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μ_{ij}-PREOPEN SETS IN BIGENERALIZED TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce μ_{ij}-preopen sets in bigeneralized topological space and investigate some of their properties.

1. INTRODUCTION

A. Csaszar [3] introduced the concepts of generalized neighbourhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized topological spaces. In [9], P. Sivagami and D. Sivaraj introduced preopen sets and studied its properties of generalized topologies. In 2010, C Boonpok [2] introduced the concept of bigeneralized topological spaces and studied (m, n)-closed sets and (m, n)- open sets in bigeneralized topological spaces. In 2019, R. Jamuna Rani and M. Anees fathima introduced the concept of semi open sets in bigeneralized topological space [1].

In this paper, our main aim is to introduce the notions of μ_{ij} -preopen sets in bigeneralized topological spaces. We study some of their properties and give its characterizations.

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2. Preliminaries

In this section, we recall some basic definitions and notations. Let X be a non empty set and denotes exp X, the power set of X. A subset μ of exp X is said to be generalized topology on X if $\emptyset \in \mu$ and an arbitrary union of elements of μ belongs to μ [3]. Let μ be a generalized topology on X, the elements of μ are called μ-open sets and the complement of μ-open sets are called μ-closed sets. In a generalized topological space (X, μ), if μ is closed under finite intersection, (X, μ) is called quasi-topological space [5].

For $A \subseteq X$, $i_\mu(A)$ is the union of all μ-open sets contained in A and $c_\mu(A)$ is the intersection of all μ-closed sets containing A [4]. In [10], the family of all μ- friendly functions, where μ is the family of all γ- open sets, is denoted by Γ_4, and (X, γ) is called a γ- space. In [6], it is established that every γ-space is a quasi-topological space and all the results established in [10] for γ-spaces are valid for quasi-topological spaces.

Definition 2.1. *Let (X, μ) be a generalized topological space. A subset M of X is said to be a μ- semi open set iff $M \subseteq c_\mu(i_\mu(M))$, μ- preopen set iff $M \subseteq i_\mu(c_\mu(M))$, $\mu\alpha$-open set iff $M \subseteq i_\mu(c_\mu(i_\mu(M)))$, and $\mu\beta$-open set iff $M \subseteq c_\mu(i_\mu(c_\mu(M)))$ [4].*

Theorem 2.1. *Let (X, μ) be a generalized topological space. Then [3]:*

(1) $c_\mu(A) = X - i_\mu(X - A)$.
(2) $i_\mu(A) = X - c_\mu(X - A)$.

Proposition 2.1. *Let (X, μ) be a generalized topological space. For subsets A and B of X, the following properties hold [8]:*

(1) $c_\mu(X - A) = X - i_\mu(A)$ *and* $i_\mu(X - A) = X - c_\mu(A)$.
(2) *If* $(X - A) \in \mu$, *then* $c_\mu(A) = A$ *and if* $A \in \mu$, *then* $i_\mu(A) = A$.
(3) *If* $A \subseteq B$, *then* $c_\mu(A \subseteq)c_\mu(B)$ *and* $i_\mu(A) \subseteq i_\mu(B)$.
(4) $A \subseteq c_\mu(A)$ *and* $i_\mu(A) \subseteq A$.
(5) $c_\mu(c_\mu(A)) = c_\mu(A)$ *and* $i_\mu(i_\mu(A)) = i_\mu(A)$.

Proposition 2.2. *If (X, μ) is a quasi topological space and $A, B \subseteq X$, then the following hold:*

(1) *If A and B are μ-open sets, then $A \cap B$ is μ-open [10].*
(2) $i_\mu(A \cap B) = i_\mu(A) \cap i_\mu(B)$, *for every subsets A and B of X [6].*
(3) $c_\mu(A \cup B) = c_\mu(A) \cup c_\mu(B)$, *for every subsets A and B of X [10].*

Proposition 2.3. *Let (X, γ) be γ- space. Then $G \cap c_\gamma(A) \subset c_\gamma(G \cap A)$, for every $A \subset X$, and γ-open set G of X, [7].*

Definition 2.2. *Let X be a nonempty set and μ_1, μ_2 be generalized topologies on X. A triple (X, μ_1, μ_2) is said to be a bigeneralized topological space [2].*

Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. The closure of A and the interior of A with respect to μ_m are denoted by $c_{\mu_m}(A)$ and $i_{\mu_m}(A)$ respectively, for $m = 1, 2$.

