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NON-SPLIT PERFECT TRIPLE CONNECTED DOMINATION NUMBER OF SEMI PRODUCT OF PATHS AND CYCLES

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ABSTRACT. Recently the concept of non-split Perfect Triple connected domination number was introduced by G. Mahadevan et.al., and obtained many interesting results along with some product related graphs. A subset *S* of *V* of a non-trivial graph *G* is said to be non-split perfect triple connected dominating set, if *S* is a triple connected dominating set and <*V-S*> is connected and has at least one perfect matching. The minimum cardinality taken over all non-split perfect triple connected dominating sets in *G* is called the non-split perfect triple connected domination number of *G* and is denoted by $\gamma_{nsptc}(G)$. In this paper, we investigate this parameter for various semi product of paths and cycles

1. INTRODUCTION

By a graph we mean a finite, simple, connected and undirected graph G(V,E), where V denotes its vertex set and E its edge set. Unless otherwise stated, the graph G has p vertices and q edges. We denote a path on *m* vertices by P_m. The concept of triple connected graphs was introduced by J. Paulraj Joseph et.al., A graph G is said to be triple connected if any three vertices lie on a path in G. A dominating set S is said to be triple connected dominating set, if the sub graph <S> is triple connected. The minimum cardinality taken over all triple

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connected dominating sets is called the triple connected domination number of a graph G and it is denoted by $\gamma_{tc}(G)$. The concept of non-split perfect triple connected domination number was introduced by G. Mahadevan et. al., A subset S of *V* of a non-trivial graph G is said to be non-split perfect triple connected dominating set, if *S* is a triple connected dominating set and the induced sub graph $< V - S >$ is connected and has a perfect matching. The minimum cardinality taken over all non-split perfect triple connected dominating sets is called the non-split perfect triple connected domination number of G and is denoted by $\gamma_{nsptc}(G)$. In [3],the authors find the Non-Split Perfect Triple Connected Domination number on Different Product of Paths. Motivated by the above, in this paper we find the non-split perfect triple connected domination number of semi product of paths and cycles.

For further reference see [1, 2, 4, 5].

2. Semi Strong Product of Paths

In this section we find the non-split perfect triple connected domination number of the semi strong product of paths. We recall the existing definition of Semi strong product of graphs: The Semi strong product of the graphs G and H is denoted by G$|\times|$H, whose vertex set is $V(G) \times V(H)$. Two vertices (g, h) and (g', h') are adjacent in $G| \times |H$ if $gg' \in E(G)$ and $hh' \in E(H)$ (or) $h = h'$and $gg' \in E(G)$. That is $V(G| \times |H)$=$\{(g, h)\ g \in (G)\ and\ h \in (H)\}$, $E(G| \times |H) = \{(g, h)(g', h') \gg' \in E(G)\ and\ hh' \in (H)\ (or)\ h = h'\ and\ gg' \in E(G)\}$. The number of vertices in the Semi Strong product of $G| \times |H$is $|V(G)||V(H)|$.

Theorem 2.1. *For any*$n \geq 3$*, we have* $\gamma_{nsptc}(P_2|\times(P_n)) = \begin{cases} n, & if\ n\ is\ even \\ n+1, & if\ n\ is\ odd \end{cases}$

Proof. Let P_2 and P_n be the paths on 2 vertices and n vertices respectively Then the semi strong product of P_2 and P_n is denoted by $P_2| \times |P_n$ has 2n vertices. Here $V(P_2| \times |P_n) = \{(u_i, v_j)/1 \leq i \leq 2, 1 \leq j \leq n\}$.

Case 1: n is even Consider the set $S = \{(u_k, v_l) : k = 1, l = 1, 3, 5, ...n-1\ and\ k = 2, l = 2, 4, 6, ...n\}$. Then $|S| = n.$

Claim: S is a non-split perfect triple connected dominating set in $P_2| \times |P_n$.

Here every vertex in $V - S$ is adjacent to some vertex in S. This gives that S is a dominating set in $P_2| \times |P_n$. Also $< S >= P_n$, S is a triple connected set. $< V - S >= P_n$,which is connected. Here n is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $P_2| \times |P_n$.

Claim: $\gamma_{nsptc}(P_2| \times |P_n) = |S|$.

For any cases, $|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $P_2| \times |P_n$. Therefore S is the non-split perfect triple connected dominating set in $P_2| \times |P_n)$which is minimum.

Case 2: n is odd.

Consider the set $S = \{(u_k, v_l) : k = 1, l = 1, 3, 5, ...n and k = 2, l = 2, 4, 6, ...n - 1, n\}$. Then $|S| = n + 1$.

