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WEAKLY NANO δG-CLOSED SETS

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ABSTRACT. Lellis Thivagar presented Na-TSs and concentrated a portion of their properties. Nano $G\delta$ closed sets and Nano δg closed sets presented by R. Vijayalakshmi et al. in Na-TSs. Point of the current paper is to present new type of closed sets is called Weakly Nano(δg)-closed sets in Na-TSs. Likewise we research a portion of their connection and portrayals.

1. INTRODUCTION

The idea of Nano topology was presented by Lellis Thivagar, [4], which was characterized as far as approximations and limit area of a subset of a universe utilizing a proportionality connection on it. Nano (δ) closed sets , Nano (δg) closed sets and Nano ($G\delta$) closed sets are presented by R. Vijayalakshmi et al. in Na-TSs and concentrated a portion of their properties, [5]. Additionally we examine the connections between the other existing Nanoclosed sets. Point of the current paper is we present more fragile type of closed set is called Weakly Nanoδg-closed sets in Na-TSs. Likewise we research a portion of their connection and portrayals.

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2. Preliminaries

Definition 2.1. *[3] Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and $\mathbf{R}^*$ be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation.*

(i) *The lower approximation of Ω with respect to $\mathbf{R}^*$ is* $\mathbf{L}_{\mathbf{R}^*}(\Omega) = \cup_{\sigma \in \mathbf{U}^*}\{\mathbf{R}^*(\sigma) : \mathbf{R}^*(\sigma) \subseteq \Omega\}$.

(ii) *The upper approximation of Ω with respect to $\mathbf{R}^*$ is* $\bigcup_{\mathbf{R}^*}(\Omega) = \cup_{\sigma \in \mathbf{U}^*}\{\mathbf{R}^*(\sigma) : \mathbf{R}^*(\sigma) \cap \Omega \neq = \emptyset\}$

(iii) *The boundary region of Ω with respect to $\mathbf{R}$ is* $\mathbf{B}_{\mathbf{R}}(\Omega) = \mathcal{U}^* R(\Omega) - \mathbf{L}_{\mathbf{R}}(\Omega)$.

Definition 2.2. *[4] Let $\mathcal{U}$ be a non-empty, finite universe of objects and $\mathbf{R}^*$ be an equivalence relation on $\mathcal{U}$. Let $\Omega \subseteq \mathcal{U}$.*

Let $\tau_{\mathbf{R}^*}(\Omega) = \mathbf{Na}^\tau = \{\mathcal{U}, \emptyset, \mathbf{L}_{\mathbf{R}^*}(\Omega), \bigcup_{\mathbf{R}^*}(\Omega), \mathbf{B}_{\mathbf{R}}(\Omega)\}$. Then $\mathbf{Na}^\tau$ is a topology on $\mathcal{U}$, called as the Nano topology with respect to Ω. Elements of the Nano topology are known as the Nano-open sets in $\mathcal{U}$ and $(\mathcal{U}, \mathbf{Na}^\tau)$ is called the Na-TS. $[\mathbf{Na}^\tau]^{\mathbf{C}}$ is called as the dual Nano topology of $\mathbf{Na}^\tau$. Elements of $[\mathbf{Na}^\tau]^{\mathbf{C}}$ are Nanoclosed sets.

Definition 2.3. *Let $(\mathcal{U}^{ast}, Na^\tau)$ be a Na-TS with respect to Ω where $\Omega \subseteq \mathcal{U}$ and if $M_1^* \subseteq \mathcal{U}$,then M_1^* is said to be*

(i) *Nano semi-open set if $M_1^* \subseteq N^{cl}(N^{int}(M_1^*))$, [5];*
(ii) *Nano-regular open set if $M_1^* = N^{cl}(N^{int}(M_1^*))$, [5];*
(iii) *Nano Pre-open set if $M_1^* \subseteq Nint(N^{cl}(M_1^*))$, [5];*
(iv) *Nano α-open set if $M_1^* \subseteq N^{int}(N^{cl}(N^{int}(M_1^*))$, [5];*
(v) *Nano β-open set if $M_1^* \subseteq N^{cl}(N^{int}(N^{cl}(M_1^*))$, [7];*
(vi) *Nano b-open set if $M_1^* \subseteq Nint(N^{cl}(M_1^*)) \cup N^{cl}(N^{int}(M_1^*))$, [6]*
(vii) *The finite union of Nano regular open sets is said to be Nano π-open, [10].*

Definition 2.4. *Let $(\mathcal{U}^*, \mathbf{Na}^\tau)$ be a Na-TS with respect to Ω where $\Omega \subseteq \mathcal{U}$ and if $\mathbf{M_1^*} \subseteq \mathcal{U}$, then $\mathbf{M_1^*}$ is said to be*

