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POWER 3 MEAN LABELING IN THE CONTEXT OF DUPLICATION OF GRAPH ELEMENTS

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ABSTRACT. In this paper, we contribute power 3 mean labeling for various graphs resulted from the Duplication of graph elements.

1. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges),then the labeling is called a vertex labeling (or an edge labeling). Here we consider simple,finite,undirected and connected graph $G = (V, E)$. For all standard terminology and notation we follow Chartrand and Lesniak [1].

Definition 1.1. *A graph G with p vertices and q edges is called a power 3 mean graph, if it is possible to label the vertices $v \in V$ with distinct labels $f(x)$ from $1, 2, \ldots, q+1$ in such a way that in each edge $e = uv$ is labelled with $f(e = uv) = \left\lceil (\frac{x^3+y^3}{2})^{\frac{1}{3}} \right\rceil$ or $\left\lfloor (\frac{x^3+y^3}{2})^{\frac{1}{3}} \right\rfloor$. Then, the edge labels are distinct. In this case, f is a power 3 mean labeling of G and G is called a power 3 Mean Graph.*

Definition 1.2. *Two vertices of a graph which are adjacent are said to be neighbours. The set of all neighbours of a vertex V is called the neighbourhood set denoted as $N(V)$.*

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Definition 1.3. *Duplication of a vertex v_k of graph G produces a new graph G' by adding a vertex v'_k with $N(v_k) = N(v'_k)$.*

Definition 1.4. *Duplication of an edge $e = uv$ of graph G produces a new graph G' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.*

Definition 1.5. *Duplication of an vertex v_k by a new edge $e' = u'v'$ in a graph G produces a new graph G' such that $N(v') = \{v_k, u'\}$ and $N(u') = \{v_k, v'\}$.*

Definition 1.6. *Duplication of an edge $e = uv$ by a vertex v' is a graph G produces a new graph G' such that $N(v') = \{u, v\}$.*

Remark 1.1. *If G is a power 3 Mean Graph then the vertices gets labels from $1, 2, \ldots, q+1$ and the edges get labels from $1, 2, \ldots, q$.*

Remark 1.2. *If G is a power 3 Mean Graph then 1 must be a label of one of the vertices of G , since an edge should get label 1.*

Remark 1.3. *If G is a k-regular graph (with $k > 3$) then G is not power 3 mean graph.*

Remark 1.4. *If u gets label 1, then any edge incident with u must get label 1 or 2. Obviously, this vertex must have a degree ≤ 2.*

Remark 1.5. *If $p > q+1$, then the graph $G = (p, q)$ is not a power 3 Mean graph, since we don't have sufficient labels from $1, 2, \ldots, q+1$ for vertices of G.*

2. Main Results

Theorem 2.1. *The graph obtained by duplication of an arbitrary vertex v_k in cycle C_n is a power 3 mean graph.*

Proof. Let $v_1 v_2 \ldots v_n$ be the vertices of a cycle C_n. Without loss of generality, we duplicate the vertex v_1 thus added vertex is v'. Now the resultant graph G will have $n+1$ vertices and $n+2$ edges.

Define a function $f : V(G) \to \{1, 2, \ldots, n+3\}$ and consider following two cases.

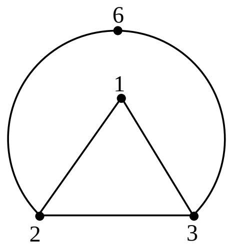

FIGURE 1

Case (i): When n=3.
The graph obtained by duplication of a vertex in cycle C_3 and its power 3 mean labeling is shown in Figure 1.

Case (ii): When $n \neq 3$. $f(v') = n + 2$ $f(v_i) = i; 1 \leq i \leq n$.
In view of the above labeling pattern, we have distinct edge labels from $\{1, 2, \ldots, n+2\}$. The graph obtained by duplication of a vertex in C_5 and its power 3 mean labeling is shown in Figure 2.

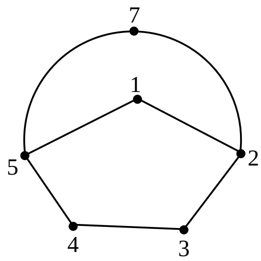

FIGURE 2

Hence from case (i) and case (ii) we have the graph obtained by duplication of an arbitrary vertex V_x in cycle C_n is power 3 mean graph. □

Theorem 2.2. *The graph obtained by duplication of an arbitrary edge e_k in cycle C_n is a power 3 mean graph.*

Proof. Let $e_1 e_2 \ldots e_n$ be the edges of a cycle C_n. Without loss of generality, if we duplicate the edge e_1 added verticesv_1' and v_2'. Now the resultant graph G will have $n + 2$ vertices and $n + 3$ edges.