Claim: S is a non-split perfect triple connected dominating set in $P_2| \times |P_n$.

Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $P_2| \times |P_n$. Also$< S >= P_{n+1}$, S is a triple connected set. $< V - S >= P_{n-1}$,which is connected. Here $n - 1$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $P_2| \times |P_n$.

Claim: $\gamma_n sptc(P_2| \times |P_n) = |S|$.

For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $P_2|\times|P_n$. Therefore S is the non-split perfect triple connected dominating set in$P_2| \times |P_n$ which is minimum. Hence

$$\gamma_{nsptc}(P_2| \times |P_n) = \begin{cases} n, & n \text{ is even} \\ n+1, & n \text{ is odd} \end{cases}.$$

□

Observation 2.1. *If there exists a Triple connected dominating set S in $P_m| \times |P_n$, then $< V - S >$ neither connected nor has a perfect matching. Hence, for any $m, n \geq 3$, $\gamma_{nsptc}(P_m| \times |P_n)$ does not exist.*

3. Semi Strong Product of Cycles

In this section we find the non-split perfect triple connected domination number of the semi strong product of cycles.

Theorem 3.1. *For any $3 \leq n$ we have $\gamma_{nsptc}(C_3| \times |C_n) = n$.*

Proof. Let C_3 and C_n be the cycles on 3 vertices and n vertices respectively Then the semi strong product of C_3 and C_n is denoted by $C_3| \times |C_n)$ has $3n$ vertices.

Here $V(C_3| \times |C_n) = \{(u_i, v_j)/1 \leq i \leq 3, 1 \leq j \leq n\}$.

Case 1: n is odd.

Consider the set $S = \{(u_k, v_l) : k = 1, l = 1, 3, 5, ...n \, and \, k = 2, l = 2, 4, 6, ...n-1\}$. Then $|S| = n$.

Claim: S is a non-split perfect triple connected dominating set in $C_3| \times |C_n)$. Here every vertex in $V - S$ is adjacent to some vertex in S. This gives that S is a dominating set in $C_3| \times |C_n$. Also $< S >= P_n$, S is a triple connected set. $< V - S >$, is connected. Here $2n$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_3| \times |C_n$.

Claim: $\gamma_{nsptc}(C_3| \times |C_n) = |S|$. For any cases, $|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_3| \times |C_n)$. Therefore S is the non-split perfect triple connected dominating set in $C_3| \times |C_n$ which is minimum.

Case 2: n is even.

Consider the set $S = \{(u_k, v_l) : k = 1, l = 1, 3, 5, ...n-1 \, and \, k = 2, l = 2, 4, 6, ...n\}$. Then $|S| = n$.

Claim: S is a non-split perfect triple connected dominating set in$C_3| \times |C_n$. Here every vertex in $V - S$ is adjacent to some vertex in S. This gives that S is a dominating set in $C_3| \times |C_n$. Also $< S >= P_n$, S is a triple connected set. $< V - S >$, is connected. Here 2n is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_3| \times |C_n$.

Claim: $\gamma_{nsptc}(C_3| \times |C_n) = |S|$. For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_3| \times |C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_3| \times |C_n$ which is minimum. Hence $\gamma_{nsptc}(C_3| \times |C_n) = n$. □

Theorem 3.2. *For any $n \geq 4$, we have*

$$\gamma_{nsptc}(C_4| \times |C_n) = \begin{cases} 2\lceil \frac{n}{2} \rceil, & n \text{ is odd} \\ 2\lceil \frac{n+1}{2} \rceil, & n \text{ is even} \end{cases}.$$

Proof. Let C_4 and C_n be the cycles on 4 vertices and n vertices respectively Then the semi strong product of C_4 and C_n is denoted by$C_4| \times |C_n$ has 4n vertices. Here$V(C_4| \times |C_n) = \{(u_i, v_j)/1 \leq i \leq 4, 1 \leq j \leq n\}$.

Case 1: n is odd.

Consider the set$S = \{(u_k, v_l) : k = 1, l = 1, 2, 3..., \lceil \frac{n}{2} \rceil - 1; k = 2, l = \lceil \frac{n}{2} \rceil; k = 3, l = \lceil \frac{n}{2} \rceil + 1 \, and \, k = 4, l = \lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil - 1, ...2\}$. Then $|S| = 2\lceil \frac{n}{2} \rceil$.

Claim: S is a non-split perfect triple connected dominating set in $C_4| \times |C_n$.