(i) *Nano δ-closed, if $\mathbf{M_1^*} = Ncl\delta(\mathbf{M_1^*})$ where* $Ncl\delta(\mathbf{M_1^*}) = \{\sigma \in \mathcal{U} : \mathbf{N^{int}}(\mathbf{N^{cl}}(Z)) \cap \mathbf{M_1^*} \neq \emptyset, Z \in N\tau$ *and* $\sigma \in Z\}$, *[10].*
(ii) *Nano δg-closed set, if $N(\delta)cl(\mathbf{M_1^*}) \subseteq Z$ whenever $\mathbf{M_1^*} \subseteq Z$, Z is Nano open in $(\mathcal{U}^*, \mathbf{Na}^\tau)$, [10].*
(iii) *Nano $G\delta$-closed set, if $\mathbf{N^{cl}}(\mathbf{M_1^*}) \subseteq Z$ whenever $\mathbf{M_1^*} \subseteq Z$, Z is $N(\delta)$-open in $(\mathcal{U}^*, \mathbf{Na}^\tau)$, [9].*

Definition 2.5. *Let $(\mathcal{U}, Na^{\tau})$ be a Na-TS with respect to Ω where $\Omega \subseteq \mathcal{U}^*$ and if $M_1^* \subseteq \mathcal{U}^*$, then M_1^* is said to be:*

(i) *Nano g-closed if Nano $cl(M_1^*) \subseteq Z$ whenever $M_1^* \subseteq Z$ and Z is Nano open, [1].*

(ii) *Nano gp-closed if Nano $pcl(M_1^*) \subseteq Z$ whenever $M_1^* \subseteq Z$ and Z is Nano open. [2].*

(iii) *Nano $g\alpha$-closed set if Nano $\alpha cl(M_1^*) \subseteq Z$ whenever $M_1^* \subseteq Z$ and Z is Nano α open. [8]*

(iv) *weakly g-closed (briefly, wg-closed) set if $N^{cl}(N^{int}(M_1^*)) \subset \mathcal{U}^*$ whenever $M_1^* \subset \mathcal{U}^*$ and $\mathcal{U}^*$ is open in $(\mathcal{U}^*, Na^{\tau})$, [8].*

(v) *Nano sg-closed if Nano $scl(M_1^*) \subseteq Q$ whenever $M_1^* \subseteq Z$ and Z is Nano open, [4].*

3. Weakly $N(\delta g)$-closed sets

In this section we introduce the definition of weakly $N(\delta g)$-closed sets in a Na-TS.

Definition 3.1. *A subset M_1^* of a Na-TS $(\mathcal{U}^*, Na^{\tau})$ is called a weakly $N(\delta g)$-closed (briefly, $N(WG\delta)$-closed) set if $N^{cl}(N^{Int}(M_1^*)) \subset \mathcal{H}$ whenever $M_1^* \subset \mathcal{H}$ and $\mathcal{H}$ is $N(\delta)$-open in $(\mathcal{U}^*, Na^{\tau})$.*

Theorem 3.1. *If a subset M_1^* of a Na-TS $(\mathcal{U}^*, Na^{\tau})$ is both closed and Nano(g)-closed, then it is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^{\tau})$.*

Proof. Let M_1^* be a Nano(αg)-closed set in $(\mathcal{U}^*,\ Na^{\tau})$ and $\mathcal{H}$ be an $N(\delta)$-open set containing M_1^*. Then $\mathcal{H}$ is Nano open containing M_1^* and so $\mathcal{H} \supset N - Cl(M_1^*) = M_1^* \cup N^{int}(N^{Int}(N^{cl}(M_1^*)))$. Since M_1^* is Nanoclosed, $\mathcal{H} \supset N^{cl}(N^{Int}(M_1^*))$ and hence M_1^* is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^{\tau})$. □

Theorem 3.2. *If a subset M_1^* of a Na-TS$(\mathcal{U}^*,\ Na^{\tau})$ is both $N(\delta)$-open and $N(WG\delta)$-closed, then it is Nanoclosed.*

Proof. Since M_1^* is both $N(\delta)$-open and $N(WG\delta)$-closed, $M_1^* \supset N^{cl}(N^{Int}(M_1^*)) = N^{cl}(M_1^*)$ and hence M_1^* is Nanoclosed in $(\mathcal{U}^*,\ Na^{\tau})$. □

Corollary 3.1. *If a subset M_1^* of a Na-TS$(\mathcal{U}^*,\ Na^{\tau})$ is both $N(\delta)$-open and $N(WG\delta)$-closed, then it is both Nano $(\mathcal{R})$ open and Nano $(\mathcal{R})$ closed in $(\mathcal{U}^*,\ Na^{\tau})$.*

