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NUMERICAL INVESTIGATION FOR THE INHOMOGENEOUS PROBLEMS BY USING LEAPFROG METHOD

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ABSTRACT. In this article, the practical and important problems have been considered, the inhomogeneous problems by an application of the Leapfrog method. The numerical solutions are presented with an example, with an initial condition (discussed by Sekar et al. [1]) having two different solutions, to illustrate and demonstrate the efficiency of the proposed method. The solution graphs are presented to highlight the efficiency of the Leapfrog method.

1. INTRODUCTION

Oscillatory IVPs frequently arise in areas such as classical mechanics, celestial mechanics, quantum mechanics, and biological sciences. The motivation governing the exponentially-fitted methods is inherent in the fact that if the frequency or a reasonable estimate of it is known in advance, these methods will be more advantageous than the polynomial based methods [2]. The goal of this session is to construct a numerical method for addressing periodic and oscillatory problems by an application of the Leapfrog method which was studied by Sekar and team of his researchers [3–5]. Recently, Sekar et al. [1] discussed the periodic and oscillatory problems using single-term Haar wavelet series method. In this paper, the same problem was considered (discussed by Sekar et al. [1]),

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but presents a different approach using the Leapfrog method with more accuracy than the single-term Haar wavelet series method.

2. Leapfrog method

In this section, we modified the method to solve the inhomogeneous equation as follows: the Euler's method for the derivative is of the form:

$$f(t,y) = y',\ y_0 = y(t_0),\ y \in R^d$$

$$\frac{[y(t+h) - y(t)]}{h} \approx y'$$

with $t_n + h = t_{n+1}$, $n = 1, 2, \dots t_0$. Hence, $N = 1$. Modifying the distinction remainder gives:

$$Y_n + hf(t_n, y_n) = y_{n+1},\ n = 0, 1, \dots t_0.$$

The proposed method we define t_n as:

$$t_n + h = t_{n+1}, n = 0, 1, \dots t_0, hy'\left(t + \frac{h}{2}\right) \approx y\left(t+h\right) - y\left(t\right)$$

$$\frac{[y(t+2h) \dots y(t)]}{h} \approx y'(t+h)$$

and then define it as follows:

$$y_{n-1} + 2hf\left(t_n, y_n\right) = y_{n+1}$$

where $n = 0, 1, \dots, t_0$. The proposed method is a linear $m = 2$- step method, with

$$a_0 = 0,\ b_0 = 2a_1 = 1, b_1 = 0 \text{ and } b_{-1} = -1$$

This circumstance recommends a potential instability present in multistep strategies, which must be tended to when we examine them two qualities y_0 and y_1, where

$$y_{n-1} + 2hf\left(t_n, y_n\right) = y_{n+1}, n = 0, 1, 2, \dots, t_0$$

$$f\left(t, y\right) = y',\ y_0 = y\left(t_0\right), y \in R^d$$

In order to illustrate the possible practical use of this method we apply the above technique to the following examples of inhomogeneous problems.

3. Inhomogeneous equation

Consider the following problem:

$$y'' = -100y + 99sinx$$

with initial conditions $y\,(0) = 1$ and $y'\,(0) = 11$, Whose analytical solution is

$$y\,(x) = cos10x + sin10x + sinx$$

The equation has been solved numerically using the STHWS and Leapfrog method and the obtained results (with step size time = 0.1) along with the exact solutions are presented in Table-1 along with absolute errors calculated between them. A graphical representation is shown for the inhomogeneous equation in the following figure, using three-dimensional effects. This result reveals the superiority of the Leapfrog method with less complexity in implementation and at the same time the error reduction is 1000 times less than the STHWS method.

