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# NEUTROSOPHIC WEAKLY $G^*$-CLOSED SETS

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ABSTRACT. In this paper are presented and explored new sort of Neutrosophic closed set which is called Neutrosophic feebly $g^*$-closed sets in NTSs and furthermore talked about properties and portrayal.

## 1. INTRODUCTION

A. Salama presented NTSs in [11, 12] by utilizing Smarandache's NSs, [5, 6]. Neutrosophic $g$ closed set presented by R. Dhavasheelan et al. in [3, 4], what's more, Neutrosophic $g^*$-closed sets introduced by A. Atkinswesley et al. in [2]. Point of this current paper is, to present and research about new sort of Neutrosophic closed set is called Neutrosophic weakly $g^*$-closed sets in Neutrosophic topological spaces and furthermore examined about properties and portrayal.

## 2. PRELIMINARIES

In this part, we review required results of Neutrosophic.

**Definition 2.1.** *[4] A Neutrosophic set $W_1^*$ is in the form*

$$W_1^* = \{< r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) >: r \in Nu_X^*\},$$

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*where* $\mu_{W_1^*}(w)$ *denotes membership function,* $\sigma_{W_1^*}(w)$ *denotes indeterminacy and* $\gamma_{W_1^*}(w)$ *denotes non-membership function.*

**Definition 2.2.** *[4] Neutrosophic set is the set*

$$W_1^* = \{< r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) >: r \in Nu_X^*\},$$

*on* $Nu_X^*$ *and* $\forall w \in Nu_X^*$. *Then complement of* $W_1^*$ *is*

$$W_1^{*C} = \{< r, \gamma_{W_1^*}((w)), 1 - \sigma_{W_1^*}(w), \mu_{W_1^*}(w) >: r \in Nu_X^*\}.$$

**Definition 2.3.** *[4] Let* $W_1^*$ *and* $W_2^*$ *are two NSs,*
$\forall w \in Nu_X^*$ $W_1^* = \{< r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) >: r \in Nu_X^*\}$
$W_2^* = \{< r, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) >: r \in Nu_X^*\}$.
*Then* $W_1^* \subseteq W_2^* \Leftrightarrow \mu_{W_1^*}(w) \leq \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \leq \sigma_{W_2^*}(w) \, \gamma_{W_1^*}(w) \geq \gamma_{W_2^*}(w)$.

**Definition 2.4.** *[4] Let* $W_1^*$ *and* $W_2^*$ *be two NSs are*
$W_1^* = \{< r, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) >: r \in Nu_X^*\}$,
$W_2^* = \{< r, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) >: r \in Nu_X^*\}$. *Then*
$W_1^* \cap W_2^* = \{< r, \mu_{W_1^*}(w) \cap \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cap \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cup \gamma_{W_2^*}(w) >: r \in Nu_X^*\}$
$W_1^* \cup W_2^* = \{< r, \mu_{W_1^*}(w) \cup \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cup \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cap \gamma_{W_2^*}(w) >: r \in Nu_X^*\}$.

**Definition 2.5.** *[10] Let* $Nu_X^*$ *be non-empty set and* $\mathrm{NS}_\tau$ *be the collection of Neutrosophic subsets of* $Nu_X^*$ *satisfying the accompanying properties:*

(1) $0_{Nu}, 1_{Nu} \in \mathrm{NS}_\tau$
(2) $Nu_{T_1} \cap Nu_{T_2} \in \mathrm{NS}_\tau$ *for any* $Nu_{T_1}, Nu_{T_2} \in Nu_\tau$
(3) $\cup Nu_{T_i} \in NS_\tau$ *for every* $Nu_{T_i} : i \in j \subseteq Nu_\tau$ .