Theorem 3.3. *Let $(\mathcal{U}^*,\ Na^\tau)$ be a $N(\delta g)$-$T_{1/2}$ space and $M_1^* \subset \mathcal{U}^*$ be $N(\delta)$-open. Then, M_1^* is $N(WG\delta)$-closed Iff if M_1^* is $N(\delta g)$-closed.*

Proof. Let M_1^* be $N(\delta g)$-closed. By theorem 3.1, it is $N(WG\delta)$-closed.
Conversely, let M_1^* be $N(WG\delta)$-closed. Since M_1^* is $N(\delta)$-open, by theorem 3.2, M_1^* is Nanoclosed. Since $\mathcal{U}^*$ is $N(\delta g)$-$T_{1/2}$, M_1^* is $N(\delta g)$-closed. □

Theorem 3.4. *A set M_1^* is $N(WG\delta)$-closed Iff if $N^{cl}(N^{Int}(M_1^*)) - M_1^*$ contains no non-empty $N(\delta)$-closed set.*

Proof. Necessity. Let $\mathcal{F}$ be a $N(\delta)$-closed set such that $\mathcal{F} \subset N^{cl}(N^{Int}(M_1^*)) - M_1^*$. Since $\mathcal{F}^C$ is $N(\delta)$-open and $M_1^* \subset \mathcal{F}^C$, by definition of $N(WG\delta)$-closed set then $N^{cl}(N^{Int}(M_1^*)) \subset \mathcal{F}^C$, i.e. $\mathcal{F} \subset (N^{cl}(N^{Int}(M_1^*)))^C$. Implies $\mathcal{F} \subset (N^{cl}(N^{Int}(M_1^*))) \subset (N^{cl}(N^{Int}(M_1^*)))^C = \emptyset$.
Sufficiency. Let $M_1^* \subset \mathcal{G}$, where $\mathcal{G}$ is $N(\delta)$-open set in $\mathcal{U}^*$. If $N^{cl}(N^{Int}(M_1^*))$ is $\not\subseteq$ in $\mathcal{G}$, then $N^{cl}(N^{Int}(M_1^*)) \subset \ \mathcal{G}^C$ is a non-empty $N(\delta)$-closed subset of $N^{cl}(N^{Int}(M_1^*)) - M_1^*$, we obtain a contradiction. This is sufficiency of theorem. □

Corollary 3.2. *A $N(WG\delta)$-closed set M_1^* is nano ($\mathcal{R}$) closed Iff if $N^{cl}(N^{Int}(M_1^*)) - M_1^*$ is $N(\delta)$-closed and $N^{cl}(N^{Int}(M_1^*)) \supset M_1^*$.*

Proof. Necessity. Since the set M_1^* is $(\mathcal{R})$ closed, $N^{cl}(N^{Int}(M_1^*)) - M_1^* = \emptyset$ is Nano $(\mathcal{R})$ closed and hence $N(\delta)$-closed.
Sufficiency. By Theorem 3.3, $N^{cl}(N^{Int}(M_1^*)) - M_1^*$ contains no non-empty $N(\delta)$-closed set. That is $N^{cl}(N^{Int}(M_1^*)) - M_1^* = \emptyset$. Therefore, M_1^* is regular closed. □

Theorem 3.5. *Let $(\mathcal{U}^*,\ Na^\tau)$ be a Na-TS and $M_2^* \subset M_1^* \subset \mathcal{U}^*$. If M_2^* is $N(WG\delta)$-closed set comparative with M_1^* and M_1^* is both open and $N(WG\delta)$-closed subset of $\mathcal{U}^*$ then M_2^* is $N(WG\delta)$-closed set comparative with $\mathcal{U}^*$.*

Proof. Let $M_2^* \subset \mathcal{H}$ and $\mathcal{H}$ be a $N(\delta)$-open in $(\mathcal{U},\ Na^\tau)$. Then $M_2^* \subset\ M_1^* \subset\ \mathcal{H}$. Since M_2^* is $N(WG\delta)$-closed comparative with M_1^*, $clA(intA(M_2^*)) \subset\ M_1^* \subset\ \mathcal{H}$. That is $M_1^* \subset\ N^{cl}(N^{Int}(M_2^*)) \subset\ M_1^* \subset\ \mathcal{H}$. We have $M_1^* \subset N^{cl}(N^{Int}(M_2^*)) \subset$ and then $[M_1^* \subset N^{cl}(N^{Int}(M_2^*))] \cup (\ N^{cl}(N^{Int}(M_2^*)))C \subset\ \mathcal{H} \cup (N^{cl}(N^{Int}(M_2^*)))^C$. Since M_1^* is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^\tau)$, we have $N^{cl}(N^{Int}(M_1^*)) \subset\ \ \mathcal{H} \cup (N^{cl}(N^{Int}(M_2^*)))^C$. Therefore $N^{cl}(N^{Int}(M_2^*)) \subset\ \mathcal{H}$ since
$N^{cl}(N^{Int}(M_2^*)) \not\subseteq (N^{cl}(N^{Int}(M_2^*)))^C$. Thus, M_2^* is $N(WG\delta)$-closed set comparative with $(\mathcal{U}^*, Na^\tau)$. □

