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# CONTRA SUPRA $G^{\#}\alpha$-CONTINUOUS FUNCTION IN SUPRA TOPOLOGICAL SPACES

N. R. BHUVANESWARI [1] AND V. KOKILAVANI

ABSTRACT. In this paper, we introduce the concept of contra supra $g^{\#}\alpha$ -continuous functions and contra supra $g^{\#}\alpha$-irresolute function. We obtain the basic properties and their relationship with other forms of contra supra continuous functions in supra topological spaces.

## 1. INTRODUCTION

In 1983, A.S.Mashhour et al [5] introduced the supra topological spaces and studied continuous functions and $s*$-continuous functions. In 1996, Dontchev [4] presented a new notion of continuous function called contra-continuity in topological spaces.

The purpose of this paper is to introduce the concept of contra supra $g^{\#}\alpha$ -continuous functions and contra supra $g^{\#}\alpha$-irresolute and study its basic properties. Also we defined almost contra supra $g^{\#}\alpha$ -continuous function, perfectly contra supra $g^{\#}\alpha$ -irresolute function and investigated their relationship to other functions in supra topological spaces.

[1] *corresponding author*

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## 2. PRELIMINARIES

**Definition 2.1.** *[5] A subfamily $\mu$ of $X$ is said to be a supra topology on $X$, if*

(1) $X$, $\emptyset \in \mu$
(2) *If $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$.*

The pair $(X, \mu)$ is called supra topological space. The elements of $\mu$ are called supra open sets in $(X, \mu)$ and complement of a supra open set is called a supra closed set.

**Definition 2.2.** *[5]*

(1) *The supra closure of a set $A$ is denoted by $cl^\mu(A)$ and is defined as $cl^\mu(A) = \cap\{B : B$ is a supra closed set and $A \subseteq B\}$.*
(2) *The supra interior of a set $A$ is denoted by $int^\mu(A)$ and defined as $int^\mu(A) = \cup\{B : B$ is a supra open set and $A \supseteq B\}$.*

**Definition 2.3.** *[5] Let $(X, \tau)$ be a topological space and $\mu$ be a supra topology on $X$. We call $\mu$ a supra topology associated with $\tau$ if $\tau \subset \mu$.*

**Definition 2.4.** *[1] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $\alpha$-open set if $A \subseteq int^\mu(\mathrm{cl}^\mu(\mathrm{int}^\mu(A)))$ . The complement of supra $\alpha$-open set is supra $\alpha$-closed set.*

**Definition 2.5.** *[6] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $g^*$-closed set if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is supra $g$-open set of $X$. The complement of supra $g$-closed set is supra $g$-open set.*

**Definition 2.6.** *[6] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called a supra $g^\#$-closed set if $\mathrm{cl}^\mu(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha g$-open set of $X$.*

**Definition 2.7.** *[4] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $g^\#\alpha$-closed set if $\alpha cl^\mu(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is supra $g$-open set of $X$. The complement of supra $g^\#\alpha$- closed set is called supra $g^\#\alpha$-open set.*

**Definition 2.8.** *Let $(X, \mu)$ and $(Y, \sigma)$ be two topological spaces and $\tau \subset \mu$. A map $f : (X, \tau) \to (Y, \sigma)$ is called*

(1) *supra continuous if the inverse image of each open set of $Y$ is a supra open set in $X$ [5].*

(2) *supra $\alpha$-continuous if the inverse image of each open set of $Y$ is a supra $\alpha$-open set in $X$ [1].*
(3) *supra $g$-continuous if the inverse image of each closed set of $Y$ is a supra $g$-closed set in $X$ [3].*
(4) *supra $g^{\#}$-continuous if the inverse image of each closed set of $Y$ is a supra $g^{\#}$-closed set in $X$ [6].*

**Definition 2.9.** *Let $(X, \mu)$ and $(Y, \sigma)$ be two topological spaces and $\tau \subset \mu$. A map $f : (X, \tau) \to (Y, \sigma)$ is called*

