

FUZZY WALK AND ITS DISTANCE OF FUZZY GRAPH

P. JAYALAKSHMI

ABSTRACT. The fuzzy adjacency relation of fuzzy graph induces the fuzzy walk on fuzzy graph. A number of concepts arising from the fuzzy adjacency leading to fuzzy walk are enumerated. The relationship between the fuzzy path, walk and trail are established. The construction of a fuzzy path from fuzzy graph is proposed. The equivalent relations for these concepts are given. An algorithm is developed to determine the fusion of vertices in fuzzy graph and is verified through example. Various distance of fuzzy walk is estimated.

1. INTRODUCTION

The primary aim of this paper is to study fuzzy walk, path and trail of fuzzy graph. A fuzzy subset of a nonempty set S is a mapping $\sigma : S \rightarrow [0, 1]$, see [5, 12, 13].

A fuzzy relation on S is a fuzzy subset of $S \times S$. If μ and ν are fuzzy relations, then $\mu \circ \nu(u, w) = \sup\{\mu(u, v) \wedge \nu(v, w) : v \in S\}$ and $\mu^k(u, v) = \sup\{\mu(u, u_1) \wedge \nu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \dots \wedge \mu(u_{k-1}, v) : u_1, u_2, \dots, u_{k-1} \in S\}$, where ' \wedge ' stands for minimum.

Later on a fuzzy graph is defined as a pair of functions $G : (\sigma, \mu)$ where $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset of non-empty set V and $\mu : V \times V \rightarrow [0, 1]$ is symmetric fuzzy relation on σ such that for all x, y in V the condition $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied for all (u, v) in E , [9]. Fuzzy adjacent matrices

¹corresponding author

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are probably the most frequently used matrix representation of a fuzzy graph. In many circumstances, it is not necessary to have a direct connection with an edge, but rather a route to be able to go from one vertex to another in some number of steps. If there is any two vertices in a given fuzzy graph which are not adjacent to each other, they might be adjacent to a common neighbor, or more generally they might be connected by a sequence of edges with membership values. This idea captured the existence of fuzzy adjacent matrix, [6,8,11].

Definition 1.1. [2] *The fuzzy adjacent matrix of fuzzy graph is defined as*

$$A_{FG}(U_i, V_j) = \begin{cases} \mu(u_i, v_j) & \text{if } u_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}.$$

These concepts provides the basis for defining the fuzzy walk on fuzzy graph.

Definition 1.2. [3] *For two (not necessarily distinct) vertices u and v in a fuzzy graph FG , a $u - v$ fuzzy walk in FG is a sequence of vertices in FG , beginning with u and ending at v such that consecutive vertices in W are adjacent in FG with $\mu(u_i, v_j) \geq 0$ such a fuzzy walk in a fuzzy graph can be expressed as:*

$$W = u_0\mu(u_0, v_1), v_1\mu(v_1, v_2), v_2\mu(v_2, v_3), \dots, v_{n-1}\mu(v_{n-1}, v_n), v_n$$

where $v_i v_{i+1} \in FG$ for $0 \leq i \leq n-1$.

The fuzzy walk W is said to contain each vertex v_i ($0 \leq i \leq n$) with $\mu(v_i, v_{i+1}) \geq 0$ and each edge $v_i v_{i+1}$ ($0 \leq i \leq n-1$). As a consequence of fuzzy walk in a fuzzy graph, fuzzy path and its trail is also refined.

Definition 1.3. *A fuzzy path in a fuzzy graph is a sequence of distinct nodes $v_0, v_1, v_2, \dots, v_n$ such that for all (v_i, v_{i+1}) , $\mu(v_i, v_{i+1}) > 0$.*

A vertex in a fuzzy graph v_i is said to be accessible or reachable from v_j if there is a fuzzy path from v_i to v_j with $\mu(v_i, v_{i+1}) \geq 0$.