Corollary 3.3. *If M_1^* is both Nano open and $N(WG\delta)$-closed and $\mathcal{F}$ is Nanoclosed in a Na-TS $(\mathcal{U}^*,\ Na^\tau)$, then $M_1^* \subset \mathcal{F}$ is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^\tau)$.*

Proof. Since $\mathcal{F}$ is Nanoclosed, we have $M_1^* \subset \mathcal{F}$ is Nanoclosed in M_1^*. Therefore $clA(M_1^* \subset \mathcal{F}) = M_1^* \subset \mathcal{F}$ in M_1^*. Let $M_1^* \subset \mathcal{F} \subset\ \mathcal{G}$, where $\mathcal{G}$ is $N(\delta)$-open in M_1^*. Then $clA(intA(M_1^* \subset \mathcal{F})) \subset\ \mathcal{G}$ and hence $M_1^* \subset \mathcal{F}$ is $N(WG\delta)$-closed in M_1^*. By theorem 3.3, $M_1^* \subset \mathcal{F}$ is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^\tau)$. □

Theorem 3.6. *If M_1^* is $N(WG\delta)$-closed and $M_1^* \subset\ M_2^* \subset N^{cl}(N^{Int}(M_1^*))$, then M_2^* is $N(WG\delta)$-closed.*

Proof. Since $M_1^* \subset\ M_2^*, N^{cl}(N^{Int}(M_2^*)) -\ M_2^* \subset N^{cl}(N^{Int}(M_1^*)) -\ M_1^*$. By Theorem 3.3 $N^{cl}(N^{Int}(M_1^*)) -\ M_1^*$ contains no non-empty $N(\delta)$-closed set and so $N^{cl}(N^{Int}(M_2^*)) - M_2^*$. Again by Theorem 3.3, M_2^* is $N(WG\delta)$-closed. □

Theorem 3.7. *Let $(\mathcal{U}^*,\ Na^\tau)$ be a Na-TS and $M_1^* \subset \mathcal{V}^* \subset \mathcal{U}^*$ and $\mathcal{V}^*$ be open. If M_1^* is $N(WG\delta)$-closed in $\mathcal{U}^*$, then M_1^* is $N(WG\delta)$-closed comparative with $\mathcal{V}^*$.*

Proof. Let $M_1^* \subset \mathcal{V}^* \subset \mathcal{G}$ where $\mathcal{G}$ is $N(\delta)$-open in $(\mathcal{U}^*,\ Na^\tau)$. Since M_1^* is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^\tau)$, $M_1^* \subset\ \mathcal{G}$ implies $N^{cl}(N^{Int}(M_1^*)) \subset\ \mathcal{G}$. That is $\mathcal{V}^* \subset (N^{cl}(N^{Int}(M_1^*))) \subset\ \mathcal{V}^* \subset\ \mathcal{G}$ where $\mathcal{V}^* \subset N^{cl}(N^{Int}(M_1^*))$ is closure of interior of M_1^* in $(\mathcal{V}^*, N\sigma)$. Thus, M_1^* is $N(WG\delta)$-closed comparative with $(\mathcal{V}^*, Na^\sigma)$. □

Theorem 3.8. *If a subset M_1^* of a Na-TS$(\mathcal{U}^*,\ Na^\tau)$ is nowhere dense, then it is $N(WG\delta)$-closed.*

Proof. Since $N^{Int}(M_1^*) \subset\ N^{Int}(N^{cl}(M_1^*))$ and M_1^* is nowhere dense, $N^{Int}(M_1^*) = \emptyset$. Therefore $N^{cl}(N^{Int}(M_1^*)) = \emptyset$ and hence M_1^* is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^\tau)$. □

Example 1. *Let $\mathcal{U}^* = \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_3}\}$ with $\mathcal{U}^*/R^* = \{\{\varrho_{a_1}\}, \{\varrho_{a_2}, \varrho_{a_3}\}\}$.*

Let $\Omega = \{\varrho_{a_1}, \varrho_{a_2}\} \subseteq \mathcal{U}^*$. Then $\mathbf{Na}^\tau = \{\mathbf{U}^*,\ \phi, \{\varrho_{\mathbf{a_1}}\}, \{\varrho_{\mathbf{a_2}}, \varrho_{\mathbf{a_3}}\}\}$. Then the set ϱ_{a_1} is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^\tau)$ but not nowhere dense in $(\mathcal{U}^*,\ Na^\tau)$.

