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INVERSE CLIQUE REGULAR DOMINATION NUMBER IN FUZZY GRAPHS

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ABSTRACT. A subset $D_{cr}(G)$ of a fuzzy graph $G = (\sigma, \mu)$ is said to be a clique regular dominating set if $V - D_{cr}(G)$ contains clique regular dominating set $D^{'}_{cr}(G)$. $D^{'}_{cr}(G)$ is called the inverse clique regular dominating set with respect to $D_{cr}(G)$. The inverse clique domination number $\gamma^{'}_{cr}(G)$ is the minimum fuzzy cardinality taken over all minimal inverse clique regular dominating sets of G.

1. INTRODUCTION

Kulli and Janakiram introduced the concept of regular domination and clique domination in graphs in [4], [5] and [6]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness, [9]. Later Mcallister, in [2] analyzed fuzzy intersection graphs. In 2001, Mordeson and Nair, also considered the fuzzy graphs in [7]. A. Somasundram and S.Somasundram discussed domination in Fuzzy graphs in [10]. In this paper we discuss the inverse clique regular domination number in fuzzy graphs and obtain some relationships with other known parameters of graphs.

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2. PRELIMINARIES

The basic definitions for graphs are given in [1,3,8].

Definition 2.1. *Let $G = (V, E)$ be a graph. A subset D of V is called a dominating set in G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number of G is the minimum cardinatliy taken over all dominating sets in G and is denoted by $\gamma(G)$.*

Definition 2.2. *Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subset V$. Define σ_1 on V_1 by $\sigma_1(u) = \sigma(u)$ for all $u \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u, v\}) = \mu(\{u, v\})$ for all $u, v \in V_1$, then (σ_1, μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $< V_1 >$.*

Definition 2.3. *The fuzzy subgraph $H = (\sigma_1, \mu_1)$ is said to be a spanning fuzzy subgraph of $G = (\sigma, \mu)$ if $\sigma_1(u) = \sigma(u)$ for all $u \in V_1$ and $\mu_1(u, v) \leq \mu(u, v)$ for all $u, v \in V$. Let $G(\sigma, \mu)$ be a fuzzy graph and σ_1 be any fuzzy subset of V_1, i.e. $\sigma_1(u) \leq \sigma(u)$ for all u.*

Definition 2.4. *Let $G = (\sigma, \mu)$ be a fuzzy graph on V. Let $u, v \in V$. We say that u dominates v in G if $\mu(\{u, v\}) = \sigma(u) \wedge (v)$. A subset D of V is called a dominating set in G if for every $v \in D$, there exists $u \in D$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$ or γ.*

Definition 2.5. *A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset $D^{'}$ of D is dominating set of G such that*

$$\sum_{v_i \in D^{'}} \sigma v_i < \sum v_i \in D\sigma(v_i)\,.$$

Definition 2.6. *The order p and size q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(\{u, v\})$.*

Definition 2.7. *An edge $e = \{u, v\}$ of a fuzzy graph is called an effective edge if $\mu(\{u, v\}) = \sigma(u) \wedge \sigma(v)$.*

$N(u) = \{v \in V | \mu(\{u, v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u] = N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by $dE(u)$. $\sum_{v \in N(u)} \sigma(v)$ is called the *neighborhood degree of u and is denoted by $dN(u)$. The minimum effective degree $\delta_E(G) = min\{dE(u)|u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = max\{dE(u)|u \in V(G)\}$.*

Definition 2.8. *A vertex u of a fuzzy graph is said to be an isolated vertex if $\mu(\{u,v\}) \leq \sigma(u) \wedge \sigma(v)$ for all $v \in V - \{u\}$, that is, $N(u) = \phi$, Thus an isolated vertex does not dominate any other vertex in G.*

Definition 2.9. *A set D of vertices of a fuzzy graph is said to be independent if $\mu(\{u,v\}) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in D$.*

