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DOUBT FUZZY KM IDEAL ON K- ALGEBRAS

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ABSTRACT. Fuzziness is used for optimizing the results of all domains like engineering, medicine, manufacturing and day to day life. In recent years fuzzy is apply for getting the solution of decision making, learning and reasoning. The main objective of this paper is to introduce doubt fuzzy KM ideal on K-algebras and also to examine few of their fundamental properties.

1. INTRODUCTION

Fuzzy set theory is applied in Artificial Intelligence (AI), control system, theory of decision, logic and operational research etc. The K-algebra $(G, ., e)$ on an abelian group (G, .) is same as the BCI-algebra $(G; , e)$ of p-semi simple and it is proved [1]. Non-associative algebra $(G, ., \odot)$ is presented and with the help of this, a group $(G, .)$, is constructed by adjoining the binary operation $\odot$ and in the group $(G, .)$ all elements are not of order 2. The author is define the function as $x \odot Y = \times \cup y < sup > -1 < /sup >= xy < sup > -1 < /sup >, \forall x, y \in G$, [2]. Homomorphic image and inverse image on fuzzy ideals of K-algebras and its characterization theorem, properties and there relativeness are studied and discussed in [3]. The extension of fuzzy KM an ideal on K-algebras is also introduced and related properties are tested in [4]. Soft set to K-algebras and abelian soft k-algebras concepts are discussed and investigated some properties in [5].

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Doubt fuzzy subalgebra, implicative and prime of double fuzzy ideal in BCK/BCI are defined and few of their properties are tested and results are presented, [6]. Doubt fuzzy BF-algebra notation is introduced and some of their basic properties are investigated in [7]. Here, the idea of KM-Ideals is apply on K algebras to introduce the notion of Doubt fuzzy KM ideal on K-algebras and establish some of their properties.

2. Preliminaries

Definition 2.1. *If satisfies $\xi(p \bigodot q) \geq min\{\xi(p), ((q)\}$ then a fuzzy set ξ in a K-Algebra is named as fuzzy sub algebra of K.*

Definition 2.2. *A fuzzy set ξ of a K-Algebra P is so-called a Doubt Fuzzy Subalgebra of P if $\xi(p \bigodot q) \leq max\{\xi(p), ((q)\}, \forall p, q \in P$.*

Definition 2.3. *Let ξ be a fuzzy set of K-Algebras P for $t \in [0, 1]$, then the sets $\xi_t = \{p \in P/\xi(p) \geq t\}$, $\xi_t = \{p \in P/\xi(p) \leq t\}$ could be empty sets. The sets $\xi_t = \phi$ (respt. $\xi^t \neq \phi$) is called the t (respt t-doubt) confidence set of ξ.*

Definition 2.4. *A fuzzy set ξ of K algebra P is called a Doubt Fuzzy(DF) ideal of P if*

(1) $\xi(e) \leq \xi(p), \forall p \in G$.
(2) $\xi(q) \leq max\{\xi(q \bigodot p), \xi(p \bigodot (p \bigodot q))\}, \forall p, q \in G$.

Theorem 2.1. *ξ is a fuzzy subalgebra of K-Algebra P iff ξ_t is empty or sub algebra of P for all $t \in [0, 1]$.*

Proof. Suppose ξ is a fuzzy subalgebra of P. Therefore

$$\xi(p \bigodot q) \geq min\{\xi(p), ((q)\}. \tag{2.1}$$

We should prove that ξ_t is a sub algebra of P. Let $p, q \in \xi_t \Rightarrow \xi(p), ((q) \geq t$.
Now from the equation (2.1) $\Rightarrow \xi(p \bigodot q) \geq min\{t, t\} = t$ i.e. $p \bigodot q \in \xi_t$.
Conversely, let ξ_t is a subalgebra of P. To prove ξ is a fuzzy subalgebra of P. Let $p, q \in \xi$ such that $\xi(p) = t$ and $\xi(q) = s$ where $t \leq s$. Then $p, q \in \xi_t$ and so $(p \bigodot q) \in \xi_t[\xi_t$ is a subalgebra of $P]$
$\Rightarrow \xi(p \bigodot q) \geq t = min\{\xi(p), ((q)\}$.
Hence ξ is a fuzzy subalgebra of P. □