Remark 3.1. *If any subsets M_1^* and M_2^* of Na-TS $\mathcal{U}^*$ are $N(WG\delta)$-closed, then their intersection need not be $N(WG\delta)$-closed.*

Example 2. *Let $\mathcal{U}^* = \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_3}, \varrho_{a_4}\}$, with $\mathcal{U}^*/\mathcal{R}^* = \{\{\varrho_{a_1}\}, \{\varrho_{a_3}\}, \{\varrho_{a_2},\ \varrho_{a_4}\}\}$. Let $\Omega = \{\varrho_{a_1}, \varrho_{a_2}\} \subseteq \mathcal{U}^*$. Then $\mathbf{Na}^\tau = \{\mathbf{U}^*,\ \phi, \{\varrho_{\mathbf{a_1}}\}, \{\varrho_{\mathbf{a_2}}, \varrho_{\mathbf{a_4}}\}, \{\varrho_{\mathbf{a_1}},\ \varrho_{\mathbf{a_2}},\ \varrho_{\mathbf{a_4}}\}\}$.*

In this Na-TS the subsets $\{\varrho_{a_1},\ \varrho_{a_3}\}$ *and* $\{\varrho_{a_1},\ \varrho_{a_4}\}$ *are* $N(WG\delta)$*-closed but their intersection* $\{\varrho_{a_1}\}$ *is not* $N(WG\delta)$*-closed in* $(\mathcal{U}^*,\ Na^\tau)$*.*

Proposition 3.1. *Every Nano* g*-closed set is* $N(WG\delta)$*-closed.*

Proof. Let M_1^* be any Nano (g)-closed subset of $(\mathcal{U}^*,\ Na^\tau)$ and let $\mathcal{H}$ be an $N(\delta)$-open set containing M_1^*. Then $\mathcal{H}$ is Nano(α)-open set containing M_1^*. Now $\mathcal{G} \supset N\alpha - Cl(M_1^*) \supset N^{cl}(N^{Int}(N^{cl}(M_1^*))) \supset N^{cl}(N^{Int}(M_1^*))$. Thus, M_1^* is $N(WG\delta)$-closed in $(\mathcal{U}^*,\ Na^\tau)$. □

Example 3. *Let* $\mathcal{U}^* = \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_3}\}$ *with* $\mathcal{U}^*/R^* = \{\{\varrho_{a_1},\ \varrho_{a_2}\}, \{\varrho_{a_3}\}\}$*. Let* $\Omega = \{\varrho_{a_1}, \varrho_{a_2}\} \subseteq \mathcal{U}^*$*. Then* $Na^\tau = \{\mathcal{U}^*, \phi, \{\varrho_{a_1}, \varrho_{a_2}\}\}$*. Then the set* $\{\varrho_{a_1}, \varrho_{a_2}\}$ *is* $N(WG\delta)$*-closed but not Nano* g*-closed in* $(\mathcal{U}^*,\ Na^\tau)$*.*

Remark 3.2. $N(WG\delta)$*-closedness is independent of Nano semi-closedness, Nano* β*-closedness, Nano* b*-closedness and Nano* sg*-closedness in* $(\mathcal{U}^*,\ Na^\tau)$*.*

Example 4. *Let* $\mathcal{U}^* = \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_3}, \varrho_{a_4}\}$*, with* $\mathcal{U}^*/R^* = \{\{\varrho_{a_1},\ \varrho_{a_2}\}, \{\varrho_{a_3},\ \varrho_{a_4}\}\}$*.*
Let $\Omega = \{\varrho_{a_1}, \varrho_{a_2}\} \subseteq \mathcal{U}^*$*. Then* $\mathbf{Na}^\tau = \{\mathbf{U}^*,\ \phi, \{\boldsymbol{\varrho}_{\mathbf{a_1}}, \boldsymbol{\varrho}_{\mathbf{a_2}}\}\}$
Then the set $\{\varrho_{a_1}, \varrho_{a_2}\}$ *is* $N(WG\delta)$*-closed in* $(\mathcal{U}^*,\ Na^\tau)$ *but not Nano semi-closed, Nano* (β)*-closed, Nano (b)-closed,and Nano*(sg)*-closed.*

Example 5. *Let* $\mathcal{U}^* = \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_3}, \varrho_{a_4}\}$*, with* $\mathcal{U}^*/R^* = \{\{\varrho_{a_1}\}, \{\varrho_{a_3}\}, \{\varrho_{a_2},\ \varrho_{a_4}\}\}$*. Let* $\Omega = \{\varrho_{a_1}, \varrho_{a_2}\} \subseteq \mathcal{U}^*$*. Then* $Na^\tau = \{\mathcal{U}^*, \phi, \{\varrho_{a_1}\}, \{\varrho_{a_2},\ \varrho_{a_4}\}, \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_4}\}\}$*. Then the set* $\{\varrho_{a_1}\}$ *is Nano semi-closed, Nano* (β)*-closed, Nano (b)-closed, Nano* (sg)*-closed closed in* $(\mathcal{U}^*,\ Na^\tau)$ *but not* $N(WG\delta)$*-closed in* $(\mathcal{U}^*,\ Na^\tau)$*.*