*Then the space* $(Nu_X^*, \mathrm{NS}_\tau)$*, is called a* $NTS(NS - T - S)$. *The component of* $\mathrm{NS}_\tau$ *are called NS-OS (Neutrosophic open set) and its complement is NS-CS(Neutrosophic closed set)*

**Example 1.** *Let* $Nu_X^* = \{w\}$ *and* $\forall w \in Nu_X^*$
$W_1^* = \left\langle w, \frac{6}{10}, \frac{6}{10}, \frac{5}{10} \right\rangle, W_2^* = \left\langle w, \frac{5}{10}, \frac{7}{10}, \frac{9}{10} \right\rangle$
$W_3^* = \left\langle w, \frac{6}{10}, \frac{7}{10}, \frac{5}{10} \right\rangle, W_4^* = \left\langle w, \frac{5}{10}, \frac{6}{10}, \frac{9}{10} \right\rangle$
*Then collection* $\mathrm{NS}_\tau = \{0_{Nu}, W_1^*, W_2^*, W_3^*, W_4^* 1_{Nu}\}$ *is called a NS-T-S on* $Nu_X^*$.

**Definition 2.6.** *[10] Let* $(Nu_X^*, Nu._\tau)$ *be NTS., Then Neutrosophic closure of* $W_1^*$ *is Nu-cl*$(W_1^*) = \cap\{K : K$ *is a Neutrosophic closed set in* $Nu_X^* and W_1^* \subseteq K\}$ .

*Neutrosophic interior of $W_1^*$ is:*
*Nu-int*$(W_1^*) = \cup\{\ G_1^* :\ G_1^*$ *is a Neutrosophic open set in* $Nu_X^*$ *and* $G_1^* \subseteq\ W_1^*\}$.

**Definition 2.7.** *Let* $(Nu_X^*, Nu._\tau)$ *be a NTS. Then* $W_1^*$ *is called*

(1) *Neutrosophic regular Closed set (Neu-RCS ) if* $W_1^* =$ *Neu-Cl(Neu-Int*$(W_1^*)$), *[1];*
(2) *Neutrosophic $\alpha$-Closed set (Neu-$\alpha$ CS ) if Neu-Cl(Neu-Int(Neu-Cl*$(W_1^*))) \subseteq W_1^*$, *[1];*
(3) *Neutrosophic semi Closed set (Neu-SCS ) if Neu-Int(Neu-Cl*$(W_1^*)) \subseteq\ W_1^*$, *[7];*
(4) *Neutrosophic pre Closed set (Neu-PCS ) if Neu-Cl(Neu-Int*$(W_1^*)) \subseteq\ W_1^*$, *[15];*

**Definition 2.8.** *Let (Nu*$_X^*, Nu._\tau$) *be a NTS. Then* $W_1^*$ *is called:*

(1) *Neutrosophic (regular open) set Neu-ROS ) if* $W_1^* =$ *Neu-Int(Neu-Cl*$(W_1^*)$), *[1];*
(2) *Neutrosophic ($\alpha$-open )set (Neu-$\alpha$ OS ) if* $W_1^* \subseteq$ *Neu-Int(Neu-Cl(Neu-Int*$(W_1^*)$))), *[1];*
(3) *Neutrosophic (semi open) set (Neu-SOS) if* $W_1^* \subseteq$ *Neu-Cl(Neu-Int*$(W_1^*)$), *[7];*
(4) *Neutrosophic (pre open) set (Neu-POS) if* $W_1^* \subseteq$ *Neu-Int(Neu-Cl*$(W_1^*)$), *[15].*

**Definition 2.9.** *A Neutrosophic set* $W_1^*$ *of a NTS (Nu*$_X^*, Nu_\tau$) *is called*

(1) *Neutrosophic(g-closed) if Nu-cl*$(W_1^*) \subseteq G_1^*$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic open, [3];*
(2) *Neutrosophic (sg-closed) if Nu-(S)Cl*$(W_1^*) \subseteq G_1^*$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic semi open, [14];*
(3) *Neutrosophic ($g^*$-closed) if Nu-cl*$(W_1^*) \subseteq G_1^*$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic g-open, [2];*
(4) *Neutrosophic ($\alpha g$-closed) if Nu-($\alpha$)cl*$(W_1^*) \subseteq G$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic - open, [8];*
(5) *Neutrosophic ($g\alpha$-closed) if Nu-($\alpha$)cl*$(W_1^*) \subseteq G_1^*$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic $\alpha$- open, [4];*
(6) *Neutrosophic (w-closed) if Nu-cl*$(W_1^*) \subseteq G$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic semi open, [13];*

(7) *Neutrosophic (gP-closed) if Nu-(P)Cl*$(W_1^*) \subseteq G_1^*$ *whenever* $W_1^* \subseteq G$ *and* $G_1^*$ *is Neutrosophic open, [9];*
(8) *Neutrosophic (gs-closed) if Nu-(S)Cl*$(W_1^*) \subseteq G_1^*$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic open, [14].*

The complements of the above mentioned closed set are their respective open sets.