Theorem 2.2. *ξ is a fuzzy KM ideal of K-algebra P iff ξ_t is KM ideal of P, $t \in [0,1]$.*

Proof. Assume ξ is a fuzzy ideal of P.
Here $\xi_t = \{q \in Q/\xi(q) \geq t\}$. Clearly $0 \in \xi_t$ since $\xi(0) \geq t$.
Let $p \bigodot q, p \bigodot (p \bigodot q), q \in \xi_t$,
$\xi(q) \geq min\{\xi(q \bigodot p), \xi(p \bigodot (p \bigodot q))\} \geq min\{t, t\}$.
$\Rightarrow q \in \xi_t$. Therefore $p \bigodot q, q \in \xi_t \Rightarrow p \in \xi_t \Rightarrow \xi_t$ is a KM ideal of K Algebra P.
Conversely, Let ξ_t is a KM ideal, to prove ξ is a fuzzy KM ideal.
Let $p, q \in P$ such that
$\xi(p \bigodot q) = t$ and $\xi(p \bigodot (p \bigodot q)) = s$ where $t \leq s$
Then $p \bigodot q, q \in \xi_t$. Hence $p \in \xi_t$ [since ξ_t is KM ideal] $\Rightarrow \xi_t \geq tmin\{t, s\}$ $= min\{\xi(p \bigodot q), \xi(p \bigodot (p \bigodot q))\}$. Therefore ξ is a fuzzy KM ideal of P. ☐

Theorem 2.3. *A fuzzy subset ξ of K algebra P is a fuzzy KM ideal of P iff its complements ξ^c is DF KM ideal of P.*

Proof. Consider ξ be a fuzzy ideal of P to prove ξ^c is a KM ideal of doubt fuzzy. Let $p, q \in P$, $\xi^c(0) = 1 - \xi(0) \leq 1 - \xi(p) = \xi^c(p)$ i.e. $\xi^c(0) \leq \xi^c(p)$. Since $\xi(0) \geq \xi(p), \forall p \in P$ it follows that

$$\begin{aligned}\xi^c(p) = 1 - \xi(p) \leq 1 - min\{\xi(q \odot p), \xi(p \odot (p \odot q))\} \leq \\ \leq 1 - min\{1 - \xi^c(q \odot p), 1 - \xi^c(q \odot (p \odot q))\} \leq \\ \leq max\{\xi^c(q \odot p), \xi^c(q \odot (p \odot q))\}\end{aligned}$$

$\Rightarrow \xi^c$ is doubt fuzzy Km ideal of P.
Conversely, let ξ^c is doubt fuzzy Km ideal of P.
To prove ξ is fuzzy KM ideal of P.
$\xi^c(0) \leq \xi^c(p)$ $\xi^c(p) \leq max\{\xi^c(q \bigodot p), \xi^c(q \bigodot (p \bigodot q))\}$
(i) $\Rightarrow 1 - \xi(0) \leq 1 - \xi(p)$
$\Rightarrow \xi(0) \ \geq \xi(p)$
(ii)

$$\begin{aligned}\Rightarrow 1 - \xi(p) \leq max\{1 - \xi^c(q \odot p), 1 - \xi^c(q \odot (p \odot q))\} \leq \\ \leq 1 - min\{\xi^c(q \odot p), \xi^c(q \odot (p \odot q))\} - \xi(p) \leq \\ \leq -min\{\xi(q \odot p), \xi(q \odot (p \odot q))\}\xi(p) \geq \\ \geq min\{\xi(q \odot p), \xi(q \odot (p \odot q))\}\end{aligned}$$

$\Rightarrow \xi$ is fuzzy KM ideal. ☐

Theorem 2.4. *Let ξ be a fuzzy subset of a K algebra P. If ξ is a Doubt Fuzzy KM ideal of P, then the lower level cut ξ_t is an KM ideal of P for all $t \in [0,1], t \geq \xi(0)$.*