Definition 2.3. *A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called (m, n)-closed if and only if $c_{\mu_m}(c_{\mu_n}(A)) = A$, where $m, n = 1, 2$ and $m \neq n$. The complement of (m, n)-closed set is called (m, n)-open [2].*

Proposition 2.4. *Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. Then A is (m, n)-closed if A is both μ-closed in (X, μ_m) and (X, μ_n), [2].*

Proposition 2.5. *Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. Then A is (m, n)-open if and only if $i_{\mu_m}(i_{\mu_n}(A)) = A$, [2].*

3. μ_{ij}-PREOPEN SETS

Definition 3.1. *Let X be a nonempty set. Let (X, μ_1, μ_2) be a bigeneralized topological space and $A \subset X$. Then A is said to be a μ_{ij}-preopen set if $A \subset i_{\mu_j}(c_{\mu_j}(A))$, where $i, j = 1, 2$ and $i \neq j$. The collection of all μ_{ij}-preopen sets is denoted by $\pi_{ij}(\mu)$.*

Example 1. *Let $X = \{a, b, c, d\}$,*
$\mu_1 = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$,
$\mu_2 = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ *on X. Then $\{a, b, c\}$ is a μ_{12}-preopen set and $\{a, b, d\}$ is not a μ_{12}-preopen set.*

The following theorem gives some of the properties of μ_{ij} -preopen sets in bigeneralized topological spaces.

Theorem 3.1. *Let (X, μ_1, μ_2) be a bigeneralized topological space. Then the following hold.*

(a) *Every μ_i-open set (respectively μ_j) is a μ_{ij}-preopen set.*

(b) *Arbitrary union of μ_{ij}-preopen sets is a μ_{ij}-preopen set.*

Proof. (a) Let $A \subset X$ be a μ_i-open set. Then $A = i_{\mu_i}(A) \subset i_{\mu_j}(c_{\mu_j}(A))$ and so A is μ_{ij}- preopen.

(b) Let $\{A_\alpha/\alpha \in \Delta\}$ be a family of μ_{ij}-preopen sets and $A = \cup\{A_\alpha/\alpha \in \Delta\}$. Since $A_\alpha \subset A$, $A_\alpha \subset i_{\mu_i}(C_{\mu_j}(A_\alpha)) \subset i_{\mu_i}(c_{\mu_j}(A))$ for every α and so $A = \cup A_\alpha \subset i_{\mu_i}(c_{\mu_j}(A))$ and therefore, A is μ_{ij}-preopen. □

Remark 3.1. (i) *It is clear that $\emptyset$ is a μ_i-open set and hence by Theorem 3.1 (a) it is a μ_{ij}- preopen set.*

(ii) *Let A and B be any two μ_{ij}-preopen sets in a bigeneralized topological space X, then $A \cap B$ need not be a μ_{ij}-preopen set. It can be seen from the following example.*

Example 2. *Let $X = \{a, b, c, d\}$, $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, $\mu_2 = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ on X. Then $\{a, c\}$ and $\{b, c\}$ are μ_{12}- preopen sets but $\{a, c\} \cap \{b, c\} = \{c\}$ which is not a μ_{12}-preopen set.*

Definition 3.2. *Let (X, μ_1, μ_2) be a bigeneralized topological space. Then the union of all μ_{ij}-preopen sets contained in A is called the μ_{ij}-preinterior of A and is denoted by $i_{\pi_{ij}}(A)$.*

The following theorem gives some of the properties of μ_{ij}-preinterior operator $i_{\pi_{ij}}(A)$.