(1) *supra closed if the image of each closed set of $X$ is a supra closed set in $Y$ [5].*
(2) *supra $\alpha$-closed if the image of each closed set of $X$ is a supra $\alpha$-closed set in $Y$ [1].*
(3) *supra $g$-closed if the image of each closed set of $X$ is a supra $g$-closed set in $Y$ [3].*
(4) *supra $g^{\#}$-closed if the image of each closed set of $X$ is a supra $g^{\#}$-closed set in $Y$ [6].*

**Definition 2.10.** *Let $(X, \mu)$ and $(Y, \sigma)$ be two topological spaces and $\tau \subset \mu$. A map $f : (X, \tau) \to (Y, \sigma)$ is called*

(1) *supra irresolute if $f^{-1}(V)$ is supra closed in $X$ for every supra closed set $V$ of $Y$ [5].*
(2) *supra $\alpha$-irresolute if $f^{-1}(V)$ is supra $\alpha$-closed in $X$ for every supra $\alpha$-closed set $V$ of $Y$ [1].*
(3) *supra $g$-irresolute if $f^{-1}(V)$ is supra $g^{\#}$-closed in $X$ for every supra $g^{\#}$-closed set $V$ of $Y$ [3].*
(4) *supra $g^{\#}$-irresolute if $f^{-1}(V)$ is supra $g^{\#}$-closed in $X$ for every supra $g^{\#}$-closed set $V$ of $Y$ [6].*

**Definition 2.11.** *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(1) *Supra $g^{\#}\alpha$-continuous if $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X, \tau)$ for every supra closed set $V$ of $(Y, \sigma)$ [4].*
(2) *Supra $g^{\#}\alpha$-irresolute if $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X, \tau)$ for every supra $g^{\#}\alpha$-closed set $V$ of $(Y, \sigma)$ [4].*
(3) *Contra continuous if $f^{-1}(V)$ is closed in $(X, \tau)$ for every open set $V$ of $(Y, \sigma)$ [2].*

**Definition 2.12.** *[2] A function $f : (X, \tau) \to (Y, \sigma)$ is called contra-continuous functions if $f^{-1}(V)$ is supra-closed in $(X, \tau)$ for every supra open set $V$ of $(Y, \sigma)$.*

## 3. Contra supra $g^{\#}\alpha$-continuous function

**Definition 3.1.** *A function $f : (X, \tau) \to (Y, \sigma)$ is called contra supra $g^{\#}\alpha$-continuous function if $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X, \tau)$ for every supra open set $V$ of $(Y, \sigma)$.*

**Theorem 3.1.** *Every contra continuous function is contra supra $g^{\#}\alpha$-continuous.*

*Proof.* Let $f : X \to Y$ be contra continuous. Let $V$ be any supra open in $Y$. Then the inverse image $f^{-1}(V)$ is supra closed in $X$. Since every supra closed is supra $g^{\#}\alpha$-closed, $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $X$. Therefore $f$ is contra supra $g^{\#}\alpha$-continuous. □

**Remark 3.1.** *The converse of the above theorem is not true and it is shown by the following example.*

**Example 1.** *Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Here $f$ is contra supra $g^{\#}\alpha$-continuous function and not contra continuous. Since $V = \{a\}$ is supra open set in $(Y, \sigma)$, $f^{-1}(\{a\}) = \{a\}$ is not supra closed in $(X, \tau)$.*

**Remark 3.2.** *The composition of two contra supra $g^{\#}\alpha$-continuous mappings need not be contra supra $g^{\#}\alpha$-continuous. Let us prove the remark by the following example.*

**Example 2.** *Let $X = Y = \{a, b, c\}$. Let $\tau = \{X, \emptyset, \{b\}, \{a, b\}\}$, $\sigma = \{Y, \emptyset, \{a\}\}$ and $\nu = \{Z, \emptyset, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \nu)$. Define $f(a) = a$, $f(b) = b$, $f(c) = c$ and $g(a) = c$, $g(b) = b$, $g(c) = a$. Both $f$ and $g$ are contra supra $g^{\#}\alpha$-continuous. Define $g \circ f : (X, \tau) \to (Z, \nu)$. Hence $\{b\}$ is a supra open set of $(Z, \nu)$. Therefore $(g \circ f)^{-1} = (g \circ f)^{-1}(\{b\}) = g^{-1}(f^{-1}(\{b\})) = g^{-1}(\{b\}) = \{b\}$ is not a supra $g^{\#}\alpha$-closed set of $(X, \tau)$. Hence $(g \circ f)$ is not contra supra $g^{\#}\alpha$-continuous.*

