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CONCURRENT PRODUCTION AND REWORK INVENTORY MANAGEMENT OF ELECTRICAL COMPONENTS

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ABSTRACT. Amidst innumerable inventory models catering to the needs of the decision makers in handling the crisis pertaining to production and rework scenario, the proposal of concurrent inventory model involving simultaneous production and rework activities will certainly gain momentum as it reflects the realistic industrial set up and incorporates the associated costs parameters. The digitalized and mechanized production processes have provided space for synchronized screening, segregating and reworking of the defective items. Reverse logistics is characterized by rework; the secondary production process occupies a prime role in waste management. Reworking consumes high costs to rectify the defective products by demanding the participation of markets to greater extent. To avert these cyclic activities, concurrency can be encouraged in production sectors. Inventory models with contemporaneous production and rework will certainly enhance the time efficiency and allay the burden of handling waste. The proposed concurrent inventory model is discussed in the context of inventory management of the electrical components and it can be validated with the real time data.

1. INTRODUCTION

Inventory management is indispensable in the economic sustenance of the production industries. The flow of products in the global markets contributes to

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the Sensex score of each nation. Building a country's economy is not an independent and easy task; rather it is a dependent and difficult job which demands proper planning and execution. Customer's choice and taste towards the products is the common denomination for product production and propagation. The customers who are the consumers devour the products based on their perception which makes the customer centric markets march ahead with streaming of products. The production sectors take new avatars to satisfy the needs of the customers to the fullest by accomplishing the tasks related to product design, product production, product testing, product development, product delivery, but it is not the end, the cycle continues in the reverse logistics mechanism, the extended endeavor of these manufacturing firms.

The delivered products are of customer's expected quality, the production process ends, if not the product recovery commences with the phases of return and rework. Based on the rate of defectiveness of the products, they can be subjected to rework, refurbish, reuse and disposal. The effectiveness of the machinery and the quality of the input decides the eminence of the product. If any of the one fails, then the production task becomes incomplete. Reverse Logistics can be tagged with the attempts and measures of greening the world. As the production and rework processes revolve around inventory control, costs minimization and waste mitigation, the industries are in need of inventory models to determine the optimal production and rework quantity to optimize the costs and time.

The history of inventory models dates back to 1913, when Harris in [1], framed the economic order quantity model for the first time. He is the pioneer of inventory models. Economic production quantity model, the next inventory model was modeled by Taft in [2] in 1918. These two are the mother models for the present generation of inventory models. More than a decade, researchers still explore the field of inventory management with their inventory models catering to the specific needs of the industrial sectors. The conventional or the classical inventory models comprised of the predominant costs such as costs of setup, production, holding, shortage, [3,4]. The inventory models were modified and the academicians framed inventory models accommodating the changes in the demand patterns. The leavening of the deterministic nature of the inventory models by probabalisticity, fuzziness and stochasticity has opened the gate ways for the entry of innovative inventory models consisting of various costs pertaining to the functioning of the production sectors.

The production sectors are naturally bonded with economic constraints, but the scenario of global warming, pollution, waste generation have made these production firms to tie knot with environmental regulations, as a result of which the environmental costs have crept into the total inventory costs, [5]. This has paved the beginning of the era of environmental oriented inventory models, a transition from a stereotypic formulation of inventory models. Economic order and production inventory models took the forms of socially responsible and eco âĂŞ conscious inventory models. These newly emerged models were so realistic and highly reflective in nature. Researchers began to compose production and rework inventory models with the inclusion of environmental costs by varying the demand pattern.

To the best of our knowledge, reverse logistic production inventory models encompassed two independent activities, rework followed by production with the inclusion of separate costs for both the process. But presently the manufacturing sectors are very cautious in upholding its branding and customer's confidence on their product's competency. The current technological advancement also supports these industries to minimize the rate of defective products by providing the platform to perform the production and rework process simultaneously, especially the industries that produce electrical components always screens the products, segregates the defective products and subject them to rework. This has motivated us to frame a concurrent production and rework inventory model.

This research work aims in constructing concurrent inventory model with the inclusion of the associated costs of synchronized production and rework. The paper is designed as follows: section 2 presents the model development; section 3 discusses the proposed model and the last section concludes the work.

2. Model Development

2.1. **Problem description.** A manufacturing firm of electrical components employs advanced technology to produce electrical parts in the time period 0 to t_1, with the assumption that the production rate exceeds the demand rate. The produced electrical parts are subjected to screening, segregation and rework of the defective items and a portion say α of P-D gets accumulated to the production rate in the time period [t_1,t_2]. The inventory level gets declined in the period [t_2, T]. One of the underlying assumptions is that all the defective items are restored. Both production and rework takes place during the time period [t_1,t_2].