Proof. Let ξ be a doubt fuzzy KM ideal of P.
Therefore, we have: $\xi(0) \leq \xi(p)$ and $\xi(q) \leq max\{\xi(q \bigodot p), \xi(p \bigodot (p \bigodot q))\}$.
To prove ξ_t is an ideal of P. We know that $\xi_t = \{p \in P/\xi(p) \leq t\}$.
Let $p, q \in \xi_t$. Since $\xi(0) \leq \xi(p) \leq t \Rightarrow 0 \in \xi_t, \forall t \in [0,1]$.
Again let $p \bigodot q, p \bigodot (p \bigodot q) \in \xi_t$. Therefore $\xi(p \bigodot q) \leq\ t, \xi(p \bigodot (p \bigodot q)) \leq t$.

$$\begin{aligned}\xi(q) &\leq max\{\xi(q \odot p), \xi(p \odot (p \odot q))\} \leq \\ &\leq max\{t, t\} = t.\end{aligned}$$

Hence $\xi(p) \leq t \Rightarrow p \in \xi_t$.
$p \bigodot q, q \in \xi_t \Rightarrow p \in \xi_t$. Therefore ξ_t is an ideal. □

Theorem 2.5. *Let ξ_1 and ξ_2 be two Doubt Fuzzy KM ideal of K algebra P. Then $\xi_1 \cup \xi_2$ is also a doubt fuzzy KM ideal of P.*

Proof. Let $p, q \in P$. Now,

$$\begin{aligned}(\xi_1 \cup \xi_2)(0) = max\{\xi_1(0), \xi_2(0)\} &\leq \\ \leq max\{\xi_1(x), \xi_2(x)\} &= (\xi_1 \cup \xi_2)(x).\end{aligned}$$

Therefore $(\xi_1 \cup \xi_2)(0) \leq\ (\xi_1 \cup \xi_2)(x)$. Again

$$\begin{aligned}&(\xi_1 \cup \xi_2)(x) = max\{\xi_1(x), \xi_2(x)\} \leq \\ &\leq max\{max\{\xi_1(q \odot p), \xi_1(p \odot (p \odot q))\}, max\{\xi_2(q \odot p), \xi_2(p \odot (p \odot q))\}\} \\ &\leq max\{max\{\xi_1(q \odot p), \xi_2(q \odot p)\}, max\{\xi_1(p \odot (p \odot q)), \xi_2(p \odot (p \odot q))\}\} \\ &= max\{(\xi_1 \cup \xi_2)(q \odot p), (\xi_1 \cup \xi_2)(p \odot (p \odot q))\}.\end{aligned}$$

Therefore $\xi_1 \cup \xi_2$ is a Doubt Fuzzy KM ideal of P. □

Theorem 2.6. *Let ξ be a Doubt fuzzy (DF) KM ideal of a K- algebra P. Then the following holds:*

(1) *If $q \leq p$ then $\xi(q) \leq \xi(p)$, i.e. ξ preserves order.*
(2) *If $\xi(q \bigodot p) = 0$ then $\xi(q) \leq \xi(p)$*
(3) *If $q \bigodot p \leq r$ then $\xi(q) \leq max\{\xi(r), \xi(p \bigodot (p \bigodot q))\}$ for all $p, q, r \in P$.*

Proof. (i) Let $q \leq p$ then $q \bigodot p = 0$.
Since ξ is a DF KM ideal and since $\xi(0) \leq \xi(p)$ for DF KM ideal:

$$\begin{aligned}\xi(q) &\leq max\{\xi(q \odot p), \xi(p \odot (p \odot\ q))\} = \\ &= max\{\xi(0), \xi(p)\} = \xi(p)\end{aligned}$$

i.e. $\xi(q) \leq \xi(p)$, i.e. ξ preserves order.
(ii) If $\xi(q \bigodot p) = 0$ then we have, since ξ is a DF KM ideal and since $\xi(0) \leq \xi(p)$ for DF KM ideal:

$$\xi(q) \leq max\{\xi(q \odot p), \xi(p \odot (p \odot \ q))\} = max\{\xi(0), \xi(p)\} = \xi(p)$$

i.e. $\xi(q) \leq \xi(p)$.
(iii) If $q \bigodot p \leq r$, then $(q \bigodot p) \bigodot r = 0$ Now

$$\xi(q) \leq max\{\xi(q \bigodot p), \xi(p \bigodot (p \bigodot \ q))\} \tag{2.2}$$