**Theorem 3.2.** *If $f : (X, \tau) \to (Y, \sigma)$ is contra supra $g^{\#}\alpha$-continuous function and $g : (Y, \sigma) \to (Z, \nu)$ is supra continuous function then the composition $(g \circ f)$ is contra supra $g^{\#}\alpha$-continuous function.*

*Proof.* Let $V$ be supra open set in $Z$. Since $g$ is supra continuous, then $g^{-1}(V)$ is supra open in $Y$. Since $f$ is contra supra $g^{\#}\alpha$-continuous function, then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $X$. Therefore $(g \circ f)$ is contra supra $g^{\#}\alpha$-continuous function. □

**Theorem 3.3.** *If $f : (X, \tau) \to (Y, \sigma)$ is supra $g^{\#}\alpha$- irresolute function and $g : (Y, \sigma) \to (Z, \nu)$ is contra supra $g^{\#}\alpha$-continuous function then the composition $(g \circ f)$ is contra supra $g^{\#}\alpha$-continuous function.*

*Proof.* Let $V$ be supra open set in $Z$. Since $g$ is contra supra $g^{\#}\alpha$-continuous function, then $g^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $Y$. Since $f$ is supra $g^{\#}\alpha$-irresolute function, then $f^{-1}(g^{-1}(V))$ is supra $g^{\#}\alpha$-closed in $X$. Therefore $(g \circ f)$ is contra supra $g^{\#}\alpha$-continuous function. □

**Remark 3.3.** *The concept of supra $g^{\#}\alpha$-continuity and contra supra $g^{\#}\alpha$-continuity are independent as shown in the following example.*

**Example 3.** *Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. $f : (X, \tau) \to (Y, \sigma)$ be the function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Here $f$ is contra supra $g^{\#}\alpha$-continuous but not supra $g^{\#}\alpha$-continuous function, since $V = \{b, c\}$ is supra closed set in $Y$ but $f^{-1}(\{b, c\}) = \{a, b\}$ is not supra $g^{\#}\alpha$-closed set in $X$.*

**Theorem 3.4.** *If $f : (X, \tau) \to (Y, \sigma)$ is contra supra $g^{\#}\alpha$-continuous function and $X$ supra $g^{\#}\alpha Tc$ is -space, then $f$ is contra supra continuous.*

*Proof.* Let $V$ be supra open set in $Y$. Since $f$ is contra supra $g^{\#}\alpha$-continuous function, then $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $X$. Since $X$ is supra $g^{\#}\alpha Tc$-space, we have every supra $g^{\#}\alpha$-closed set is supra closed in $X$, then $f^{-1}(V)$ is supra closed in $X$. Therefore $f$ is contra supra continuous function. □

**Definition 3.2.** *A map $f : (X, \tau) \to (Y, \sigma)$ is called almost contra supra $g^{\#}\alpha$-continuous function if $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X, \tau)$ for every supra regular open set $V$ in $(Y, \sigma)$.*

**Theorem 3.5.** *Every contra supra continuous function is almost contra supra $g^{\#}\alpha$-continuous function.*

*Proof.* Let $f : (X, \tau) \to (Y, \sigma)$ be a contra supra continuous function. Let $V$ be a supra regular open set in $(Y, \sigma)$. We know that every supra regular open set is supra open, then $V$ is supra open in $(Y,\sigma)$. Since $f$ is contra supra continuous function, $f^{-1}(V)$ is supra closed in $(X,\tau)$. We know that every supra closed set is supra $g^{\#}\alpha$-closed, which implies $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X,\tau)$. Therefore $f$ is almost contra supra $g^{\#}\alpha$-continuous function. $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.** *Let $X = Y = \{a,b,c\}$ and $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. $f : (X,\tau) \to (Y,\sigma)$ be the identity. Here $f$ is almost contra supra $g^{\#}\alpha$-continuous but it is not contra supra continuous function, since $V = \{a\}$ is supra open in $Y$ but $f^{-1}(\{a\}) = \{a\}$ is not supra closed set in $X$.*