Definition 2.10. *The complement of a fuzzy graph G, denoted by $\overline{G}$ is defined to be $\overline{G} = (\sigma, \overline{\mu})$ where $\overline{\mu}(\{u,v\}) = \sigma(u) \wedge \sigma(v) - \mu(\{u,v\})$.*

Definition 2.11. *Let $\sigma : V \to [0,1]$ be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ, μ) where $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$ for all $uv \in E$ and is denoted by K_σ.*

Definition 2.12. *A fuzzy graph $G = (\sigma, \mu)$ is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(V_1, V_2) = 0$ if $V_1, V_2 \in V_1$ or $V_1, V_2 \in V_2$. Further, if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are the restrictions of σ to V_1 and V_2 respectively.*

Definition 2.13. *Let $G = (\sigma, \mu)$ be a regular fuzzy graph on $G* = (V, E)$. If $d_G(v) = k$ for all $v \in V$, (i.e ,) if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph. Where $G* = (V, E)$ is an underlying crisp graph.*

Remark 2.1. *G is k-regular graph iff $\delta = \Delta = k$.*

Definition 2.14. *Let $G = (\sigma, \mu)$ be a fuzzy graph. The total degree of a vertex $u \in V$ is defined by $td_G(u) = d_G(u) + \sigma(u) = \sum_{uv \in E} \mu(uv) + \sigma(u)$. If each vertex of G has the same total degree k then G is said to be a totally regular fuzzy graph of total degree k or k-totally regular fuzzy graph*

Definition 2.15. *A set of fuzzy vertex which covers all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and denoted by $\beta(G)$.*

Definition 2.16. *Let $G = (\sigma, \mu)$ be a fuzzy graph on D and $D \subseteq E$ then the fuzzy edge cardinality of D is defined to be $\sum_{e \in D} \mu(e)$.*

Definition 2.17. *The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident of $'u'$ and is denoted by $dE(u)$. $\sigma(v)$ is called the neighbourhood of u and is denoted by $dN(u)$.*

Definition 2.18. *The minimum effective degree $\delta_E(G) = min\{dE(u)|u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = max\{dE(u)|u \in V(G)\}$.*

3. Main Results

Definition 3.1. *Let $G = (\sigma, \mu)$ be a fuzzy graph without isolated vertices. A subset $D_{cr}(G)$ of V is said to be a clique regular dominating set if $V - D_{cr}(G)$ contains clique regular dominating set $D'_{cr}(G)$ then $D'_{cr}(G)$ is called the inverse clique regular dominating set with respect to $D_{cr}(G)$. The inverse clique domination number $\gamma'_{cr}(G)$ is the minimum fuzzy cardinality taken over all minimal inverse clique regular dominating sets of G.*

Example 1. $D_{cr}(G) = \{v_1, v_2, v_3\} \gamma_{cr}(G) = 0.6$
$D'_{cr}(G) = \{v_4, v_5, v_6\} \gamma'_{cr}(G) = 0.6$

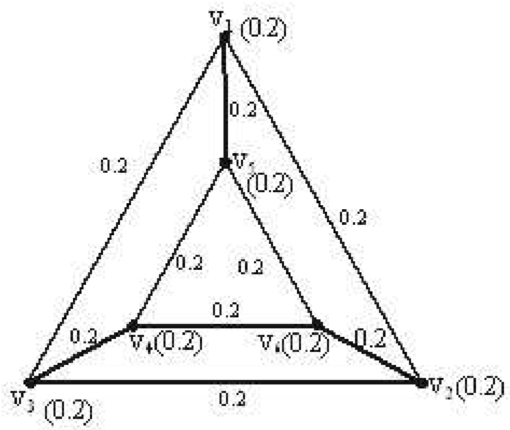

Figure 1

Theorem 3.1. *If $G = (\sigma, \mu)$ is a complete fuzzy graph $K\sigma$ with $n \geq 2$ then*

(1) $< N(D_{cr}(G)) >$ *is a fuzzy complete graph with* $(n-1)$ *vertices.*
(2) $< N(D'_{cr}(G)) >$ *is a fuzzy complete graph with* $(n-2)$ *vertices*