2.2. **Notations.** The below notations are used in developing the concurrent inventory model.

D Demand per unit of time
P Production per unit of time
H Holding cost per unit per unit of time
F Set up cost
P Production cost
R Rework cost of defective items
K Screening cost of the items
J Cost associated with segregation of defective items from non-defective.

Let $q(t)$ be the inventory level at any $t \in$ [0,T]. The differential equations reflecting the concurrent modelling of the inventory are represented as below.

$$\begin{aligned} \frac{dq}{dt} &= P - D \quad 0 \le t \le t_1 \\ \frac{dq}{dt} &= P - D(1+\alpha) \quad t_1 \le t \le t_2 \\ \frac{dq}{dt} &= -D \quad t_2 \le t \le T\,. \end{aligned}$$

with the initial condition $q(0) = 0$ and the boundary conditions $q(T) = 0$.

Let $q(t_2) = I_M$ be the maximum inventory level. Using the above conditions, the values of $q(t)$ for the given conditions are determined as follows:

$$\begin{aligned} q(t) &= (P-D)t \quad 0 \le t \le t_1 \\ q(t) &= (P-D)[(1+\alpha)t - \alpha t_1] \quad t_1 \le t \le t_2 \\ q(t) &= D(T-t) \quad t_2 \le t \le T \\ I_M &= (P-D)[(1+\alpha)t_2 - \alpha t_1] = (P-D)t_2 = D(T-t_2) \end{aligned}$$

$$\frac{I_M}{(P-D)} = t_2 + \alpha(t_2 - t_1)\,, \quad \frac{I_M}{(P-D)} = t_2\,, \quad \frac{I_M}{D} = (T - t_2)\,.$$

The holding cost

$$
\begin{aligned}
& H[\int_0^{t_1} q(t)dt + \int_{t_1}^{t_2} q(t)dt + \int_{t_2}^{T} q(t)dt] \\
= \; & H[\int_0^{t_1} (P-D)(t)dt + \int_{t_1}^{t_2} (P-D)[(1+\alpha)t - \alpha t_1]dt + \int_{t_2}^{T} D(T-t)dt] \\
= \; & \frac{H}{2}[(P-D)\{t_2^2 + \alpha(t_2-t_1)^2\} + D(T-t_2)^2] \\
= \; & \frac{H}{2}[(P-D)\{t_2^2 + \alpha(t_2-t_1)^2\} + (P-D)[(1+\alpha)t_2 - \alpha t_1](T-t_2)] \,.
\end{aligned}
$$

The associated costs of rework in the time period

$$[t_1, t_2] = (R+K+J)\frac{P-D(t_2-t_1)}{2}[t_2(1+\alpha) + t_1(1-\alpha)] \,.$$

The Total Cost =

$$
\begin{aligned}
& F + P + \frac{H}{2}[(P-D)\{t_2^2 + \alpha(t_2-t_1)^2\} + (P-D)[(1+\alpha)t_2 - \alpha t_1](T-t_2)] \\
& +(R+K+J)\frac{P-D(t_2-t_1)}{2}[t_2(1+\alpha) + t_1(1-\alpha)] \,.
\end{aligned}
$$

By minimizing the total average cost, the optimal time is:

$$\sqrt{\frac{2(F+P+(R+K+J)\frac{P-D(t_2-t_1)}{2}[t_2+t_1+\alpha(t_2-t_1)]}{(P-D)[(1+\alpha)t_2 - \alpha t_1]}} \,.$$

3. Discussion

The proposed concurrent inventory model comprising of the costs associated with the simultaneous production and rework activities is an underlying model with the assumption that all the defective items are reworked, but pragmatically this supposition will sometimes fail with certain probability and this contributes to the limitations of the model. The model can be extended to production, rework and disposal concurrent model comprising of three parallel activities by fragmenting the time period [t_1,t_2] and [t_2, T] to discuss the disposal activities. Inventory models with production, rework and disposal were formulated earlier, but with the coexistence of these three activities will set a new archetype in inventory modeling. The facet of eco-conscious can also be embraced in these concurrent inventory models.

4. Conclusion

Concurrent inventory models will support the decision makers of the production sectors in handling the situations of parallel production and rework tasks by determining the optimal values of the desired parameters. The degree of certainty of inaugurating new vistas by these concurrent inventory models is very high. The extension of these concurrent models will open the platform for the researchers to frame synchronized inventory models focusing on multi activities and incorporation of varied costs. Concurrency in the production and rework activities can be encouraged as it paves way for costs minimization in the context of setting up and initialization of production and rework.

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