Definition 1.4. *A fuzzy walk in a fuzzy graph in which $\mu(v_i, v_{i+1}) \geq 0$ is no repeated is a trail in a fuzzy graph.*

The fusion of vertices under maxmin composition is found using an algorithm. The distance of a fuzzy walk is determined and it induces metric on the vertex set. In this paper the existence of fuzzy walk and its varieties are established.

2. FUZZY WALK AND ITS DISTANCE IN FUZZY GRAPH

Fuzzy walk theory is rapidly moving into the main stream of fuzzy graph theory. Many applications of fuzzy graphs involve ‘getting from one vertex to another’, see [1, 4, 6, 7, 10].

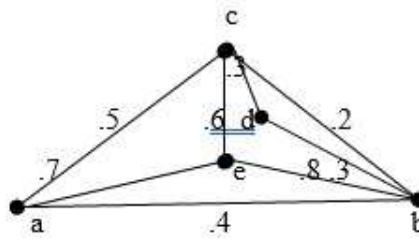


FIGURE 1

On the figure 1 the walk is given by

$$W = \sigma(a) (.5) \sigma(c) (.2) \sigma(b) (.8) \sigma(d) (.3) \sigma(c) (.6) \sigma(e).$$

Let $\sigma(u_1) = \sigma(u) = \sigma(a)$,

$\sigma(u_2) = \sigma(c)$, $\sigma(u_3) = \sigma(b)$, $\sigma(u_4) = \sigma(d)$, $\sigma(u_5) = \sigma(c)$, $\sigma(u_6) = \sigma(e) = \sigma(v)$.

In this process $\sigma(u_2) = \sigma(u_5)$. Hence delete $\sigma(u_5)$. The deletion of $\sigma(u_5)$ gives

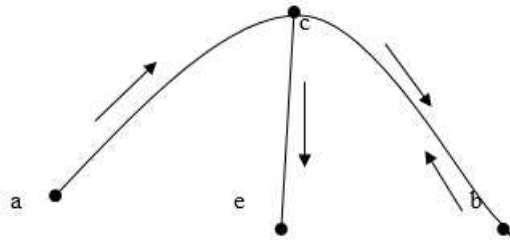


FIGURE 2

the walk on Figure 2,

$$W = \sigma(a) (.5) \sigma(c) (.2) \sigma(b) (.8) \sigma(d) (.3) \sigma(e).$$

This is nothing but the path P of the above graph.

The above example leads to the following theorem:

Theorem 2.1. *Let u and v be the vertices of the fuzzy graph $G(\sigma, \mu)$. Every $\sigma(u) - \sigma(v)$ walk in G contains $\sigma(u) - \sigma(v)$ path.*

Proof. Let $W = \sigma(u) \mu(e_1) \sigma(v_1) \mu(e_2) \dots \sigma(v_{k-1}) \mu(e_k) \sigma(v)$ be the given walk. If $\sigma(u) = \sigma(v)$ then W is closed. Then there will be a trivial path where $P = u$. Suppose $\sigma(u) \neq \sigma(v)$ then W is open.

Let the vertices of W be given in the order as

$$\sigma(u) = \sigma(u_0) \sigma(u_1) \sigma(u_2) \dots \sigma(u_{k-1}) \sigma(u_k) = \sigma(v).$$

If no vertices of $F(G)$ occurs more than once then W is a $\sigma(u) - \sigma(v)$ path.

Therefore $P = W$.

Suppose that there are vertices in $F(G)$ that occurs in W twice or more then there are distinct j, k with $j < k$ such that $\sigma(u_j) = \sigma(u_k)$.

If $\sigma(u_j), \sigma(u_{j+1}), \dots, \sigma(u_{k-1})$ are deleted from W then $\sigma(u) - \sigma(v)$ walk W_1 is obtained having fewer vertices than W .