Theorem 3.2. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $A \subset X$. Then the following hold.*

(a) *$i_{\pi_{ij}}(A)$ is the largest μ_{ij}-preopen set contained in A.*
(b) *A is μ_{ij}-preopen if and only if $A = i_{\pi_{ij}}(A)$.*
(c) *A is μ_{ij}-preopen if and only if A is $i_{\mu_i}c_{\mu_j}$-open if and only if $A = i_{i_{\mu_i}c_{\mu_j}}(A)$.*
(d) *$x \in i_{\pi_{ij}}(A)$ if and only if there exists a μ_{ij}-preopen set G containing x such that $G \subset A$.*
(e) *$i_{\pi_{ij}} = i_{i_{\mu_i}}c_{\mu_j}$.*
(f) *$i_{\pi_{ij}} \in \Gamma_{02-}$.*

Proof. (a) Since arbitrary union of μ_{ij}-preopen sets is a μ_{ij}-preopen set and by the definition of $i_{\pi_{ij}}$, the proof follows.

(b) Clearly, $i_{\pi_{ij}}(A) \subset A$. Since A is μ_{ij}-preopen, $A = i_{\mu_{ij}} c_{\mu_j}(A) \subset i_{\pi_{ij}}(A)$. Therefore, $A \subset i_{\pi_{ij}}(A)$. Hence $A = i_{\pi_{ij}}(A)$. Conversely, suppose $A = i_{\pi_{ij}}(A)$, then by Theorem 3.1 (b), A is μ_{ij}-preopen.

(c) Suppose A is μ_{ij}-preopen, then $A \subset i_{\mu_i}(c_{\mu_j}(A))$ and hence A is $i_{\mu_i}c_{\mu_j}$-open.

Conversely, if A is $i_{\mu_i}c_{\mu_j}$-open, then $A \subset i_{i_{\mu_i}c_{\mu_j}}(A)$. Clearly, $i_{i_{\mu_i}c_{\mu_j}}(A) \subset A$ and hence $A = i_{i_{\mu_i}c_{\mu_j}}(A)$.

(d) Suppose $x \in i_{\pi_{ij}}(A)$. Let G be a μ_{ij}-preopen set containing x. Suppose $G \not\subset A$, then $x \notin A$ and hence $x \notin i_{\pi_{ij}}(A)$ which is a contradiction. So $G \subset A$.

Conversely, suppose there exists a μ_{ij}-preopen set G containing x such that $G \subset A$.

Suppose $x \notin i_{\pi_{ij}}(A)$, then x does not belong to any of the μ_{ij}-preopen set G and hence $G \not\subset A$ which is a contradiction. Therefore, $x \in i_{\pi_{ij}}(A)$.

(e) Let A be any subset of X. Let x be any element in $i_{\pi_{ij}}(A)$. Then by (d), there exists a μ_{ij}-preopen set G containing x such that $G \subset A$ and hence x belongs to a μ_{ij}-preopen set G. Now by (c), $G = i_{i_{\mu_i}c_{\mu_j}}(G) \subset i_{i_{\mu_i}c_{\mu_j}}(A)$ and hence $x \in i_{i_{\mu_i}c_{\mu_j}}(A)$.

Therefore, $i_{\pi_{ij}}(A) \subset i_{i_{\mu_i}c_{\mu_j}}(A)$. Suppose $x \in i_{i_{\mu_i}c_{\mu_j}}(A)$, then clearly $x \in i_{\pi_{ij}}(A)$ and hence $i_{i_{\mu_i}c_{\mu_j}}(A) \subset i_{\pi_{ij}}(A)$. Therefore, the result follows.

(f) Since $i_{\pi_{ij}}(A)$ is the union of all μ_{ij}-preopen set contained in A, then by (b), $i_{\pi_{ij}} \in \Gamma_2$. Also, $\emptyset$ is μ_i-open and so is μ_{ij}-preopen, then $\emptyset = i_{\pi_{ij}}(\phi)$. Hence $i_{\pi_{ij}} \in \Gamma_0$. By the definition of $i_{\pi_{ij}}$, $i_{\pi_{ij}}(A) \subset A$ for every subset A of X. Therefore, $i_{\pi_{ij}} \in \Gamma_-$. □

Definition 3.3. *Let (X, μ_1, μ_2) be a bigeneralized topological space. Then the complement of μ_{ij}-preopen set is called μ_{ij}-preclosed set. The intersection of all μ_{ij}-preclosed sets containing A is called the μ_{ij}-preclosure of A and is denoted by $c_{\pi_{ij}}(A)$.*

It is clear that X is μ_{ij}-preclosed since $\emptyset$ is μ_{ij}-preopen and the following theorem gives some of the properties of μ_{ij}-preclosure operator $i_{\pi_{ij}}(A)$.