To define $f : V(G) \to \{1, 2, \ldots, n + 4\}$, we consider following two cases.

Case (i): When n=3.

The graph obtained by duplication of an edge is cycle C_3 and its power 3 mean labeling is shown in Figure 3.

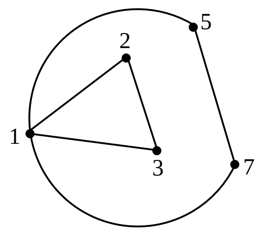

FIGURE 3

Case (ii): When $n \neq 3$. $f(v_1') = n+2$, $f(v_2') = n+4$, $f(v_i) = i; 1 \leq i \leq n$.

In view of the above labeling pattern, we have distinct edge labels. The graph obtained by duplication of a vertex in C_6 and its power 3 mean labeling is shown in Figure 4.

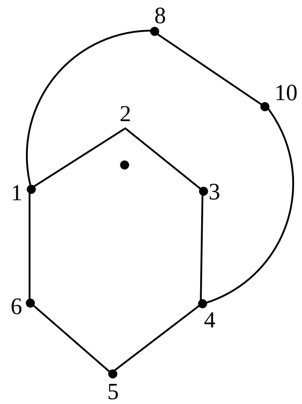

FIGURE 4

Hence from case (i) and case (ii) we have the graph obtained by duplication of an arbitrary edge e_k is cycle C_n is power 3 mean graph. □

Theorem 2.3. *The graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n is a power 3 mean graph.*

Proof. Let $v_1 v_2 \dots v_n$ be the vertices of a cycle C_n. Without loss of generality we duplicate the vertex v_n by an edge e_{n+1} with end vertices as v_1' and v_2'. Now the resultant graph G will have $n+2$ vertices and $n+3$ edges. We define $f : V(G) \to \{1, 2, \ldots, n+4\}$ as follows

$f(v'_1) = n+2,$
$f(v'_2) = n+4,$
$f(v_i) = i; 1 \leq i \leq n.$

In view of the above labeling pattern, we have distinct edge labels from $\{1, 2, \ldots, n+3\}$.

The graph obtained by duplication of a vertex by a new edge in C_5 and its power 3 mean labeling is shown in Figure 5.

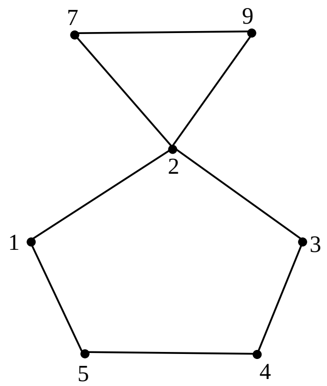

FIGURE 5

Hence the graph obtained by duplicating an arbitrary vertex by a new edge in cycle C_n is power 3 mean graph. □

Theorem 2.4. *The graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n is a power 3 mean graph.*

Proof. Let $v_1 v_2 \ldots v_n$ be the vertices and $e_1 e_2 \ldots e_n$ be the edges of a cycle C_n. Without loss of generality, we duplicate the edge $v_n v_1$ by a vertex v'.

Now the resultant graph G will have n+1 vertices and n+2 edges. Define a function $f : V(G) \to \{1, 2, \ldots, n+3\}$, we consider following two cases.

Case (i): When n=3.
The graph and power 3 mean labeling is shown in Figure 6.

Case (ii): When $n \neq 3$. $f(v'_1) = n+2$, $f(v'_2) = n+4$, $f(v_i) = i; 1 \leq i \leq n.$

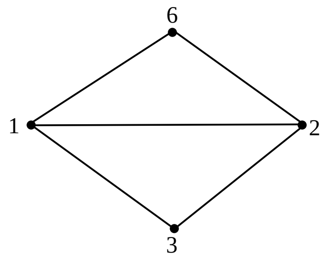

FIGURE 6

In view of the above labeling pattern we have distinct edge labels. The graph obtained by duplication of a vertex in C_6 and its power 3 mean labeling is shown in Figure 7.