But if the vertices are not repeated in W_1 then W_1 is a $\sigma(u) - \sigma(v)$ path. Hence $P = W_1$. If this is not the case then the process is repeated by deletion procedure until arriving at the $\sigma(u) - \sigma(v)$ walk which is a path. \square

Theorem 2.2. *If $G(\sigma, \mu)$ is the fuzzy graph with n vertices $\sigma(v_1), \sigma(v_2), \sigma(v_3), \dots, \sigma(v_n)$ contains a $\sigma(u) - \sigma(v)$ walk of length l then G contains $\sigma(u) - \sigma(v)$ path of atmost length l .*

Proof. The proof is by contradiction.

Among all the $\sigma(u) - \sigma(v)$ walks in G , let $W : \sigma(u) = \sigma(v_0), \sigma(v_1), \sigma(v_2), \dots, \sigma(v_k) = \sigma(v)$ be a $\sigma(u) - \sigma(v)$ walk of smallest length k . Therefore $k \leq l$.

Claim: W is a $\sigma(u) - \sigma(v)$ path.

Assume on the contrary that this is not the case.

Then some vertex in G must be repeated in W say $\sigma(u_i) = u_j$ for some i and j with $0 \leq i < j \leq k$.

Deletion of the vertices $\sigma(u_{i+1}), \sigma(u_{i+2}), \dots, \sigma(u_j)$ from W , $\sigma(u) - \sigma(v)$ walk given by $\sigma(u_0), \sigma(u_1), \sigma(u_2), \dots, \sigma(u_{i-1}) \sigma(u_i), \sigma(u_{i+1}), \sigma(u_{i+2}), \dots, \sigma(u_{j-1}), \sigma(u_j), \sigma(u_{j+1}), \sigma(u_{j+2}), \dots, \sigma(u_n)$ is arrived whose length is less than k which is impossible.

Therefore W is a $\sigma(u) - \sigma(v)$ path of length $k \leq l$. \square

Theorem 2.3. *If $G(\sigma, \mu)$ is the fuzzy graph with n vertices $\sigma(v_1), \sigma(v_2), \sigma(v_3), \dots, \sigma(v_n)$ then there is a $\sigma(u) - \sigma(v)$ trail iff there is a $\sigma(u) - \sigma(v)$ path.*

Proof. Since every path is a trail, if there is a $\sigma(u)$ to $\sigma(v)$ path it is automatic that there is a $\sigma(u)$ to $\sigma(v)$ trail. Therefore it suffices to prove that if there is a $\sigma(u)$ to $\sigma(v)$ trail then there is a $\sigma(u)$ to $\sigma(v)$ path.

Assume that there is a $\sigma(u)$ to $\sigma(v)$ trail in G .

Among all the trails choose a trail of minimum length and denote it by

$\sigma(v_0), \sigma(v_1), \sigma(v_2), \dots, \sigma(v_n)$ where $\sigma(v_0) = u$ and $\sigma(v_n) = v$.

If there is only one $\sigma(u)$ to $\sigma(v)$ trail, it will be the one with minimum length.

If in the trail $\sigma(v_0), \sigma(v_1), \sigma(v_2), \dots, \sigma(v_n)$, no vertex is repeated then it is a path from $\sigma(u) - \sigma(v)$. This completes the proof.

Otherwise the trail $\sigma(v_0), \sigma(v_1), \sigma(v_2), \dots, \sigma(v_n)$ will be of the form

$\sigma(v_0), \sigma(v_1), \sigma(v_2), \dots, \sigma(v_{i-1}), \sigma(v_i), \sigma(v_{i+1}), \sigma(v_{i+2}), \dots, \sigma(v_{j-1}), \sigma(v_j), \sigma(v_{j+1}), \sigma(v_{j+2}), \dots, \sigma(v_n)$ where $\sigma(v_j) = v_i$ for some v_i and v_j .