Proof. Let $G = (\sigma, \mu)$ be a complete fuzzy graph $K\sigma$ with $\sigma(v_i) = c$, for every $v_i \in V$ and $n \geq 2$. $D_{cr}(G)$ is the fuzzy clique regular dominating set. Clearly $D_{cr}(G) = \{v_i\}$ and $< N(D_{rc}(G)) >$ is a complete fuzzy graph with $(n-1)$ fuzzy vertices. Clearly $< N(Drc(G)) >$ is a complete fuzzy graph with $(n-1)$ fuzzy vertices., further $V - D_{cr}(G) = V - \{v_i\} = \{v_1, v_2, \ldots v_{i-1}, v_{i+1}, \ldots, v_n\}$.
Let $D'_{cr}(G) \subseteq V - D_{cr}(G)$ is the fuzzy inverse clique regular dominating set then $D'_{cr}(G) = \{v_j/\sigma(v_i)$ is minimum, $j \neq i\}$, also $< V - D'_{cr}(G) >$ is regular with vertices of degree $(n-2)c$. Moreover, $< N(D'rc(G)) >$ is complete with $(n-2)$ fuzzy vertices. Therefore, $< N(D'_{cr}(G)) >$ is a complete fuzzy graph with $(n-2)$ intuitionistic fuzzy vertices. $\square$

Theorem 3.2. *If* $G = (\sigma, \mu)$ *is a fuzzy cycle with equal fuzzy vertex cardinality and* $\gamma'_{cr}(G)$*-set exist, then* $< N(D_{cr}(G)) >$ *is a fuzzy complete graph with two vertices.*

Proof. $G = (\sigma, \mu)$ be a fuzzy cycle with vertex set

$$V = \{v_1, v_2, \ldots v_i, v_{i+1}, \ldots, v_n = v_0\}$$

such that v_i is adjacent with $v_{(i-1)mod\ n}$ and $v_{(i+1)mod\ n} 1 \leq i \leq n$. Moreover, v_i dominates $v_{(i-1)\ mod\ n}$ and $v_{(i+1)\ mod\ n}$. Let $D_{cr}(G)$ be the clique regular dominating set with $(n-2)$ vertices such that $< N(D_{rc}(G)) >$ is regular and also complete graph with two fuzzy vertices. Therefore, $< N(D_{rc}(G)) >$ is a fuzzy complete graph with two vertices. $\square$

Theorem 3.3. *If* $G = (\sigma, \mu)$ *is a fuzzy cycle and* $\mu(v_i)'s$ *are constant with* $\mu(v_i, v_j) = min\{\sigma(v_i), \sigma(v_j)\}$ *then* $\gamma'_{cr}(G) = (n-2)\sigma(v_i)$*.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy cycle with vertex set

$$V = \{v_1, v_2, \ldots v_i, v_{i+1}, \ldots, v_n = v_0\}$$

such that v_i is adjacent with $v_{(i-1)}\ mod\ n$ and $v_{(i+1)}\ mod\ n 1 \leq i \leq n$. Moreover, v_i dominates $v_{(i-1)}\ mod\ n$ and $v_{(i+1)}\ mod\ n$ and $\sigma(v_i) = c$ for every $v_i \in G$ with $\mu(v_i, v_j) = min\{\sigma(v_i), \sigma(v_j)\}$, then $< N(D_{cr}) >$ is a fuzzy complete graph with two vertices, clearly $D'_{cr}(G)$ has $(n-2)$ fuzzy vertices. Therefore, the fuzzy clique regular domination number $\gamma'_{cr}(G) = (n-2)\sigma(v_i)$. $\square$