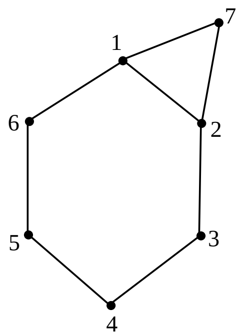

FIGURE 7

Hence from case (i) and case (ii), we have the graph obtained by duplication of an arbitrary edge e_k is cycle C_n is power 3 mean graph. □

Theorem 2.5. *The graph obtained by duplication of an arbitrary vertex v_k in path P_n is a power 3 mean graph.*

Proof. Let $v_1v_2\dots v_n$ be the vertices of path P_n. We can duplicate the vertex v_k thus added vertex v'. We consider following two cases.

Case (i):
When pendant vertex is duplicated.Then the resultant graph G will have $n+1$ vertices and n edges. We define $f: V(G) \to \{1, 2, \dots, n+1\}$ as follows

$f(v') = n+1,$

$f(v_i) = i; 1 \leq i \leq n.$

The graph obtained by duplication of a pendant vertex in P_5 and its power 3 mean labeling is shown in Figure 8.

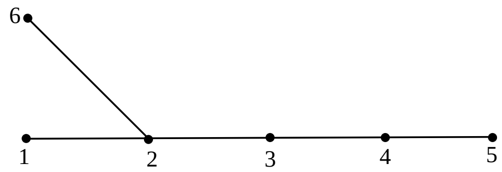

FIGURE 8

Case (ii): When other than pendant vertex is duplicated. Then the resultant graph G will have n+1 vertices and n+1 edges. We define $f : V(G) \to \{1, 2, \ldots, n+1\}$ as follows:

$f(v') = n + 2$,

$f(v_i) = i; 1 \leq i \leq n$.

In view of the above labeling pattern we have distinct edge labels. The graph obtained by duplication of other than pendant vertex in P_5 and its power 3 mean labeling is shown in Figure 9.

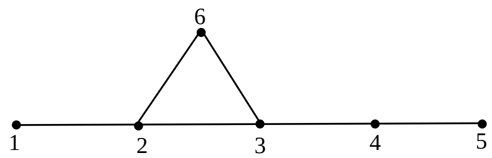

FIGURE 9

Hence from case (i) and case (ii), we have the graph obtained by duplication of an arbitrary vertex v_k is path P_n is power 3 mean graph. □

Theorem 2.6. *The graph obtained by duplication of an arbitrary edge e_k in path P_n is a power 3 mean graph.*

Proof. Let $e_1 e_2 \ldots e_n$ be the edges of a path P_n. We duplicate the edge e_k thus added vertices are v_1' and v_2'. We consider following two cases.

Case (i): When pendant edge is duplicated. Then the resultant graph G will have n+1vertices and n+1 edges.

We define $f : V(G) \to \{1, 2, \ldots, n+2\}$ as follows:

$f(v'_1) = n + 1,$
$f(v'_2) = n + 2,$
$f(v_i) = i; 1 \le i \le n.$

The graph obtained by duplication of a pendant edge in P_5 and its power 3 mean labeling is shown in Figure:10

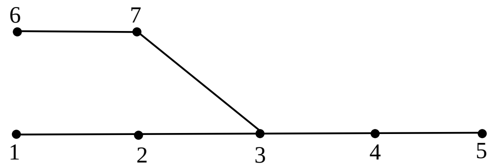

FIGURE 10

Case (ii): When other than pendant edge is duplicated. Then the resultant graph G will have n+2 vertices and n+2 edges. We define $f : V(G) \to \{1, 2, \ldots, n + 2\}$ as follows:

$f(v'_1) = n + 1,$
$f(v'_2) = n + 2,$
$f(v_i) = i; 1 \le i \le n.$

In view of the above labeling pattern, we have distinct edge labels. The graph obtained by duplication of an edge other than pendant edge in P_5 and its power 3 mean labeling is shown in Figure 11.

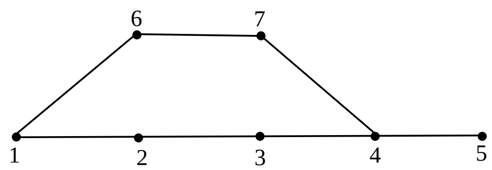

FIGURE 11

Hence from case (i) and case (ii) we have the graph obtained by duplication of an arbitrary edge e_k in path P_n is power 3 mean graph. $\square$

3. Conclusion

Here we discuss power 3 mean labeling in the context of Duplication of graph elements. The results are demonstrated by means of sufficient illustrations which provide better understanding.

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