Consider the trail

$\sigma(v_0), \sigma(v_1), \sigma(v_2), \dots, \sigma(v_{i-1}), \sigma(v_i), \sigma(v_j), \sigma(v_{j+1}), \sigma(v_{j+2}), \dots, \sigma(v_n)$ which is got by skipping the vertices $\sigma(v_{i+1}), \sigma(v_{i+2}), \dots, \sigma(v_{j-1})$ together with all edges preceding them. Evidently the trail is shorter than

$\sigma(v_0), \sigma(v_1), \sigma(v_2), \dots, \sigma(v_{i-1}), \sigma(v_i), \sigma(v_{i+1}), \sigma(v_{i+2}), \dots, \sigma(v_{j-1}), \sigma(v_j), \sigma(v_{j+1}), \sigma(v_{j+2}), \dots, \sigma(v_n)$ which is a contradiction.

Hence the trail with minimum length has to be a path. \square

Theorem 2.4. Let $\sigma(u)$ and $\sigma(v)$ be the vertices of the fuzzy graph $G(\sigma, \mu)$. If $\sigma(u) \neq \sigma(v)$ then the following statements are equivalent:

- (1) There is a fuzzy walk from $\sigma(u)$ to $\sigma(v)$.
- (2) There is a fuzzy trail from $\sigma(u)$ to $\sigma(v)$.
- (3) There is a fuzzy path from $\sigma(u)$ to $\sigma(v)$.

Furthermore given a fuzzy walk from $\sigma(u)$ to $\sigma(v)$ there is a fuzzy path from $\sigma(u)$ to $\sigma(v)$ all of whose edges are in the fuzzy walk.

Proof. Since every fuzzy path is a trial, (3) \Rightarrow (2).

Since every fuzzy trail is a fuzzy walk, (2) \Rightarrow (1).

Thus it suffices to prove (1) \Rightarrow (2).

Let $\mu(e_1) \mu(e_2) \mu(e_3), \dots, \mu(e_k)$ be a fuzzy walk from $\sigma(u)$ to $\sigma(v)$. Let n be the number of repeated vertices in a fuzzy walk.

The induction on ' n ' is used.

If the fuzzy walk has no repeated vertices, it is a fuzzy path. This starts the induction on $n = 0$.

Suppose $n > 0$.

Let $\sigma(r)$ be the repeated vertex. Suppose it first appears in edge $\mu(e_i)$ and last appears on $\mu(e_j)$.

If $\sigma(r) = \sigma(u)$ then $\mu(e_j) \mu(e_{j+1}) \mu(e_{j+2}), \dots, \mu(e_k)$ is the fuzzy walk from $\sigma(u)$ to $\sigma(v)$ in which $\sigma(r)$ is not a repeated vertex.

Again if $\sigma(r) = \sigma(v)$ then $\mu(e_1) \mu(e_2) \mu(e_3), \dots, \mu(e_i)$ is the fuzzy walk from $\sigma(u)$ to $\sigma(v)$ in which $\sigma(r)$ is not a repeated vertex.

Otherwise, $\mu(e_1) \mu(e_2) \mu(e_3), \dots, \mu(e_i) \mu(e_j) \mu(e_{j+1}) \mu(e_{j+2}), \dots, \mu(e_k)$ is a fuzzy walk from $\sigma(u)$ to $\sigma(v)$ in which $\sigma(r)$ is not a repeated vertex.

Hence there are less than n repeated vertices in this fuzzy walk from $\sigma(u)$ to $\sigma(v)$ and so there is a fuzzy path by induction. Since the fuzzy path is constructed by removing edges from the fuzzy walk the last statement of the theorem follows. \square

Theorem 2.5. *Let $\sigma(u)$ and $\sigma(v)$ be the vertices of the fuzzy graph $G(\sigma, \mu)$. Two vertices $\sigma(u) \neq \sigma(v)$ are on a fuzzy cycle of fuzzy graph iff there are at least two fuzzy paths from $\sigma(u)$ to $\sigma(v)$ that have no vertices in common except the endpoint $\sigma(u)$ and $\sigma(v)$.*

Proof. Consider $\sigma(u)$ and $\sigma(v)$ are on the fuzzy cycle.