**Definition 2.10.** *[3] If* $W_1^*$ *is a Neutrosophic set in NTS(Nu*$_X^*, Nu_\tau)$ *then*

(1) *Nu-(S)Cl*$(W_1^*) = \cap\{F_1^* : W_1^* \subseteq F_1^*, F_1^*$ *is Nu(S)CS*$\}$.
(2) *Nu-(P)Cl*$(W_1^*) = \cap\{F_1^* : W_1^* \subseteq F_1^*, F_1^*$ *is Nu(P)CS*$\}$.
(3) *Nu-*$(\alpha)$*cl*$(W_1^*) = \cap\{F_1^* : W_1^* \subseteq F_1^*, F_1^*$ *is Nu*$(\alpha)$*CS*$\}$.

**Remark 2.1.** (1) *Every NuCS is Nu(g)CS.*
(2) *Every Nu*$(\alpha)$*CS is Nu*$(\alpha$ *g)CS.*
(3) *Every Nu(g)CS is Nu(g*$\alpha$*)CS.*
(4) *Every Nu*$(\alpha$*g)CS is Nu(g*$\alpha$*)CS.*
(5) *Every Nu(w)CS is Nu(g)CS.*
(6) *Every Nu(w)CS is Nu(sg)CS.*
(7) *Every Nu(sg)CS is Nu(gs)CS.*

**Lemma 2.1.** *[7] Let* $W_1^*$ *and* $W_2^*$ *be any two NSs of a NTS (Nu*$_X^*, Nu_\tau$*). Then:*

(a) $W_1^*$ *is a NuCS in Nu*$_X^*$ $\Leftrightarrow$ *Nu-cl*$(W_1^*) = W_1^*$
(b) $W_1^*$ *is a NuOS in Nu*$_X^*$ $\Leftrightarrow$ *Nu-int*$(W_1^*) = W_1^*$.
(c) *Nu-cl*$(W_1^{*c})$ =*(Nu-int*$(W_1^*))C$.
(d) *Nu-int*$(W_1^{*c})$ =*(Nu-cl*$(W_1^*))C$.
(e) $W_1^* \subseteq W_2^* \Rightarrow$ *Nu-int*$(W_1^*) \subseteq$ *Nu-int*$(W_2^*)$.
(f) $W_1^* \subseteq W_2^* \Rightarrow$ *Nu-cl*$(W_1^*) \subseteq$ *Nu-cl*$(W_2^*)$.
(g) *Nu-cl*$(W_1^* \cup W_2^*)$ = *Nu-cl*$(W_1^*)\cup$ *Nu-cl*$(W_2^*)$.
(h) *Nu-int*$(W_1^* \cap W_2^*)$ = *Nu-int*$(W_1^*)\cap$ *Nu-int*$(W_2^*)$.

## 3. NEUTROSOPHIC WEAKLY $g^*$-CLOSED

**Definition 3.1.** *A Neutrosophic set* $W_1^*$ *of a NTS(Nu*$_X^*, Nu_\tau)$ *is called Nu(*$wg^*$*)CS Neutrosophic weakly* $g^*$*-closed if Nu-cl(Nu-int(*$W_1^*$*))* $\subseteq G_1^*$ *whenever* $W_1^* \subseteq G_1^*$ *and* $G_1^*$ *is Neutrosophic g-open in Nu*$_X^*$.