Theorem 3.3. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $A \subset X$. Then the following hold.*

(a) *$c_{\pi_{ij}}(A)$ is the smallest μ_{ij}-preclosed set containing A.*
(b) *A is μ_{ij}-preclosed if and only if $A = c_{\pi_{ij}}(A)$.*

(c) *A is μ_{ij}-preclosed if and only if A is $i_{\mu_i}c_{\mu_j}$-closed if and only if $A = c_{i_{\mu_i}c_{\mu_j}}(A)$.*
(d) *$x \in c_{\pi_{ij}}(A)$ if and only if every μ_{ij}-preclosed set G containing x, $G \cap A \neq \emptyset$.*
(e) *$c_{\pi_{ij}} = c_{i_{\mu_i}}c_{\mu_j}$.*
(f) *$c_{\pi_{ij}} \in \Gamma_{12+}$.*

Proof. Since μ_{ij}-preclosed set is the complement of μ_{ij}-preopen set, then by Theorem 3.2, the results follows. □

Theorem 3.4. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $A \subset X$. Then the following hold.*

(a) $(i_{\pi_{ij}})^* = c_{\pi_{ij}}$.
(b) $(c_{\pi_{ij}})^* = i_{\pi_{ij}}$.
(c) $i_{\pi_{ij}}(X - A) = X - c_{\pi_{ij}}(A)$.
(d) $c_{\pi_{ij}}(X - A) = X - i_{\pi_{ij}}(A)$.

Proof. (a) Let A be any subset of X. Then, $(i_{\pi_{ij}})^*(A) = X - i_{\pi_{ij}}(X - A)$. Since, $i_{\pi_{ij}}(X - A)$ is the largest μ_{ij}-preopen set contained in $X - A$, $X - i_{\pi_{ij}}(X - A)$ is the smallest μ_{ij}- preclosed set containing A and $X - i_{\pi_{ij}}(X - A) = c_{\pi_{ij}}(A)$. Hence, $(i_{\pi_{ij}})^* = c_{\pi_{ij}}$.

(b) $(c_{\pi_{ij}})^* = (i^*_{\pi_{ij}})^* = i_{\pi_{ij}}$, this proves (b).

(c) If A is a subset of X, then, $(i_{\pi_{ij}})^*(A) = X - i_{\pi_{ij}}(X - A)$ and so by (i) $(c_{\pi_{ij}})(A) = X - i_{\pi_{ij}}(X - A)$ which implies $i_{\pi_{ij}}(X - A) = X - c_{\pi_{ij}}(A)$, for every subset A of X.

(d) The proof of (d) is similar to the proof of (c). □

The following theorem discusses the intersection of μ_i-open set and μ_{ij}-preopen set is a μ_{ij}-preopen set.

Theorem 3.5. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $\mu_i, \mu_j \in \Gamma_4$. If A is a μ_i-open set and B is a μ_{ij}-preopen set, then $A \cap B$ is a μ_{ij}-preopen set.*

Proof. Since B is μ_{ij}-preopen set, then $B \subset i_{\mu_i}(c_{\mu_j}(B))$ and so

$$A \cap B \subset A \cap i_{\mu_i}(c_{\mu_j}(B)) \subset i_{\mu_i}(A \cap (c_{\mu_j}(B))) \subset i_{\mu_i}(c_{\mu_j}(A \cap B)).$$

Hence the result. □

Theorem 3.6. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $\mu_i, \mu_j \in \Gamma_4$, then $c_{\pi_{ij}}$ and $i_{\pi_{ij}} \in \Gamma_4$.*

Proof. Let G be a μ_i-open set and A be any subset of X. Then $G \cap i_{\pi_{ij}}(A)$ is a μ_{ij}- preopen set by Theorem 3.5 and $G \cap i_{\pi_{ij}}(A) \subset G \cap A$.