The fuzzy cycle from $\sigma(u)$ to $\sigma(v)$ is followed to obtain one fuzzy path.

Then the fuzzy cycle is followed from $\sigma(v)$ to $\sigma(u)$ to obtain another.

Since the fuzzy cycle has no repeated vertices, the only vertices that lie on both the fuzzy paths are $\sigma(u)$ and $\sigma(v)$.

On the other hand, a fuzzy path from $\sigma(u)$ to $\sigma(v)$ is followed by a fuzzy path $\sigma(v)$ to $\sigma(u)$ is a fuzzy cycle if the fuzzy paths have no common vertices other than $\sigma(u)$ and $\sigma(v)$. \square

Theorem 2.6. *Let $G(\sigma, \mu)$ be the fuzzy graph with n vertices $v_1, v_2, v_3, \dots, v_n$ and Adj_{FG} be the fuzzy adjacent matrix of G with respect to this listing of this vertices. Let k be any positive integer and Adj_{FG}^k denote the max-min composition of k copies of Adj_{FG} . Then the $(i, j)^{th}$ entry of Adj_{FG}^k is the $v_i - v_j$ walk with distinct $\mu(v_i, v_j)$ in G .*

Proof. The proof is by mathematical induction on k .

For $k = 1$, the $(i, j)^{th}$ entry of Adj_{FG} is the $v_i - v_j$ walk with distinct $\mu(v_i, v_j)$ in G .

Assume that the result is true for Adj_{FG}^{k-1} where $k > 1$ and prove the result for Adj_{FG}^k .

Let $Adj_{FG}^{k-1} = b_{ij}$ where b_{ij} is the $v_i - v_j$ walk with distinct $\mu(v_i, v_j)$ in G .

If $Adj_{FG}^k = c_{ij}$ where c_{ij} is the $v_i - v_j$ walk with distinct $\mu(v_i, v_j)$ in G ,

Then $Adj_{FG}^k = Adj_{FG}^{k-1} \circ Adj_{FG}$

$$= \sum_{t=1}^n [(i, t)^{th} \text{ element of } Adj_{FG}^{k-1}] \circ [t, j^{th} \text{ element of } Adj_{FG}] = \sum_{t=1}^n [b_{it} \circ atj]$$

Now every $v_i - v_j$ walk consists of $v_i - v_t$ walk with distinct $\mu(v_i, v_j)$ where v_t is adjacent to v_j followed by an edge $v_t v_j$.

Since there are b_{it} and a_{tj} such walks for each vertex v_t .

Therefore the total number of all $v_i - v_j$ walk is $\sum_{t=1}^n [b_{it} \circ atj]$ which is nothing but Adj_{FG}^k . \square

Theorem 2.7. *If $G(\sigma, \mu)$ is the fuzzy graph without any fuzzy cycles then $G(\sigma, \mu)$ has atleast one pendant vertex.*

Proof. Consider a fuzzy path in G which has a maximum number of vertices. Let $\sigma(u)$ be the end vertex of P . This implies every neighbour of $\sigma(u)$ belongs to P . Suppose a neighbour x of $\sigma(u)$ does not belong to P then a path $P1$ is obtained by extending P to x , then P will be longer a path with maximum number of vertices. Hence every neighbour of $\sigma(u)$ belongs to P .

If u has atleast two neighbours, say y and z , then y and z both belong to P and then the edges $(u, y), (y, z), (z, u)$ form a cycle. This is impossible as G has no cycles. Hence u can have only one neighbour. Accordingly u is a pendant vertex. Thus G has atleast one pendant vertex. \square

Theorem 2.8. *Let $G(\sigma, \mu)$ be the fuzzy graph with n vertices $v_1, v_2, v_3, \dots, v_k$ and $P = v_1 v_2 v_3, \dots, v_k$ be the fuzzy path in G and u be any vertex in $V - V(P)$. If there is no $v_1 - v_k$ fuzzy path with vertex set $V(P) \cup \{u\}$ then $|\{u, V(P)\}| \leq k + 1$ when $\{u, V(P)\}$ is the set of all arcs in P with one end in u or $V(P)$ and the other end in $V(P)$ or u respectively.*

Proof. By the assumption on P , for any $u \in V - V(P)$ there is no $u_i u u_{i+1}$ in P .