**Theorem 3.1.** *Every Nu(w)CS set is Nu(wg$^*$)CS.*

*Proof.* Let $W_1^*$ is Nu(w)CS. Let $W_1^* \subseteq H_1^*$ and $H_1^*$ Nu(S)OS in Nu$_X^*$. Since every Nu(S)OS is Nu(g)OS H$_1^*$ is Nu(g)OS. using definition Nu(w)CS Nu-cl$(W_1^*) \subseteq H_1^*$. But Nu-cl(Nu-int$(W_1^*)) \subseteq$ Nu-cl$(W_1^*) \subseteq H_1^*$. We have Nu-cl(Nu-int$(W_1^*)) \subseteq H_1^*$ whenever $W_1^* \subseteq H_1^*$ and $H_1^*$ is Nu(g)OS in Nu$_X^*$. Therefore $W_1^*$ is Nu(w$g^*$)CS. □

**Remark 3.1.** *Every Nu(wg$^*$)CS is not Nu(w)CS set.*

**Example 2.** *Let Nu$_X^*$ $= \{a, b\}$ and $Nu_\tau = \{0_{Nu}, W_1^*, 1_{Nu}\}$ is Neutrosophic topology on Nu$_X^*$, where $W_1^* = \left\langle w, \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right)\right\rangle$. Then $W_2^* = \left\langle w, \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right)\right\rangle$ is Nu(wg$^*$)CS but not Nu(w)CS.*

**Theorem 3.2.** *Every Nu(g$^*$)CS is Nu(wg$^*$)CS.*

*Proof.* Let $W_1^*$ is Nu($g^*$)CS. Let $W_1^* \subseteq H_1^*$ and $H_1^*$ is Nu(g)OS in Nu$_X^*$. using definition Nu($g^*$)CS Nu-cl$(W_1^*) \subseteq H_1^*$. But Nu-cl(Nu-int$(W_1^*)) \subseteq$ Nu-cl$(W_1^*) \subseteq H_1^*$. We have Nu-cl(Nu-int$(W_1^*)) \subseteq H_1^*$ whenever $W_1^* \subseteq H_1^*$ and $H_1^*$ is Nu(g)OS in Nu$_X^*$. Therefore $W_1^*$ is Nu(w$g^*$)CS. □

**Remark 3.2.** *Every Nu(wg$^*$)CS is not Nu(g$^*$)CS.*

**Example 3.** *Let Nu$_X^*$ $= \{w_1, w_2, \ w_3, w_4\}$ and NSs $W_1^*, \ W_2^*, W_3^*, \ W_4^*$ defined as*
$W_1^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$W_2^* = \left\langle w, \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$W_3^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$W_4^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$Nu_\tau = \{0_{Nu}, W_1^*, \ W_2^*, W_3^*, \ W_4^*, 1_{Nu}\}$ *be a NT on Nu$_X^*$. Then*
$W_1^* = \left\langle w, \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
*is Nu(wg$^*$)CS but it is not Nu(g$^*$)CS.*

**Theorem 3.3.** *Every Nu(g)CS is Nu(wg$^*$)CS.*

*Proof.* Let $W_1^*$ is Nu(g)CS. Let $W_1^* \subseteq H_1^*$ and $H_1^*$ NuOS in Nu$_X^*$. Since every NuOS is Nu(g)OS $H_1^*$ is Nu(g)OS. Presently using definition Nu(g)CSs Nu-cl$(W_1^*) \subseteq H_1^*$. But Nu-cl(Nu-int$(W_1^*)) \subseteq$ Nu-cl$(W_1^*) \subseteq H_1^*$. We have Nu-cl(Nu-int$(W_1^*)) \subseteq H_1^*$ whenever $W_1^* \subseteq H_1^*$ and $H_1^*$ is Nu(g)OS in Nu$_X^*$. Therefore $W_1^*$ is Nu(w$g^*$)CS set. □

**Remark 3.3.** *Every Nu($wg^*$)CS is not Nu(g)CS.*

**Example 4.** *Let* $Nu_X^* = \{w_1, w_2, w_3\}$ *and NSs*$W_1^*, W_2^*, W_3^*$*, defined as*
$W_1^* = \langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle$
$W_2^* = \langle w, \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle$
$W_3^* = \langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle$
*Let* $Nu_\tau = \{0_{Nu}, W_1^*,\ W_2^*, W_3^*, 1_{Nu}\}$ *is Neutrosophic topology on* $Nu_X^*$*. Then the Neutrosophic set*
$W_4^* = \langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle$
*is Nu($wg^*$)CS but it is not Nu(g)CS.*