Definition 3.2. *A subset* M_1^* *of a Na-TS*$\mathcal{U}^*$ *is called* $N(WG\delta)$*-open set if* M_1^{*C} *is* $N(WG\delta)$*-closed in* $\mathcal{U}^*$*.*

Proposition 3.2. (i) *Every* $N(\delta g)$*-open set is* $N(WG\delta)$ *-open;*
(ii) *Every Nano g-open set is* $N(WG\delta)$ *-open.*

Theorem 3.9. *A subset* M_1^* *of a Na-TS*$\mathcal{U}^*$ *is* $N(WG\delta)$*-open if* $\mathcal{G} \subset N^{Int}(N^{cl}(M_1^*))$ *whenever* $\mathcal{G} \subset\ M_1^*$ *and* $\mathcal{G}$ *is* $N(\delta)$*-closed.*

Proof. Let M_1^* be any $N(WG\delta)$-open. Then M_1^{*C} is $N(WG\delta)$-closed. Let $\mathcal{G}$ be a $N(\delta)$-closed set contained in M_1^*. Then $\mathcal{G}^C$ is a $N(\delta)$-open set in $\mathcal{U}^*$ containing M_1^{*C}. Since M_1^{*C} is $N(WG\delta)$-closed we have $N^{cl}(N^{Int}(M_1^{*C})) \subset \mathcal{G}^C$. Therefore $\mathcal{G} \subset N^{Int}(N^{cl}(M_1^*))$.

Conversely, we suppose that $\mathcal{G} \subset N^{Int}(N^{cl}(M_1^*))$ whenever $\mathcal{G} \subset M_1^*$ and $\mathcal{G}$ is $N(\delta)$-closed. Then $\mathcal{G}^C$ is a $N(\delta)$-open set containing M_1^{*C} and
$\mathcal{G}^C \supset (N^{Int}(N^{cl}(M_1^*)))^C$. It follows that $\mathcal{G}^C \supset N^{cl}(N^{Int}(M_1^{*C}))$. Hence M_1^{*C} is $N(WG\delta)$-closed and so M_1^* is $N(WG\delta)$-open. □

4. WEAKLY $G\delta$-CONTINUOUS FUNCTION

Definition 4.1. *Let $\mathcal{U}^*$ and $\mathcal{V}^*$ be Na-T-S. A map $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ is called weakly $N(\delta g)$-CTS (briefly, $N(WG\delta)$-CTS) if $Na_f^{-1}(\mathcal{H})$ is a $N(WG\delta)$-open set in $\mathcal{U}^*$, for each open set $\mathcal{H}$ in $\mathcal{V}^*$.*

Example 6. *Let $\mathcal{U}^* = \mathcal{V}^* = \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_3}\}$, with $\mathcal{U}^*/R^* = \{\{\varrho_{a_1}\}, \{\varrho_{a_2}, \varrho_{a_3}\}\}$. Let $\Omega = \{\varrho_{a_1}, \varrho_{a_2}\} \subseteq \mathcal{U}^*$. Then $\mathbf{Na}^\tau = \{\mathbf{U}^*, \ \phi, \{\varrho_{\mathbf{a_1}}\}, \{\varrho_{\mathbf{a_2}}, \ \varrho_{\mathbf{a_3}}\}\}$ with $\mathcal{V}^*/R^* = \{\{\varrho_{a_1}\}, \{\varrho_{a_2}, \ \varrho_{a_3}\}\}$. Let $\Omega = \{\varrho_{a_1}\} \subseteq \mathcal{U}^*$. $Na^\sigma = \{f, \{\varrho_{a_1}\}, \mathcal{V}^*\}$. The map $Na_f : (\mathcal{U}^*, \ Na^\tau) \to (\ \mathcal{V}^*, Na^\sigma)$ defined by the identity map is $N(WG\delta)$-CTS, because every subset of $\mathcal{U}^*$ is $N(WG\delta)$-closed.*