Therefore for each $i, 1 \leq i \leq k - 1, |\{u_i, u\}| + |\{u, u_{i+1}\}| \leq 1$

$$\text{Thus } |\{u, V(P)\}| = \sum_{i=1}^{k-1} [|\{u_i, u\}| + |\{u, u_{i+1}\}|] + |\{u, u_1\}| + |\{u, u_k\}| \\ \leq (k - 1) 1 + 2 |\{u, V(P)\}| \leq k + 1. \quad \square$$

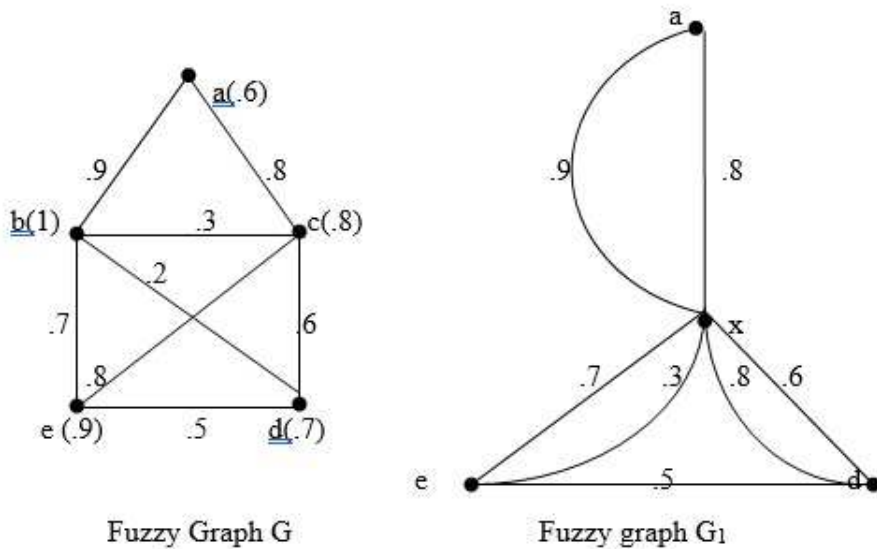
3. FUSION OF VERTICES IN FUZZY GRAPH

In this section is given an algorithm for obtaining the adjacency matrix of the fuzzy graph.

ALGORITHM

Step 1: Change u 's row to the $\max(u, v)$ row and symmetrically change u 's column to the $\max(u, v)$ column.

Step 2: Delete the row and column corresponding to v if u is maximized or u if v is maximized. The resulting matrix is the adjacency matrix of new graph G_1 .



The fuzzy adjacency matrix of the above fuzzy graph is:

$$Adj_{FG} = \begin{bmatrix} 0 & .9 & .8 & 0 & 0 \\ .9 & 0 & .3 & .7 & .8 \\ .8 & .3 & 0 & .2 & .6 \\ 0 & .7 & .2 & 0 & .5 \\ 0 & .8 & .6 & .5 & 0 \end{bmatrix}.$$

Step 1: Changing b 's row to the $\max(b, c)$ row and symmetrically change b 's column to the $\max(b, c)$ column the resulting matrix is:

$$\text{Adj}_{FG} = \begin{bmatrix} 0 & .9 & .8 & 0 & 0 \\ .9 & .3 & .3 & .7 & .8 \\ .8 & .3 & 0 & .2 & .6 \\ 0 & .7 & .2 & 0 & .5 \\ 0 & .8 & .6 & .5 & 0 \end{bmatrix}.$$

Step 2: Deleting the row and column corresponding to c as b is maximized the resulting matrix is:

$$\text{Adj}_{FG1} = \begin{bmatrix} 0 & .9 & 0 & 0 \\ .9 & .3 & .7 & .8 \\ 0 & .7 & 0 & .5 \\ 0 & .8 & .5 & 0 \end{bmatrix}.$$

The resulting matrix is the adjacency matrix of new graph G_1 .

