-1}}_{pu}(G, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_3\})\} \in S\lambda O(X)$ but is not soft open. Thus f is soft λ-continuous but not soft continuous.*

Theorem 4.2. *Let $f : X \to Y$ be a soft mapping. Then the following statements are equivalent:*

(1) *f is soft λ-continuous.*
(2) *The inverse image of each soft closed set in Y is softλ-closed set in X.*

Proof. (i) $\Rightarrow$ (ii) Let (G, A) be a soft closed set in Y. Then $(G, A)^C$ is soft open set in Y. Thus $f^{-1}((G, A)^C) \in S\lambda O(X)$. That is $X - f^{-1}((G, A)) \in S\lambda O(X)$. Hence $f^{-1}((G, A))$ is a soft λ-closed set in X.

(ii) $\Rightarrow$ (i) Let (G, B) be a soft open set in Y, then $(G, B)^C$ is soft closed set in Y and according to (ii) $f^{-1}((G, B)^C) \in S\lambda C(X)$. That is $X - f^{-1}((G, B)) \in S\lambda C(X)$. Hence $f^{-1}((G, B))$ is a soft λ-open set in X. Therefore f is soft λ-continuous function. □

Definition 4.5. *A soft mapping $f : X \to Y$ is said to be soft λ-irresolute if the inverse image of every is softλ- closed set in Y is soft λ-closed set in X.*

Theorem 4.3. *Every soft λ-irresolute mapping is soft λ-continuous function.*

Proof. Let $f : X \to Y$ be a softλ-irresolute mapping. Let (F, K) be a soft open set in Y. Then (F, K) is soft λ-open in Y. Therefore $f^{-1}((F, K))$ is soft λ-open in X. Hence f is softλ-continuous. □

Remark 4.2. *Converse of the above theorem is invalid in general.*

Theorem 4.4. *Let $f : (X, \tau, E) \to (Y, \sigma, K)$ and $g : (Y, \sigma, K) \to (Z, \gamma, T)$ be two functions. Then*

(1) *$g \circ f : X \to Z$ is soft λ-continuous, iff f is soft λ-continuous and g is soft continuous.*
(2) *$g \circ f : X \to Z$ is soft λ-irresolute, iff f and g are soft λ-irresolute.*
(3) *$g \circ f : X \to Z$ is soft λ-continuous, iff f is soft λ-irresolute and g is softλ-continuous.*

Proof. (1) Consider (F, K) a soft closed set in Z. Since $g : (Y, \sigma, K) \to (Z, \gamma, T)$ is soft continuous, by definition $g^{-1}((F, K))$ is soft closed in Y. Now $f : (X, \tau, E) \to (Y, \sigma, K)$ is soft λ-continuous and $g^{-1}((F, K))$ is soft closed set in Y, then $f^{-1}(g^{-1}((F, K))) = (g \circ f)^{-1}((F, K))$ is soft λ-closed in X. Hence $g \circ f : X \to Z$ is soft λ-continuous.

(2) Let $g : (Y, \sigma, K) \to (Z, \gamma, T)$ be soft λ-irresolute and let (F, K) be a soft λ-closed set in Z. Since g is softλ-irresolute, $g^{-1}((F, K))$ is softλ-closed in Y. Also f is soft λ-irresolute. So $f^{-1}(g^{-1}((F, K))) = (g \circ f)^{-1}((F, K))$ is soft λ-closed in X. Hence $g \circ f : X \to Z$ is soft λ-irresolute.

(3) Consider (F, K) a soft open set in Z. Since $g : (Y, \sigma, K) \to (Z, \gamma, T)$ is soft λ-continuous, $g^{-1}((F, K))$ is soft λ-open in Y. Now $f : (X, \tau, E) \to (Y, \sigma, K)$ is soft λ-irresolute and $g^{-1}((F, K))$ is soft λ-open set in Y, then $f^{-1}(g^{-1}((F, K))) = (g \circ f)^{-1}((F, K))$ is soft λ-open in X. Hence $g \circ f : X \to Z$ is soft λ-continuous.

□

Definition 4.6. *A soft mapping $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be soft λ-open map if the image of every soft open set in X is soft λ-open in Y.*

Definition 4.7. *A soft mapping $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be soft λ-closed map if the image of every soft closed set in X is soft λ-closed in Y.*

Theorem 4.5. *If $f : (X, \tau, E) \to (Y, \sigma, K)$ is soft closed function and $g : (Y, \sigma, K) \to (Z, \gamma, T)$ is soft λ-closed function, then $g \circ f$ is soft λ-closed function.*

Proof. Let (F, K) be a soft closed set in X. Then, $f((F, K))$ is soft closed in Y. Subsequently, $g : (Y, \sigma, K) \to (Z, \gamma, T)$ is soft λ-closed function, $g(f((F, K)))$ is soft λ-closed in Z. $g(f((F,K))) = (g \circ f)((F,K))$ is soft λ-closed in Z. Therefore, $g \circ f$ is soft λ-closed function. $\square$

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