Theorem 3.4. *If $G = (\sigma, \mu)$ is a fuzzy path with all effective edges then $\gamma'_{cr}(G) = p - max\{\sigma(v_i) + \sigma(v_{i+1})/i \neq 1$ or $n\}$.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy path with vertex set

$$V = \{_1, v_2, \dots v_i, v_{i+1}, \dots, v_n\}$$

and having all effective edges, v_i 's are adjacent with v_{i+1} also v_i dominates v_{i+1}, $i = 1$ to $n - 1$. Let $D_{cr}(G)$ be the clique regular dominating set which contains $\{v_i/v_i \in G\}$ such that $< N(D_{cr}(G)) >$ is regular. The minimum fuzzy clique regular domination number $\gamma'_{cr}(G) = p - max\{\sigma(v_i) + \sigma(v_{i+1})/i \neq 1$ or $n\}$. □

Theorem 3.5. *If $G = (\sigma, \mu)$ is a fuzzy wheel W_{n+1} with $\sigma(v_i) = c$, for every $v_i \in V$ and all edges are effective then $\gamma'_{cr}(G) = \{\sigma(v)/v$ is the centre vertex of the fuzzy wheel$\}$.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy wheel W_{n+1} with $\sigma(v_i) = c$, for every $v_i \in V$ and having all effective edges. The vertex set of G is

$$\{v, v_1, v_2, \dots v_i, v_{i+1}, \dots, v_n\}$$

where v is the centre vertex of the fuzzy wheel, v is adjacent with v_i, $i = 1$ to n also v dominates $v_i, i = 1$ to n. Further, v_i is adjacent with $v_{(i-1)\ mod\ n}$ and $v_{(i+1)\ mod\ n} 1 \leq i \leq n$ and v_i dominates $v_{(i-1\ mod\ n)}$ and $v_{(i+1\ mod\ n)} 1 \leq i \leq n$. Let $D_{cr}(G)$ be the clique regular dominating set which contains $\{v/v$ is the centre vertex of the fuzzy wheel$\}$ such that $< N(D_{cr}) >$ is regular. Therefore, The minimum fuzzy clique regular domination number $\gamma_{cr}(G) = \sigma(v) = c, v$ is the centre vertex of the fuzzy wheel. □

Theorem 3.6. *If $G = (\sigma, \mu)$ is a complete fuzzy graph then $\gamma_{cr}(G)$-set and $\gamma'_{cr}(G)$-set exists but converse need not be true.*

Proof. If $G = (\sigma, \mu)$ is a complete fuzzy graph $K\sigma$ with vertex set

$$\{v_1, v_2, \dots v_i, v_{i+1}, \dots, v_n\}$$

and every $v \in V$ is adjacent to the other vertices. Further, every v dominates the other vertices. Suppose the degree of all $v_i \in V$ are equal. Then the fuzzy clique regular dominating set exists, therefore $\gamma_{cr}(G)$-set exists.

Converse need not be true since $\gamma_{cr}(G)$- set exists, the degree of $v_i \in V$ are equal, but all v_i's need not adjacent with other vertices, further all v_i's need not dominate the other vertices. By definition of complete graphs, G is not a fuzzy complete graph. $\square$

Example 2.
$D_{cr}(G) = \{v\}$, $\gamma_{cr}(G) = 0.1$
$D'_{cr}(G) = \{v_3, v_4, v_5\}$, $\gamma'_{cr}(G) = 0.3$
Therefore $\gamma_{cr}(G)$ and $\gamma'_{cr}(G)$ exist but G is not a complete graph.

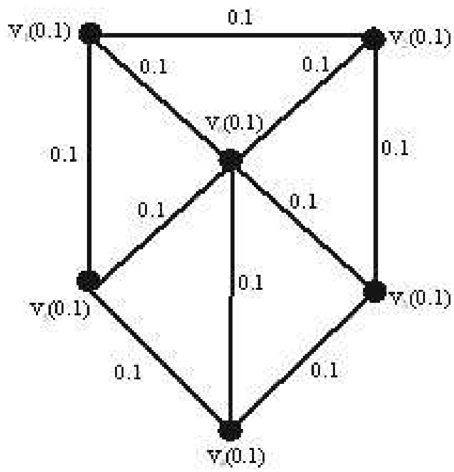

FIGURE 2

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