**Theorem 3.4.** *Every Nu($\alpha$ g)CS is Nu($wg^*$)CS.*

*Proof.* Let $W_1^*$ is Nu($\alpha$ g)CS. Let $W_1^* \subseteq H_1^*$ and $H_1^*$ NuOS in Nu$_X^*$. Since every NuOS is Nu(g)OS $H_1^*$ is Nu(g)OS. Presently using definition Nu($\alpha$ g)CS, Nu-($\alpha$)cl($W_1^*$) $\subseteq H_1^*$. But Nu-($\alpha$)cl($W_1^*$) $\subseteq$ Nu-cl($W_1^*$) therefore Nu-cl($W_1^*$) $\subseteq W_1^*$. Now Nu-cl(Nu-int($W_1^*$)) $\subseteq$ Nu-cl($W_1^*$) $\subseteq H_1^*$. We have Nu-cl(Nu-int($W_1^*$)) $\subseteq\ H_1^*$ whenever $W_1^* \subseteq H_1^*$ and $H_1^*$ is Nu(g)OS in Nu$_X^*$. Therefore $W_1^*$ is Nu(w$g^*$)CS. □

**Remark 3.4.** *Every Nu($wg^*$)CS is not Nu($\alpha$ g)CS .*

**Example 5.** *Let* $Nu_X^* = \{w_1, w_2\}$ *and NSs* $W_1^*,\ W_2^*$ *defined as*
$W_1^* = \langle w, \left(\frac{7}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle$
$W_2^* = \langle w, \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{1}{10}\right)\rangle$
*Let* $Nu_\tau = \{0_{Nu}, W_1^*, W_2^*, 1_{Nu}\}$ *be a Neutrosophic topology on* $Nu_X^*$*.*
*Then Neutrosophic set*
$W_3^* = \langle w, \left(\frac{7}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\rangle$ *is Nu($wg^*$)CS but it is not Nu($\alpha$ g)CS.*

**Theorem 3.5.** *Every Nu(g $\alpha$)CS is Nu($wg^*$)CS.*

*Proof.* From theorem 3.4 we get every Nu(g $\alpha$)CS is Nu($\alpha$ g)CS. □

**Theorem 3.6.** *Every Nu(gP)CS is Nu($wg^*$)CS.*

*Proof.* Let $W_1^*$ is Nu(gP)CS. Let $W_1^* \subseteq H_1^*$ and $H_1^*$ NuOS in Nu$_X^*$. Since every NuOS is Nu(g)OS $H_1^*$ is Nu(g)OS. Presently using definition Nu(gP)CS Nu-(P)Cl($W_1^*$) $\subseteq H_1^*$. But Nu-(P)Cl($W_1^*$) $\subseteq$ Nu-cl($W_1^*$) therefore Nu-cl($W_1^*$) $\subseteq W_1^*$. Now Nu-cl(Nu-int($W_1^*$)) $\subseteq$ Nu-cl($W_1^*$) $\subseteq H_1^*$. We have Nu-cl(Nu-int($W_1^*$)) $\subseteq\ H_1^*$

whenever $W_1^* \subseteq H_1^*$ and $H_1^*$ is Nu(g)OS in Nu$_X^*$. Therefore $W_1^*$ is Nu(w$g^*$)CS. □

**Remark 3.5.** *Every Nu(w$g^*$)CS is not Nu(gP)CS.*

**Example 6.** *Let Nu$_X^*$ $= \{w_1, w_2\}$ and $Nu_\tau = \{0_{Nu},\ W_1^*,\ 1_{Nu}\}$ be a NTon Nu$_X^*$, where*
$W_1^* \left\langle w, \left(\frac{4}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right)\right\rangle$*. Then*
$W_2^* = \left\langle w, \left(\frac{4}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{1}{10}\right)\right\rangle$ *is Nu(w$g^*$)CS but it is not Nu(gP)CS.*

**Corollary 3.1.** (1) *Every NuCS is Nu(w$g^*$)CS.*
(2) *Every Nu($\alpha$)CS is Nu(w$g^*$)CS.*
(3) *Every Nu(P)CS is Nu(w$g^*$)CS.*
(4) *Every Nu(R)CS is Nu(w$g^*$)CS.*