**Theorem 3.6.** *Every contra supra $g^{\#}\alpha$-continuous function is almost contra supra $g^{\#}\alpha$-continuous function.*

*Proof.* Let $f : (X,\tau) \to (Y,\sigma)$ be a contra supra $g^{\#}\alpha$-continuous function. Let $V$ be a supra regular open set in $(Y,\sigma)$. We know that every supra regular open set is supra open, then $V$ is supra open in $(Y,\sigma)$. Since $f$ is contra supra $g^{\#}\alpha$-continuous function, $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X,\tau)$. Therefore $f$ is almost contra supra $g^{\#}\alpha$-continuous function. $\square$

The converse of the above theorem need not be true. It is shown by the following example.

**Example 5.** *Let $X = Y = \{a,b,c\}$ and $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{a,b\}\}$. $f : (X,\tau) \to (Y,\sigma)$ be the identity function. Here $f$ is almost contra supra $g^{\#}\alpha$-continuous, but it is not contra supra $g^{\#}\alpha$-continuous, since $V = \{a\}$ is open in $Y$ but $f^{-1}(\{a\}) = \{a\}$ is not supra $g^{\#}\alpha$ closed in $X$.*

**Definition 3.3.** *A Space $(X,\tau)$ is supra $g^{\#}\alpha$-locally indiscrete if every supra $g^{\#}\alpha$-open (supra $g^{\#}\alpha$-closed) set is supra closed (supra open) in $(X,\tau)$.*

**Theorem 3.7.** *If $f : (X,\tau) \to (Y,\sigma)$ is supra $g^{\#}\alpha$-continuous function and $X$ is supra $g^{\#}\alpha$-locally indiscrete, then $f$ is contra supra $g^{\#}\alpha$-continuous.*

*Proof.* Let $V$ be supra open set in $Y$. Since $f$ is supra $g^{\#}\alpha$-continuous function, then $f^{-1}(V)$ is supra $g^{\#}\alpha$-open in $X$. Since $X$ is supra $g^{\#}\alpha$-locally indiscrete, then $f^{-1}(V)$ is supra closed set in $X$. We know that every supra closed set is supra $g^{\#}\alpha$-closed set . Therefore $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed set in $X$. Hence $f$ is contra supra $g^{\#}\alpha$-continuous function. $\square$

**Theorem 3.8.** *Let $f : (X,\tau) \to (Y,\sigma)$ be a surjective supra $g^{\#}\alpha$-irresolute. $g : (Y,\sigma) \to (Z,\nu)$ is a function such that $(g \circ f) : (X,\tau) \to (Z,\nu)$ is contra supra $g^{\#}\alpha$-continuous function, iff $g$ is contra supra $g^{\#}\alpha$ -continuous.*

*Proof.* Suppose $(g \circ f)$ is contra supra $g^{\#}\alpha$-continuous. Let $V$ be a supra closed set in $Z$, then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is supra $g^{\#}\alpha$-open in $(X,\tau)$. Since $f$ is surjective and supra $g^{\#}\alpha$-irresolute , then $f((g \circ f)^{-1}) = f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is supra $g^{\#}\alpha$-open in $(Y,\sigma)$. Hence $g$ is contra supra $g^{\#}\alpha$-continuous function.