Proposition 4.1. *Every N(Gδ)-CTS map is $N(WG\delta)$-CTS.*

Proof. It follows from Proposition 3.2 (i). □

Example 7. *Let $\mathcal{U}^* = \mathcal{V}^* = \{\varrho_{a_1}, \varrho_{a_2}, \varrho_{a_3}, \ \varrho_{a_4}\}$, with $\mathcal{U}^*/R^* = \{\{\varrho_{a_1}\}, \{\varrho_{a_3}\}, \{\varrho_{a_2}, \ \varrho_{a_4}\}\}$. Let $\Omega = \{\varrho_{a_1}, \varrho_{a_2}\} \subseteq \mathcal{U}^*$. Then $\mathbf{Na}^\tau = \{\mathcal{U}, \phi, \{\varrho_{\mathbf{a_1}}\}, \{ \ \varrho_{\mathbf{a_2}}, \ \varrho_{\mathbf{a_4}}\}, \{\varrho_{\mathbf{a_1}}, \ \varrho_{\mathbf{a_2}}, \ \varrho_{\mathbf{a_4}}\}\}$ with $\mathcal{V}^*/R^* = \{\{ \ \varrho_{a_2}\}, \{\varrho_{a_1}, \ \varrho_{a_3}\}\}$. Let $\Omega = \{ \ \varrho_{a_2}\} \subseteq \mathcal{U}^*$. $\mathbf{Na}^\sigma = \{f, \{\varrho_{\mathbf{a_2}}\}, \mathbf{V}^*\}$. Let the map $Na_f : (\mathcal{U}^*, \ Na^\tau) \to (\mathcal{V}^*, Na^\sigma)$ be the identity map. Then Na_f is $N(WG\delta)$-CTS but it is not N(Gδ)-CTS.*

Theorem 4.1. *A map $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ is called $N(WG\delta)$-CTS Iff if $Na_f^{-1}(\mathcal{H})$ is a $N(WG\delta)$-closed set in $\mathcal{U}^*$ for each Nanoclosed set $\mathcal{H}$ in $\mathcal{V}^*$.*

Proof. Let $\mathcal{H}$ be any nano closed set in $\mathcal{V}^*$. By assumption $Na_f^{-1}((\mathcal{U}^*C)) = \mathcal{U}^* \ Na_f^{-1}(\mathcal{H})$ is $N(WG\delta)$-open in $\mathcal{U}^*$, so $Na_f^{-1}(\mathcal{H})$ is $N(WG\delta)$-closed in $\mathcal{U}^*$. □

Theorem 4.2. *Suppose that $\mathcal{U}^*$ and $\mathcal{V}^*$ are spaces and the family of $N(\delta g)$-open sets of $\mathcal{U}^*$ is nano closed under arbitrary unions. If a map $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ is contra $N(\delta g)$-CTS and $\mathcal{V}^*$ is $N(\delta)$, then Na_f is $N(WG\delta)$-CTS.*

Proof. Let $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ be contra $N(\delta g)$-CTS and $\mathcal{V}^*$ be $N(\delta)$. then, Na_f is $N(\delta g)$-CTS. Hence, Na_f is $N(WG\delta)$-CTS. □

Theorem 4.3. *Let $Na_f : (\mathcal{U}^*, Na^\tau) \to (\mathcal{V}^*, Na^\sigma)$ be a map. If Na_f is contra $N(\delta g)$-CTS and $(\mathcal{U}^*, Na^\tau)$ is locally $N(\delta g)$-indiscrete, then Na_f is $N(WG\delta)$-CTS.*

Proof. Let $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ be contra $N(\delta g)$-CTS and $(\mathcal{U}^*, Na^\tau)$ be locally $N(\delta g)$-indiscrete., Na_f is CTS. Hence, Na_f is $N(WG\delta)$-CTS. □

Definition 4.2. *A Na-TS $\mathcal{U}^*$ is weakly $G\delta$-compact (briefly, $N(WG\delta)$-compact) if every $N(WG\delta)$-open cover of $\mathcal{U}^*$ has a finite subcover.*

Remark 4.1. *Every $N(WG\delta)$-compact space is N($G\delta$)-compact.*

Theorem 4.4. *Let $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ be a surjective $N(WG\delta)$-CTS map. If $\mathcal{U}^*$ is $N(WG\delta)$-compact, then $\mathcal{V}^*$ is compact.*

Proof. Let $\{M_i^* : i \in I\}$ be an open cover of $\mathcal{V}^*$. Then $\{Na_f^{-1}(M_i^*) : i \in I\}$ is a $N(WG\delta)$-open cover of $\mathcal{U}^*$. Since $\mathcal{U}^*$ is $N(WG\delta)$-compact, it has a finite subcover, say $\{Na_f^{-1}(M_1^*), Na_f^{-1}(M_2^*), \ldots, Na_f^{-1}(M_n^*)\}$. Since Na_f is surjective $\{M_1^*, M_2^*, \ldots, M_n^*\}$ is a finite subcover of $\mathcal{V}^*$ and hence $\mathcal{V}^*$ is compact. □