*Proof.* Obvious. □

**Remark 3.6.** *The intersection of two Nu(w$g^*$)CS is a NTS (Nu$_X^*$, $Nu_\tau$) may not be Nu(w$g^*$)CS.*

**Example 7.** *Let Nu$_X^*$ $= \{w_1, w_2,\ w_3, w_4\}$ and NSs $W_1^*, W_2^*, W_3^*, W_4^*$ defined as*
$W_1^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$W_2^* = \left\langle w, \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$W_3^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$W_4^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$\tau_{Nu} = \{0_{Nu}, W_1^*, W_2^*, W_3^*, W_4^* 1_{Nu}\}$ *is Neutrosophic topology on Nu$_X^*$. Then*
$W_1^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right)\right\rangle$
$W_2^* = \left\langle w, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{10}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right)\right\rangle$ *are Nu(w$g^*$)CS in (Nu$_X^*$, $Nu_\tau$) but $W_1^* \cap W_2^*$ is not Nu(w$g^*$)CS.*

**Theorem 3.7.** *Let $W_1^*$ is Nu(w$g^*$)CS is a NTS (Nu$_X^*$, $Nu_\tau$) and $W_1^* \subseteq W_2^* \subseteq$ Nu-cl (Nu-int($W_1^*$)). Then $W_2^*$ is Nu(w$g^*$)CS in Nu$_X^*$.*

*Proof.* Let $G_1^*$ is Nu(g)OS in Nu$_X^*$ such that $W_2^* \subseteq G_1^*$. Then $W_1^* \subseteq G_1^*$ and since $W_1^*$ is Nu(w$g^*$)CS, Nu-cl(Nu-int($W_1^*$)) $\subseteq G_1^*$. Now $W_2^* \subseteq$ Nu-cl (Nu-int($W_1^*$)) $\Rightarrow$ Nu-cl(Nu-int($W_2^*$)) $\subseteq$ Nu-cl(Nu-int(Nu-cl(Int($W_1^*$)))) $=$ Nu-cl(Nu-int($W_1^*$)), Nu-cl(Nu-int($W_2^*$)) $\subseteq$ Nu-cl(Nu-int($W_1^*$)) $\subseteq G_1^*$. Consequently $W_2^*$ is Nu(w$g^*$)CS. □

**Definition 3.2.** *A Neutrosophic set $W_1^*$ of a NTS (Nu$_X^*, Nu_\tau$) is called Nu($g^*$)OS iff $W_1^{*c}$ is Nu(w$g^*$)CS.*

**Remark 3.7.** *Every Nu(w)OS is Nu(w$g^*$)OS.*

**Example 8.** *Let Nu$_X^* = \{w_1, w_2\}$ and $Nu_\tau = \{0_{Nu}, W_1^*, 1_{Nu}\}$ is Neutrosophic topology on Nu$_X^*$, where $W_1^* \left\langle w, \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right)\right\rangle$.*
*Then $W_2^* = \left\langle w, \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{1}{10}, \frac{5}{10}, \frac{8}{10}\right)\right\rangle$*
*is Nu(w$g^*$)OS in (Nu$_X^*, Nu_\tau$) but it is not Nu(w)OS in (Nu$_X^*, Nu_\tau$).*

**Theorem 3.8.** *A Neutrosophic set $W_1^*$ of a NTS (Nu$_X^*, Nu_\tau$) Nu(w$g^*$)OS if $F_1^* \subseteq$ Nu-cl(Nu-int$(W_1^*)$) whenever $F_1^*$ is Nu(g)CS and $F_1^* \subseteq W_1^*$.*

*Proof.* Follows from Definition 3.1 and Lemma 2.1. □

**Theorem 3.9.** *Let $W_1^*$ is Nu(w$g^*$)OS of a NTS (Nu$_X^*, Nu_\tau$) and Nu-cl(Nu-int$(W_1^*)$) $\subseteq W_2^* \subseteq W_1^*$. Then $W_2^*$ is Nu(w$g^*$)OS.*