Conversely, suppose $g$ is contra supra $g^{\#}\alpha$-continuous. Let $V$ be supra closed in $Z$, then $g^{-1}(V)$ is supra $g^{\#}\alpha$-open in $Y$. Since $f$ is surjective and supra $g^{\#}\alpha$-irresolute, then $f^{-1}(g^{-1}(V))$ is supra $g^{\#}\alpha$-open in $X$. Hence $(g \circ f)$ is contra supra $g^{\#}\alpha$-continuous function. □

**Theorem 3.9.** *If $f : (X,\tau) \to (Y,\sigma)$ is a supra $g^{\#}\alpha$ -continuous and $g : (Y,\sigma) \to (Z,\nu)$ is contra supra $g^{\#}\alpha$ -continuous function and $(Y,\sigma)$ is supra $g^{\#}\alpha Tc$-space, then $(g \circ f) : (X,\tau) \to (Z,\nu)$ is contra supra $g^{\#}\alpha$-continuous function.*

*Proof.* Let $V$ be any supra open set in $Z$, then $g^{-1}(V)$ is supra $g^{\#}\alpha$-closed set in $Y$. Since $Y$ is supra $g^{\#}\alpha Tc$-space, $g^{-1}(V)$ is supra closed set in $Y$. Since $f$ is supra $g^{\#}\alpha$-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra $g^{\#}\alpha$-closed set in $X$. Hence $(g \circ f)$ is contra supra $g^{\#}\alpha$-continuous. □

## 4. Contra supra $g^{\#}\alpha$- irresolute function

**Definition 4.1.** *A function $f : (X,\tau) \to (Y,\sigma)$ is called contra supra $g^{\#}\alpha$-irresolute function if $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X,\tau)$ for every supra $g^{\#}\alpha$-open set $V$ in $(Y,\tau)$.*

**Definition 4.2.** *A function $f : (X,\tau) \to (Y,\sigma)$ is called perfectly contra supra $g^{\#}\alpha$-irresolute is function if $f^{-1}(V)$ supra $g^{\#}\alpha$-closed and supra $g^{\#}\alpha$-open in $(X,\tau)$ for every supra $g^{\#}\alpha$-open set $V$ in $(Y,\sigma)$.*

**Theorem 4.1.** *Every contra supra $g^{\#}\alpha$-irresolute function is contra supra $g^{\#}\alpha$-continuous.*

*Proof.* Let $f : (X,\tau) \to (Y,\sigma)$ be a contra supra $g^{\#}\alpha$- irresolute function. Let $V$ be a supra open set in $(Y,\sigma)$. We know that every supra open set is supra

$g^{\#}\alpha$-open set, then $V$ is supra $g^{\#}\alpha$-open in $(Y,\sigma)$. Since $f$ is contra supra $g^{\#}\alpha$-irresolute function, $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed in $(X,\tau)$. Therefore $f$ is contra supra $g^{\#}\alpha$-continuous function. □

The converse of the above theorem need not be true. It is shown by the following example.

**Example 6.** *Let $X = Y = \{a,b,c\}$, $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. A function $f : (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Here $f$ is contra supra $g^{\#}\alpha$-continuous but not contra supra $g^{\#}\alpha$-irresolute. Since $V = \{b,c\}$ is supra $g^{\#}\alpha$-open set in $(Y,\sigma)$ and $f^{-1}(\{b,c\}) = \{a,b\}$ is not in supra $g^{\#}\alpha$-closed set $in(X,\tau)$.*

**Theorem 4.2.** *If $f : (X,\tau) \to (Y,\sigma)$ is a supra $g^{\#}\alpha$-irresolute and $g : (Y,\sigma) \to (Z,\nu)$ is contra supra $g^{\#}\alpha$-irresolute function, then $(g \circ f) : (X,\tau) \to (Z,\nu)$ is contra supra $g^{\#}\alpha$-irresolute function.*

*Proof.* Let $V$ be any supra $g^{\#}\alpha$-open set in $Z$. Since $g$ is contra supra $g^{\#}\alpha$-irresolute then $g^{-1}(V)$ is supra $g^{\#}\alpha$-closed set in $Y$. Since $f$ is supra $g^{\#}\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra $g^{\#}\alpha$-closed set in $X$. Hence $(g \circ f)$ is contra supra $g^{\#}\alpha$-irresolute function. □