Here every vertex in V$-$S is adjacent to some vertex in S. This gives that S is a dominating set in$C_4| \times |C_n$. Also $< S >= P_2\lceil \frac{n}{2} \rceil$, S is a triple connected set. $< V - S >$ is connected. Here $4n - 2\lceil \frac{n}{2} \rceil$ is even, $< V - S >$ has a perfect matching. Therefore S is a non-split perfect triple connected dominating set in $C_4| \times |C_n$.

Claim: $\gamma_{nsptc}(C_4| \times |C_n) = |S|$. For any cases, $|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_4| \times |C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_4| \times |C_n$ which is minimum.

Case 2: n is even.

Consider the set $S = \{(u_k, v_l) : k = 1, l = 1, 2, 3..., \lceil \frac{n}{2} \rceil; k = 2, l = \lceil \frac{n}{2} \rceil + 1; k = 3, l = \lceil \frac{n}{2} \rceil + 2 \ and \ k = 4, l = \lceil \frac{n}{2} \rceil + 1, \lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil - 1, ...2\}$. Then $|S| = 2\lceil \frac{n+1}{2} \rceil$.

Claim: S is a non-split perfect triple connected dominating set in $C_4| \times |C_n$. Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $C_4| \times |C_n$. Also $< S >= P_2\lceil \frac{n+1}{2} \rceil$, S is a triple connected set. $< V - S >$ is connected. Here $4n - 2\lceil \frac{n+1}{2} \rceil$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_4| \times |C_n$.

Claim: $\gamma_{nsptc}(C_4| \times |C_n) = |S|$. For any cases, $|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_4| \times |C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_4| \times |C_n$ which is minimum. Hence

$$\gamma_{nsptc}(C_4| \times |C_n) = \begin{cases} 2\lceil \frac{n}{2} \rceil, & n \text{ is odd} \\ 2\lceil \frac{n+1}{2} \rceil, & n \text{ is even} \end{cases}.$$

□

Theorem 3.3. *For any $m, n \geq 5$ we have,*

$$\gamma_{nsptc}(C_m| \times |C_n) = \begin{cases} \lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil) + 1, & m, n \textit{ are odd and } m \equiv 1 (mod\ 3) \\ \lceil \frac{m}{2} \rceil (\lceil \frac{n+1}{2} \rceil), & m \textit{ is odd} \\ \lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil), & \textit{otherwise} \end{cases}.$$

Proof. Let C_m and C_n be the cycles on m vertices and n vertices respectively Then the semi strong product of C_m and C_n is denoted by$C_m| \times |C_n$ has mn vertices. Here$V(C_m| \times |C_n) = \{(u_i, v_j)/1 \leq i \leq m, 1 \leq j \leq n\}$.

Case 1: m,n are odd and $m \equiv 1(mod\ 3)$.

Consider the set$S = \{(u_k, v_l) : k = 1, l = 1, 2, 3..., \lceil \frac{n}{2} - 1 \rceil; k = 2, l = \lceil \frac{n}{2} \rceil; k = 3, l = \lceil \frac{n}{2} + 1 \rceil; k = 4, l = \lceil \frac{n}{2} \rceil; \lceil \frac{n}{2} - 1 \rceil, ...2, 1; ..... \ and \ k = m, l = \lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} - 1, ...2 \rceil\}$.

Then $|S| = \lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil) + 1$.

Claim: S is a non-split perfect triple connected dominating set in $C_m| \times |C_n$.

Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $C_m| \times |C_n$. Also $< S >= P_{\lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil) + 1}$, S is a triple connected set. $< V - S >$, is connected. Here $mn - [\lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil) + 1]$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_m| \times |C_n$.

Claim: $\gamma_{nsptc}(C_m| \times |C_n) = |S|$.

For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_m| \times |C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_m| \times |C_n$ which is minimum.

Case 2: n is even.

Consider the se t$S = \{(u_k, v_l) : k = 1, l = 1, 2, 3..., \lceil \frac{n}{2} - 1 \rceil; k = 2, l = \lceil \frac{n}{2} \rceil; k = 3, l = \lceil \frac{n}{2} + 1 \rceil; k = 4, l = \lceil \frac{n}{2} \rceil; \lceil \frac{n}{2} - 1 \rceil, ...2, 1; ..... \; and \; k = m, l = \lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} - 1, ...2 \rceil\}$.

Then $|S| = \lceil \frac{m}{2} \rceil (\lceil \frac{n+1}{2} \rceil)$.

Claim: S is a non-split perfect triple connected dominating set in $C_m| \times |C_n$.

Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $C_m| \times |C_n$. Also $< S >= P_{\lceil \frac{m}{2} \rceil (\lceil \frac{n+1}{2} \rceil)}$, S is a triple connected set. $< V - S >$, is connected. Here $mn - [\lceil \frac{m}{2} \rceil (\lceil \frac{n+1}{2} \rceil)]$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_m| \times |C_n$.

Claim: $\gamma_{nsptc}(C_m| \times |C_n) = |S|$.

For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_m| \times |C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_m| \times |C_n$ which is minimum.

Case 3:

Consider the se t$S = \{(u_k, v_l) : k = 1, l = 1, 2, 3..., \lceil \frac{n}{2} - 1 \rceil; k = 2, l = \lceil \frac{n}{2} \rceil; k = 3, l = \lceil \frac{n}{2} + 1 \rceil; k = 4, l = \lceil \frac{n}{2} \rceil; \lceil \frac{n}{2} - 1 \rceil, ...2, 1; ..... \; and \; k = m, l = \lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} - 1, ...2 \rceil\}$.

Then $|S| = \lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil)$.

Claim: S is a non-split perfect triple connected dominating set in $C_m| \times |C_n$.

Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $C_m| \times |C_n$. Also $< S >= P_{\lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil)}$, S is a triple connected set. $< V - S >$, is connected. Here $mn - [\lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil)]$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_m| \times |C_n$.

Claim: $\gamma_{nsptc}(C_m| \times |C_n) = |S|$
For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_m| \times |C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_m| \times |C_n$ which is minimum Hence

$$\gamma_{nsptc}(C_m| \times |C_n) = \begin{cases} \lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil) + 1, & m, n \text{ are odd and } m \equiv 1 (mod\ 3) \\ \lceil \frac{m}{2} \rceil (\lceil \frac{n+1}{2} \rceil), & m \text{ is odd} \\ \lceil \frac{m}{2} \rceil (\lceil \frac{n}{2} \rceil), & \text{otherwise} \end{cases} .$$

□

4. Semi Lexico graphic product of Paths

In this section we find the non-split perfect triple connected domination number of the semi lexico graphic product of paths. We recall the existing definition of Semi lexico graphic product of graphs: The Semi lexico graphic product of the graphs G and H is denoted by G◯H, whose vertex set is $V(G) \times V(H)$. Two vertices (g, h) and (g', h') are adjacent in $G \bigcirc H$ if $gg' \in E(G)$ (or) $h = h'$ and $gg' \in E(G)$. That is $V(G \bigcirc H) = \{(g, h)\ g \in (G)\ and\ h \in (H)\}$, $E(G \bigcirc H) = \{(g, h)(g', h') : gg' \in E(G)\ (or)\ h = h'\ and\ gg' \in E(G)\}$. The number of vertices in the Semi lexico graphic product of $G \bigcirc H$is $|V(G)||V(H)|$.

Theorem 4.1. *For any $n \geq 3$, we have $\gamma_{nsptc}(P_2 \bigcirc P_n) = 4$.*

Proof. Let P_2 and P_n be the paths on 2 vertices and n vertices respectively Then the semi lexicographic product of P_2 and P_n is denoted by $P_2 \bigcirc P_n$ has 2n vertices. Here $V(P_2 \bigcirc P_n) = \{(u_i, v_j)/1 \leq i \leq 2, 1 \leq j \leq n\}$.
Consider the set $S = \{(u_1, v_1), (u_1, v_2), (u_2, v_1), (u_2, v_2)\}$. Then $|S| = 4$.
Claim: S is a non-split perfect triple connected dominating set in $P_2 \bigcirc P_n$. Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $P_2 \bigcirc P_n$. Also $< S >= C_4$, S is a triple connected set. $< V - S >$ is connected. Here$2n - 4$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $P_2 \bigcirc P_n$.
Claim: $\gamma_{nsptc}(P_2 \bigcirc P_n) = |S|$.
For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $P_2 \bigcirc P_n$. Therefore S is the non-split perfect triple connected dominating set in $P_2 \bigcirc P_n$ which is minimum. Hence $\gamma_{nsptc}(P_2 \bigcirc P_n) = 4$. □

Theorem 4.2. *For any $3 \leq m \leq n$ and m is even , we have $\gamma_{nsptc}(P_m \bigcirc P_n) = m$.*