*Proof.* Suppose $W_1^*$ is a Nu(w$g^*$)OS in Nu$_X^*$ and Nu-cl(Nu-int$(W_1^*)$) $\subseteq W_2^* \subseteq W_1^*$. $\Rightarrow W_1^{*c} \subseteq W_2^{*C} \subseteq$ (Nu-cl(Nu-int$(W_1^*)$))$C \subseteq W_1^{*c} \subseteq W_2^{*C} \subseteq$ Nu-cl(Nu-int$(W_1^{*c})$ by Lemma 2.18 and $W_1^{*c}$ is Nu(w$g^*$)CS it follows from theorem that $W_2^{*C}$is Nu(w$g^*$)CS. Hence $W_2^*$ is Nu(w$g^*$)OS. □

## 4. Conclusion

The hypothesis of g-closed sets assumes a significant job when all is said in done topology. Since its initiation numerous powerless and solid types of g-closed sets have been presented by and large topology just as fuzzy topology and Neutrosophic topology. The current paper researched another type of Nu(g)CSs called Nu(w$g^*$)CS which has been contrasted and the classes of Neutrosophic closed sets, Nu(P)CS, Nu($\alpha$)CS, Nu(w)CS, Nu(gP)CS , Nu($\alpha$ g)CS, Nu(g $\alpha$)CS, Nu($g^*$)CS. A few properties and utilization of Nu(w$g^*$)CS are examined. Numerous models are given to legitimize the outcome.

## References

[1] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala: *On Some New Notions and Functions in NTSs*, Neutrosophic Sets and Systems, **16** (2017), 16–19.

[2] A. ATKINSWESTLEY, S. CHANDRASEKAR: *Neutrosophic $g*$- closed sets* (Communicated)

[3] R. DHAVASEELAN, S. JAFARI: *Generalized Neutrosophic closed sets*, New trends in Neutrosophic theory and applications, **2** (2018), 261–273.

[4] R. DHAVASEELAN, S. JAFARI, M. HANIF PAGE: *Neutrosophic Generalized $\alpha$-contra-continuity*, Creat. Math. Inform., **27**(2) (2018), 1–6.

[5] F. SMARANDACHE: *Neutrosophic and Neutrosophic Logic*, First International Conference On Neutrosophic, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA, 2002.

[6] F. SMARADACHE: *Neutrosophic Set: - A Generalization of Neutrosophic set*, Journal of Defense Resourses Management, **1** (2010), 107–114.

[7] P. ISWARYA, K. BAGEERATHI: *On Neutrosophic semi-open sets in NTSs*, International Journal of Mathematics Trends and Technology (IJMTT), **37**(3) (2016), 24–33.

[8] D. JAYANTHI: *$\alpha$-Generalized closed Sets in NTSs*, International Journal of Mathematics Trends and Technology (IJMTT)- Special Issue ICRMIT, 2018.

[9] A. M. MARGARET, M. T. PRICILLA: *Neutrosophic vague generalized pre-closed sets in neutrosophic vague topological spaces*, International journal of mathematics and its applications, **5**(4-E) (2017), 747âĂŞ 759.

[10] C. MAHESWARI, M. SATHYABAMA, S. CHANDRASEKAR: *Neutrosophic generalized b-closed Sets In NTSs*, Journal of physics Conf. Series, **1139**(1) (2018), 012065.

[11] A. A. SALAMA, S. A. ALBLOWI: *Generalized Neutrosophic Set and Generalized NTSs*, Journal computer Sci. Engineering, **2**(7) (2012), 129–132.

[12] A. A. SALAMA, S. A. ALBLOWI: *Neutrosophic set and NTS*, ISOR J. Mathematics, **3**(4) (2012), 31–35.

[13] R. SANTHI, UDHAYARANI: *N$\omega$-Closed Sets In NTSs*, Neutrosophic Sets and Systems, **12** (2016), 114–117

[14] V. K. SHANTHI, S. CHANDRASEKAR, K. SAFINA BEGAM: *Neutrosophic Generalized Semi Closed Sets In NTSs*, International Journal of Research in Advent Technology, **6**(7) (2018), 1739–1743.

[15] V. VENKATESWARA RAO, Y. SRINIVASA RAO: *Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology*, International Journal of ChemTech Research, **10**(10) (2017), 449–458.

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