**Theorem 4.3.** *If $f : (X,\tau) \to (Y,\sigma)$ is a contra supra $g^{\#}\alpha$-irresolute and $g : (Y,\sigma) \to (Z,\nu)$ is supra $g^{\#}\alpha$-irresolute function, then $(g \circ f) : (X,\tau) \to (Z,\nu)$ is contra supra $g^{\#}\alpha$-irresolute function.*

*Proof.* Let $V$ be any supra $g^{\#}\alpha$-open set in $Z$. Since $g$ is supra $g^{\#}\alpha$-irresolute then $g^{-1}(V)$ is supra $g^{\#}\alpha$-open set in $Y$. Since $f$ is contra supra $g^{\#}\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra $g^{\#}\alpha$-closed set in $X$. Hence $(g \circ f)$ is contra supra $g^{\#}\alpha$-irresolute function. □

**Theorem 4.4.** *Every perfectly contra supra $g^{\#}\alpha$-irresolute is contra supra $g^{\#}\alpha$-irresolute function.*

*Proof.* Let $f : (X,\tau) \to (Y,\sigma)$ be a perfectly contra supra $g^{\#}\alpha$-irresolute function. Let $V$ be a supra $g^{\#}\alpha$-open set in $(Y,\sigma)$. Since $f$ is perfectly contra supra $g^{\#}\alpha$-irresolute function, $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed and supra $g^{\#}\alpha$-open in $(X,\tau)$. Therefore $f$ is contra supra $g^{\#}\alpha$-irresolute function. □

The converse of the above theorem need not be true. It is shown by the following example.

**Example 7.** *Let* $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{Y, \emptyset, \{b\}, \{b, c\}, \{a, b\}\}$ *and let* $f : (X, \tau) \to (Y, \sigma)$ *be a function defined by* $f(a) = a$, $f(b) = c$, $f(c) = b$. *Here* $f$ *is contra supra* $g^{\#}\alpha$*-irresolute function but not perfectly contra supra* $g^{\#}\alpha$*-irresolute function. Since* $V = \{a, c\}$ *is supra* $g^{\#}\alpha$*-open set in* $(Y, \sigma)$, $f^{-1}(\{a, c\}) = \{a, b\}$ *is not supra* $g^{\#}\alpha$*-closed and supra* $g^{\#}\alpha$*-open set in* $(X, \tau)$.

**Theorem 4.5.** *Every perfectly contra supra* $g^{\#}\alpha$*-irresolute is contra supra* $g^{\#}\alpha$*-irresolute function.*

*Proof.* Let $f : (X, \tau) \to (Y, \sigma)$ be a perfectly contra supra $g^{\#}\alpha$-irresolute function. Let $V$ be a supra $g^{\#}\alpha$-open set in $(Y, \sigma)$. Since $f$ is perfectly contra supra $g^{\#}\alpha$-irresolute function, $f^{-1}(V)$ is supra $g^{\#}\alpha$-closed and supra $g^{\#}\alpha$-open in $(X, \tau)$. Therefore $f$ is supra $g^{\#}\alpha$-irresolute function. □

The converse of the above theorem need not be true. It is shown by the following example.

**Example 8.** *Let* $X = Y = \{a, b, c\}$ *and* $\tau = \{X, \emptyset, \{a\}\}$, $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}$, $f : (X, \tau) \to (Y, \sigma)$ *be a identity function. Here* $f$ *is supra* $g^{\#}\alpha$*-irresolute function but not perfectly contra supra* $g^{\#}\alpha$*-irresolute function. Since* $V = \{a, c\}$ *is supra* $g^{\#}\alpha$*-open set in* $(Y, \sigma)$, $f^{-1}(\{a, c\}) = \{a, c\}$ *is not supra* $g^{\#}\alpha$*-closed and supra* $g^{\#}\alpha$*-open set in* $(X, \tau)$.

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Department of Mathematics
SNS College of Technology
Coimbatore, (TN), India
*E-mail address*: ramg.phd@gmail.com

Department of Mathematics
Kongunadu Arts and Science College
Coimbatore, (TN), India