Then $G \cap i_{\pi_{ij}}(A) \subset i_{\pi_{ij}}(G \cap A)$ and so $i_{\pi_{ij}} \in \Gamma_4$. Again $i_{\pi_{ij}} \in \Gamma_4$ implies $(i_{\pi_{ij}})^* \in \Gamma_4$. By Theorem 3.4 (a), $c_{\pi_{ij}} \in \Gamma_4$. □

Theorem 3.7. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $\mu_i \in \Gamma_4$ and G be μ_i-open, then for every subset A of X, $c_{\pi_{ij}}(G \cap A) = c_{\pi_{ij}}(G \cap (c_{\pi_{ij}}(A)))$.*

Proof. Since $c_{\pi_{ij}} \in \Gamma_4$, then by Theorem 3.6, $G \cap c_{\pi_{ij}}(A) \subset c_{\pi_{ij}}(G \cap A)$ and so $c_{\pi_{ij}}(G \cap c_{\pi_{ij}}(A)) \subset c_{\pi_{ij}}(G \cap A)$.

Also, since $G \cap A \subset G \cap c_{\pi_{ij}}(A)$, we have $c_{\pi_{in}}(G \cap A) \subset c_{\pi_{ij}}(G \cap c_{\pi_{ij}}(A))$ and $c_{\pi_{in}}(G \cap A) = c_{\pi_{ij}}(G \cap c_{\pi_{ij}}(A))$. □

Theorem 3.8. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $\mu_i \in \Gamma_4$ (respectively μ_j). Then the following hold.*

(a) *If G is a μ_i-open set such that $G \subset i_{\mu_i}(c_{\mu_j}(A))$ for some subset A of X, then G is a μ_{ij}-preopen set.*
(b) *If $\mu_i \in \Gamma_1$ (respectively μ_j), then every μ_i-open set is a μ_{ij}-preopen set.*

Proof. (a)Since $G \subset i_{\mu_i}(c_{\mu_j}(A))$, we have $G = G \cap i_{\mu_i}(c_{\mu_j}(A)) \subset i_{\mu_i}(c_{\mu_j}(G \cap A)$. Therefore, $G \subset i_{\mu_i}(c_{\mu_j}(G))$ which implies G is μ_{ij}-preopen.

(b) Let G be an open set. If $\mu_i \in \Gamma_1$, then $G \subset X = i_{\mu_i}(c_{\mu_j}(X))$. By (a), G is μ_{ij}-preopen. □

Theorem 3.9. *Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A be any subset of X. Then the following are equivalent.*

(a) $c_{\pi_{ij}}(A) = X$.
(b) *If B is any μ_{ij}-preclosed subset of X such that $A \subset B$, then $B = X$.*
(c) *Every nonempty μ_{ij}-preopen set has a nonempty intersection with A.*
(d) $i_{\pi_{ij}}(X - A) = \emptyset$.

Proof. (a) $\Rightarrow$ (b)

If B is any μ_{ij}-preclosed set such that $A \subset B$, then $X = c_{\pi_{ij}}(A) \subset c_{\pi_{ij}}(B) = B$ and $B = X$.

(b) $\Rightarrow$ (c)

If G is any nonempty μ_{ij}-preopen set such that $G \cap A = \emptyset$, then $A \subset X - G$ and $X - G$ is μ_{ij}-preclosed. By hypothesis, $X - G = X$ and $G = \emptyset$ which is a contradiction.

Therefore, $G \cap A = \emptyset$.

(c) $\Rightarrow$ (d)

Suppose $i_{\pi_{ij}}(X - A) \neq \emptyset$, then $i_{\pi_{ij}}(X - A)$ is a nonempty μ_{ij}-preopen set such that $i_{\pi_{ij}}(X - A) \cap A = \emptyset$, a contradiction.

Therefore, $i_{\pi_{ij}}(X - A) = \emptyset$.

(d) $\Rightarrow$ (a)

$i_{\pi_{ij}}(X - A) = \emptyset$ implies that $X - i_{\pi_{ij}}(X - A) = X$ and $c_{\pi_{ij}}(A) = X$.

□

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