Therefore

$$\xi(q \odot p) \leq max\{\xi(q \odot p) \odot r, \xi(r)\} = max\{\xi(0), \xi(r)\} = \xi(r),$$

since $\xi(0) \leq \xi(r)$ for DF KM ideal. Therefore $\xi(q \bigodot p) \leq \xi(r)$. Therefore

$$max\{\xi(q \bigodot p), \xi(p \bigodot (p \bigodot \ q))\} \leq max\{\xi(r\ ), \xi(p \bigodot (p \bigodot \ q))\} \tag{2.3}$$

From (2.3) and (2.2) $\xi(q) \leq max\{\xi(r), \xi(p \bigodot (p \bigodot \ q))\}$. □

Theorem 2.7. *If ξ is a Doubt fuzzy (DF) KM ideal of a K-algebra P, then the set $P_\xi = \{p \in P / \xi(q) = \ \xi(0)\}$ is an ideal of P.*

Proof. Clearly $0 \in P_\xi$. Let $q \bigodot p, q \in P_\xi$, $\Rightarrow \xi(q \bigodot p) = \xi(q) = \xi(0)$. Since ξ is a DF KM ideal

$$\xi(q) \leq max\{\xi(q \odot p), \xi(p \odot (p \odot \ q))\} = max\{\xi(0), \xi(0)\} = \xi(0).$$

Therefore, since ξ is a DF KM ideal, $\xi(q) \leq \xi(0)$ also $\xi(0) \leq \xi(q)$. $\Longrightarrow \xi(q) = \xi(0)$.
If $q \in P_\xi$, then $q \bigodot p, q \in P_\xi \Rightarrow q \in P_\xi \Rightarrow P_\xi$ is an ideal. □

3. Product of doubt fuzzy KM ideals of K-Algebra

Definition 3.1. *Let ξ_1 and ξ_2 be two DF KM ideals of a K-algebra P. Then their Cartesian product is defined by $(\xi_1 \times \xi_2)(q,p) = max\{\xi_1(q), \xi_2(p)\}$ where $(\xi_1 \times \xi_2) :$ $X \times X \cup [0,1]$ for all $p, q \in P$.*

Theorem 3.1. *Let ξ_1 and ξ_2 be two DF KM ideals of a K-algebra. Then $\xi_1 \times \xi_2$ is also a DF KM ideal of $X \times X$.*

Proof. For any $(q,p) \in X \times X$, we have

$$(\xi_1 \times \xi_2)(0,0) = max\{\xi_1(0), \xi_2(0)\} \leq max\{\xi_1(q), \xi_2(p)\} = (\xi_1 \times \xi_2)(q,p).$$

Therefore,

$$(\xi_1 \times \xi_2)(0,0) \leq (\xi_1 \times \xi_2)(q,p) \tag{3.1}$$

Let $(q_1,q_2),(p_1,p_2) \in X \times X$
Then,

$$\begin{aligned}
&(\xi_1 \times \xi_2)(q_1,q_2) = max\{\xi_1(q_1), \xi_2(q_2)\} \leq \\
&\leq max\{max\{\xi_1(q_1 \odot p_1), \xi_1(p_1 \odot (p_1 \odot q_1))\}, max\{\xi_2(q_2 \odot p_2), \xi_2(p_2 \odot (p_2 \odot q_2))\}\} \\
&max\{max\{\xi_1(q_1 \odot p_1), \{\xi_2(q_2 \odot p_2)\}, max\{\xi_1(p_1 \odot (p_1 \odot q_1)), \xi_2(p_2 \odot (p_2 \odot q_2))\}\} \\
&= max\{(\xi_1 \times \xi_2)(q_1 \odot p_1, q_2 \odot p_2), (\xi_1 \times \xi_2)(p_1 \odot (p_1 \odot q_1)), (p_2 \odot (p_2 \odot q_2))\} \\
&= max\{(\xi_1 \times \xi_2)((q_1,q_2) \odot (p_1,p_2)), \{(\xi_1 \times \xi_2)((p_1,p_2) \odot ((p_1 \odot q_1),(p_2 \odot q_2)))\}\}.
\end{aligned}$$