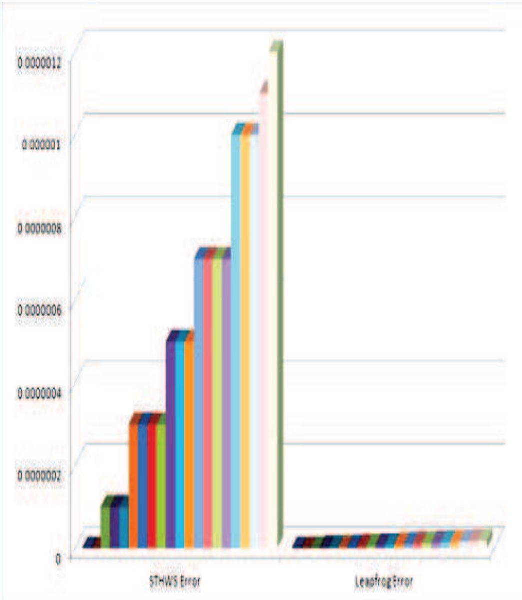

FIGURE 1. Error graph for various values of x

4. Discussions

The obtained results of the periodic and oscillatory problems using Leapfrog are very closer to these exact solutions of the problem when compared to the

STHWS method. From the tables, one can observe that for most of the time intervals, the absolute error is less in Leapfrog when compared to the STHWS method which yields a little error, along with the exact solutions. From figure, one can predict that the error is less in Leapfrog when compared to the STHWS method and especially Leapfrog method works well for the orbit problem and the two body problem. Hence, the Leapfrog method is more suitable for finding the solution of the inhomogeneous problems.

Time t	Exact Solution	Solution for STHWS		Solution for Leapfrog Method	
	Solutions	Solutions	Error	Solutions	Error
0	1.0000000	1.0000000	0	1.0000000	0
0.1	1.4816067	1.4816067	0	1.4816067	0
0.2	0.6918199	0.6918200	1E-07	0.6918199	1E-09
0.3	-0.5533524	-0.5533525	1E-07	-0.5533524	2E-09
0.4	-1.0210278	-1.0210279	1E-07	-1.0210278	3E-09
0.5	-0.1958365	-0.1958368	3E-07	-0.1958365	4E-09
0.6	1.2453975	1.2453978	3E-07	1.2453975	5E-09
0.7	2.0551066	2.0551069	3E-07	2.0551066	6E-09
0.8	1.5612134	1.5612137	3E-07	1.5612134	7E-09
0.9	0.2843139	0.2843144	5E-07	0.2843139	8E-09
1	-0.5416219	-0.5416224	5E-07	-0.5416219	9E-09
1.1	-0.1043556	-0.1043561	5E-07	-0.1043556	1E-08
1.2	1.2393225	1.2393232	7E-07	1.2393225	1.1E-08
1.3	2.2911729	2.2911736	7E-07	2.2911729	1.2E-08
1.4	2.1127925	2.1127932	7E-07	2.1127925	1.3E-08
1.5	0.8880915	0.8880922	7E-07	0.8880915	1.4E-08
1.6	-0.2459909	-0.2459919	1E-06	-0.2459909	1.5E-08
1.7	-0.2448940	-0.2448950	1E-06	-0.2448940	1.6E-08
1.8	0.8831813	0.8831823	1E-06	0.8831813	1.7E-08
1.9	2.0848846	2.0848857	1.1E-06	2.0848846	1.8E-08
2.0	2.2303235	2.2303247	1.2E-06	2.2303235	1.9E-08

FIGURE 2

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REFERENCES

[1] S. SEKAR, E. PARAMANATHAN: *A study on periodic and oscillatory problems using single-term Haar wavelet series*, International Journal of Current Research, **2**(1) (2011), 97–105.

[2] J. M. FRANCO: *Runge-Kutta-Nystrom methods adapted to the numerical integration of perturbed oscillators*, Comput. Phys. Comm., **147** (2002), 770–787.

[3] S. SEKAR, K. PRABHAVATHI: *Numerical solution of First order linear fuzzy differential equations using Leapfrog method*, IOSR Journal of Mathematics, **10** (2014), 7–12.

[4] S. SEKAR, K. PRABHAVATHI: *Numerical Solution of Second Order Fuzzy Differential Equations by Leapfrog Method*, International Journal of Mathematics Trends and Technology, **16**(2) (2014), 74–78.

[5] S. SEKAR, K. PRABHAVATHI: *Numerical Strategies for the n-th order fuzzy differential equations by Leapfrog Method*, International Journal of Mathematical Archive, **6**(1) (2014), 162–168.

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