Proof. Let P_m and P_n be the paths on m vertices and n vertices respectively Then the semi lexicographic product of P_m and P_n is denoted by $P_m \bigcirc P_n$ has mn vertices. Here $V(P_m \bigcirc P_n) = \{(u_i, v_j)/1 \leq i \leq m, 1 \leq j \leq n\}$.
Consider the set $S = \{(u_i, v_1) : 1 \leq i \leq m\}$. Then $|S| = m$.
Claim: S is a non-split perfect triple connected dominating set in $P_m \bigcirc P_n$. Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $P_m \bigcirc P_n$. Also $< S >= P_m$, S is a triple connected set. $< V - S >= P_m \bigcirc P_(n-1)$ is connected. Here $m(n-1)$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $P_m \bigcirc P_n$.
Claim: $\gamma_{nsptc}(P_m \bigcirc P_n) = |S|$.
For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $P_m \bigcirc P_n$. Therefore S is the non-split perfect triple connected dominating set in $P_m \bigcirc P_n$ which is minimum. Hence $\gamma_{nsptc}(P_m \bigcirc P_n) = m$. □

Note: If m is odd, then for any triple connected dominating set S in $P_m \bigcirc P_n$, $< V - S >$ has no perfect matching. This gives that $\gamma_{nsptc}(P_m \bigcirc P_n)$ doesn't exist.

5. Semi Lexico graphic Product of Cycles

In this section we find the non-split perfect triple connected domination number of the semi lexico graphic product of cycles.

Theorem 5.1. *For any $3 \leq m \leq n$, we have*

$$\gamma_{nsptc}(C_m \bigcirc C_n)) = \begin{cases} m+1, & m \textit{ is odd}, n \textit{ is even} \\ m, & \textit{otherwise} \end{cases}.$$

Proof. Let C_m and C_n be the cycles on m vertices and n vertices respectively Then the semi lexicographic product of C_m and C_n is denoted by $C_m \bigcirc C_n$ has mn vertices. Here $V(C_m \bigcirc C_n) = \{(u_i, v_j)/1 \leq i \leq m, 1 \leq j \leq n\}$.
Case I: m is odd and n is even.
Consider the set $S = \{(u_i, v_1), (u_2, v_n) : 1 \leq i \leq m\}$. Then $|S| = m + 1$.
Claim: S is a non-split perfect triple connected dominating set in $C_m \bigcirc C_n$.
Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $C_m \bigcirc C_n$. Also $< S >= P(m-2) \cup P_3$, S is a triple connected

set. $< V - S >= C_m \bigcirc C_(n-2) \cup P_n - 1$ is connected. Here $m(n-1)-1$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_m \bigcirc C_n$.

Claim: $\gamma_{nsptc}(C_m \bigcirc C_n) = |S|$.

For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_m \bigcirc C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_m \bigcirc C_n$ which is minimum. Hence $\gamma_{nsptc}(C_m \bigcirc C_n) = m + 1$

Case II: m is odd and n is odd.

Consider the set $S = \{(u_i, v_1) : 1 \leq i \leq m\}$. Then $|S| = m$.

Claim: S is a non-split perfect triple connected dominating set in $C_m \bigcirc C_n$.

Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $C_m \bigcirc C_n$. Also$< S >= C_m$, S is a triple connected set. $< V - S >= C_m \bigcirc C_n - 1$ is connected. Here $m(n-1)$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_m \bigcirc C_n$.

Claim: $\gamma_{nsptc}(C_m \bigcirc C_n) = |S|$.

For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_m \bigcirc C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_m \bigcirc C_n$ which is minimum. Hence $\gamma_{nsptc}(C_m \bigcirc C_n) = m$.

Case III: m is even and n is even or n is odd.

Consider the set $S = \{(u_i, v_1) : 1 \leq i \leq m\}$. Then $|S| = m$.

Claim: S is a non-split perfect triple connected dominating set in $C_m \bigcirc C_n$.

Here every vertex in V-S is adjacent to some vertex in S. This gives that S is a dominating set in $C_m \bigcirc C_n$. Also$< S >= C_m$, S is a triple connected set. $< V - S >= C_m \bigcirc C_n - 1$ is connected. Here $m(n-1)$ is even, $< V - S >$ has a perfect matching. Therefore S is non-split perfect triple connected dominating set in $C_m \bigcirc C_n$.

Claim: $\gamma_{nsptc}(C_m \bigcirc C_n) = |S|$.

For any cases,$|S'| < |S|, < V - S' >$ has an odd number of vertices, it has no perfect matching in $C_m \bigcirc C_n$. Therefore S is the non-split perfect triple connected dominating set in $C_m \bigcirc C_n$ which is minimum. Hence $\gamma_{nsptc}(C_m \bigcirc$

$C_n) = m$. Therefore

$$\gamma_{nsptc}(C_m \bigcirc C_n)) = \begin{cases} m+1, & m \text{ is odd}, n \text{ is even} \\ m, & \text{otherwise} \end{cases}.$$

□

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