From the equation (3.1) and the last equation it is shown that shows $\xi_1 \times \xi_2$ is a DF KM ideal of $X \times X$. □

4. HOMOMORPHISM AND EPIMORPHISM OD DOUBT FUZZY KM IDEALS OF K-ALGEBRA

Definition 4.1. *Let P and P' be two K- Algebras. A mapping $f : P \to P'$ is said to be homomorphism if $f(q \odot p) = f(q) \odot f(p)$ for all $p, q \in P$.*

Theorem 4.1. *Let P and P' be two K-Algebras and $f : P \to P'$ be a homomorphism. Then $f(0) = 0'$.*

Proof. Let $p \in P$. $\Rightarrow f(p) \in P'$. Now

$$f(0) = f(p \odot p) = f(p) \odot f(p) = 0 \odot 0 = 0'.$$

□

Theorem 4.2. *Let $f : P \cup P'$ be an epimorphism of K-Algerbras if v be a doubt fuzzy KM ideal of P, then the pre image of V under f is also a DF KM deal of P.*

Proof. $f^{-1}(V)$ can be defined as $f^{-1}(v)(q) = V(f(q))$.
Let ξ be the pre image of V under f then $V(f(q)) = \xi(q), \forall q \in P$. Since V is a doubt fuzzy KM ideal $V(0') = V(f(q)) = \xi(q)$.
On the other hand $V(0') = V(f(0)) = \xi(0) \Rightarrow \xi(0) \leq \xi(q), \forall q \in P$.

Again $\xi(q) = V(f(q)) \leq max\{V((f(q)) \odot p'), V(p' \odot \ (p' \odot q))\}$ for any $p' \in P'$
Let $p \in P$ such that $f(p) = p'$ then
$\xi(q) \leq max\{V((f(q)) \odot p'), V(p' \odot \ (p' \odot q))\}$
$= max\{V(f(q) \odot f(p)), V(f(p) \odot \ (f(p) \odot q))\}$
$= max\{V(f(q \odot p)), V(f(p \odot \ (p \odot q)))\}$
$= max\{\xi(q \odot p), \xi(p \odot \ (p \odot q))\}$
Therefore $\xi(q) \leq max\{\xi(q \odot p), \xi(p \odot \ (p \odot q))\}$, which is true for all $p, q \in P$.
Hence ξ is a doubt fuzzy KM ideal of P. □

Theorem 4.3. *Let $f : x \to \chi$ be an epimorphism where X and X' are two K algebras if V be a fuzzy subset of X such that $f^{-1}(v)$ is Doubt fuzzy KM ideal of X, then V is also a DF KM Ideal of A.*

Proof. Let $u, v \epsilon x'$, which implies $\exists x, y \in X$ such that $f(x) = v, \ f(y) = u$.
Let χ be the preimage of V under F, then $v(f(y)) = \mu(y)$ (since $f^{-1}(v)(y) = v(f(y))$ since μ is DF KM ideal of χ.
Therefore $\mu(0) \leq \mu(y) \Rightarrow v(f(0) \leq v(f(y))$. $\Rightarrow v\left(0'\right) \leq v(u)$ for all $u \epsilon P'$.
Again $\xi(q) \leq max\{\xi(q \odot p), \xi(p \odot \ (p \odot q))\}, \forall p, q \in P$.
$\Rightarrow V((f(q)) \leq \ max\{V(f(q \odot p)), V(f(p \odot \ (p \odot q)))\}$
$\Rightarrow V(u) \leq \ max\{V(f(q) \odot f(p)), V(f(p) \odot \ (f(p) \odot q))\}$
$= max\{V(u \ \odot v), V(f(p) \odot \ (f(p) \odot q))\}$
$= max\{V(u \ \odot v), V(v \odot \ (v \odot u))\}$
$V(u) \leq max\{V(u \ \odot v), V(v \odot \ (v \odot u))\}$ for all $u, v \in P'$
Hence V is a doubt fuzzy KM ideal of P. □

5. Conclusion

In this paper, Doubt Fuzzy KM ideal on K-algebras is introduced and the symbols are studied. Doubt fuzzy KM ideal on K-algebras is examined by few of their properties. Contribution of this work is very useful for the researcher for doing the research under real time application of decision making problems.

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