4. DISTANCE OF WALK IN FUZZY GRAPH

Example 1. The walk is given by $W = \sigma(a) (.8) \sigma(c) (.6) \sigma(e) (.5) \sigma(d) (.7) \sigma(b)$
The following are the distances obtained for the walk in fuzzy graph

- The Chebyshev distance of two vertices in a walk W is $d(x, y) = \max |x_i - y_i|$ Example: $d(a, b) = \max\{|.8 - .6|, |.6 - .5|, |.7 - .5|\}$
 $= \max\{.2, .2, .2\}$
 $d(a, b) = .2$
- Euclidean distance: $d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$
Example: $d(a, b) = \sqrt{(.8 - .6)^2 + (.5 - .6)^2 + (.5 - .7)^2}$
 $= \sqrt{.09}$
 $d(a, b) = 0.3$
- Squared Euclidean distance: $d(x, y) = \sum_{i=1}^n (x_i - y_i)^2$
Example: $d(a, b) = (.8 - .6)^2 + (.5 - .6)^2 + (.5 - .7)^2$
 $d(a, b) = 0.09$

- *Manhattan distance:* $d(x, y) = \sum |x_i - y_j|$
Example: $d(a, b) = \sum [|.8 - .6| + |.5 - .6| + |.5 - .7|]$
 $= .2 + .1 + .2$
 $d(a, b) = .5$
- *Canberra distance:* $d(x, y) = \sum \frac{|x_i - y_j|}{|x_i + y_j|}$
Example: $d(a, b) = \frac{|.8 - .6|}{|.8 + .6|} + \frac{|.5 - .6|}{|.5 + .6|} + \frac{|.5 - .7|}{|.5 + .7|}$ $d(a, b) = 0.634199134$
- *Bray Curtis distance:* $d(x, y) = \frac{\sum |x_i - y_j|}{\sum |x_i + y_j|}$
Example: $d(a, b) = \frac{|.8 - .6| + |.5 - .6| + |.5 - .7|}{|.8 + .6| + |.5 + .6| + |.5 + .7|}$
 $d(a, b) = 0.135135135$

Theorem 4.1. *In a fuzzy graph $G : (\sigma, \mu)$, $d : V \times V \rightarrow [0, 1]$ is a metric on V i.e $\forall u, v, w \in V$*

- (1) $d(u, v) \geq 0, u, v \in V$
- (2) $d(u, v) = 0$ iff $u = v$.
- (3) $d(u, v) = d(v, u)$
- (4) $d(u, v) \leq d(u, w) + d(w, v)$.

Proof. (1) and (2) follows from definition. Next a path from u to v is a strong path from v to u . Let P_{1FG} be the $u - w$ path and P_{2FG} be the $w - v$ path whose length is atmost $d(u, w) \cup d(w, v)$. Therefore $d(u, v) \leq d(u, w) + d(w, v)$. \square

5. CONCLUSION

The key property of random walk on fuzzy graph is its degree which represents the number of links it has to other nodes. The degree distribution provides the probability of randomly selected nodes in a network. This enables to determine many network phenomenon from network robustness to the spread of viruses. In mobile call networks the values in the interval $[0, 1]$ represents the total number of minutes the individual talk with each others on the phone, on the power grid it is the amount of current flowing through a transmission line. Fuzzy walk themselves used to infer the structural properties of networks. It has a wide application in the design and analysis of online random algorithm, resistance, continuous scheduling etc. It has the remarkable application in queuing,

networks, traversal sequences, interacting practical systems and physical systems etc.

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SRI GVG VISALAKSHI COLLEGE FOR WOMEN (AUTONOMOUS)

UDUMALPET

E-mail address: jayagvg@gmail.com