Definition 4.3. *A Na-TS $\mathcal{U}^*$ is Nanoweakly $G\delta$-connected (briefly, $N(WG\delta)$-connected) if $\mathcal{U}^*$ cannot be written as the disjoint union of two non-empty $N(WG\delta)$-open sets.*

Theorem 4.5. *If a Na-TS $\mathcal{U}^*$ is $N(WG\delta)$-connected, then $\mathcal{U}^*$ is almost connected and $G\delta$-connected.*

Proof. It follows from each $N(\delta)$ open set and each N($G\delta$)-open set is $N(WG\delta)$-open. □

Theorem 4.6. *For a Na-TS $\mathcal{U}^*$ the following statements are equivalent:*

(i) *$\mathcal{U}^*$ is $N(WG\delta)$-connected.*
(ii) *The empty set f and $\mathcal{U}^*$ are only subsets which are both $N(WG\delta)$-open and $N(WG\delta)$-closed.*
(iii) *Each $N(WG\delta)$-CTS map from $\mathcal{U}^*$ into a discrete space $\mathcal{V}^*$ which has at least two points is a constant map.*

Proof. (i) ⇒ (ii). Let $S^* \subset \mathcal{U}^*$ be any proper subset, which is both $N(WG\delta)$-open and $N(WG\delta)$-closed. Its complement $\mathcal{U}^*\ S^*$ is also $N(WG\delta)$-open and $N(WG\delta)$-closed. Then $\mathcal{U}^* = S^* \cup (\mathcal{U}^*\ S^*)$ is a disjoint union of two non-empty $N(WG\delta)$-open sets logical inconsistency with the fact that $\mathcal{U}^*$ is $N(WG\delta)$-connected. Hence, $S^* = f$ or $\mathcal{U}^*$.

(ii) $\Rightarrow$ (i). Let $\mathcal{U}^* = M_1^* \cup M_2^*$ where $M_1^* \cap M_2^* = f$, $M_1^* \neq \emptyset$, $M_2^* \neq \emptyset$ and M_1^*, M_2^* are $N(WG\delta)$-open. Since $M_1^* = \mathcal{U}^*\ M_2^*, M_1^*$ is $N(WG\delta)$-closed. According to the assumption $M_1^* = f$, logical inconsistency.
(ii) $\Rightarrow$ (iii). Let $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ be a $N(WG\delta)$-CTS map where $\mathcal{V}^*$ is a discrete space with at least two points. Then $Na_f^{-1}(v^*)$ is $N(WG\delta)$-closed and $N(WG\delta)$-open for each $v^* \in \mathcal{V}^*$ and $\mathcal{U}^* = \cup\{Na_f^{-1}(v^*) \cap v^* \in \mathcal{V}^*$. by assumption, $Na_f^{-1}(v^*) = f$ or $Na_f^{-1}(v^*) = \mathcal{U}^*$. If $Na_f^{-1}(v^*) = f$ for all $v^* \in \mathcal{V}^*, Na_f$ will not be a map. Also there is no exist more than one $v^* \in \mathcal{V}^*$ such that $Na_f^{-1}(v^*) = \mathcal{U}^*$. Hence, there exists only one $v^* \in \mathcal{V}^*$ such that $Na_f^{-1}(v^*) = \mathcal{U}^*$ and $Na_f^{-1}(v1) = f$ where $v^* \neq v1 \in \mathcal{V}^*$. This shows Na_f is a constant map.
(iii) $\Rightarrow$ (ii). Let $S^* \neq \emptyset$ be both $N(WG\delta)$-open and $N(WG\delta)$-closed in $\mathcal{U}^*$. Let $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ be a $N(WG\delta)$-CTS map defined by $Na_f(S^*) = \{a\}$ and $Na_f(\mathcal{U}^*\ S^*) = \{b\}$ where $a \neq b$. Since Na_f is constant map we get $S^* = \mathcal{U}^*$. □

Theorem 4.7. *Let $Na_f : \mathcal{U}^* \to \mathcal{V}^*$ be a $N(WG\delta)$-CTS surjective map. If $\mathcal{U}^*$ is $N(WG\delta)$-connected, then $\mathcal{V}^*$ is connected.*

Proof. We suppose that $\mathcal{V}^*$ is not connected. Then $\mathcal{V}^* = M_1^* \cup M_2^*$ where $M_1^* \cap M_2^* = f$, $M_1^* \neq \emptyset$, $M_2^* \neq \emptyset$ and M_1^*, M_2^* are open sets in $\mathcal{V}^*$. Since Na_f is $N(WG\delta)$-CTS surjective map $\mathcal{U}^* = Na_f^{-1}(M_1^*) \cup Na_f^{-1}(M_2^*)$ are disjoint union of two non-empty $N(WG\delta)$-open subsets. This is logical inconsistency with the way that $\mathcal{U}^*$ is $N(WG\delta)$-connected. □

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