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JORDAN $*$ DERIVATIONS IN SEMIPRIME INVERSE SEMIRING WITH INVOLUTION

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ABSTRACT. We define and study $*$derivations and Jordan $*$derivation of inverse $*$semiring. In our main result we show that the results concerning Jordan $*$derivation in semiprime rings with involution due to Vukman [14] are considered and classified in the direction of inverse $*$semirings.

1. INTRODUCTION

Let $(S,+,.)$ be a semiring with $(S,+)$ and $(S,.)$ are semigroups with absorbing Zero i.e., $u+0=u=0+u$ and $u0=0=0u$ for all $u\in S$, and multiplication is distributes over addition on both sides [5]. Karvellas [10] introduced, S is an inverse semiring if for all $u\in S$ there exists a unique element $u^{'}\in S$ satisfying the condition $u+u^{'}+u=u$ and $u^{'}+u+u^{'}=u^{'}$, $u^{'}$ is called pseudo inverse of u. In [8] Inverse semiring S which satisfies A_2 condition of [2] is studied as MA Semiring. That is $u+u^{'}$ lies in the center $Z(S)$ of S, for every $u\in S$. It is independently studied in [9]. For examples we refer readers [8, 9, 11, 13]. Karvellas [10] proved that $(u,v)^{'}=u^{'}.v=u.v^{'}$ and $u^{'}v^{'}=uv$, for all $u,v\in S$. Recall that if S is prime then $uSv=(0)$ in case $u=0$ or $v=0$ and semiprime if $uSu=(0)$ in case $u=0$. S is said to be 2-torsion free if $2u=0$, $u\in S$ in case $u=0$. Following [1,7] S is said to be a semiring with an involution $*$ if there exists an mapping S into itself, then for all $u,v\in S$, $(u+v)^*=v^*+u^*,(uv)^*=v^*u^*,(u^*)^*=u$. Semiring equipped with an involution is called a semiring with involution or $*$ semiring.

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Jordan derivation in prime rings are introduced by Herstein [6]. Then Bresar [4] developed Herstein$'s$ work in semiprime rings. We introduce and study Jordan $*$ derivation and we will establish some results of [3, 14] in inverse $*$semiring. We continue to define the notion of $*$derivation and Jordan $*$ derivation as J is a additive mapping of S into itself, if for all $x, y\in S$, $J(xy)+J^{'}(x)y^{*}+x^{'}J(y)=0$ and $J(x^2)+J^{'}(x)x^{*}+x^{'}J(x)=0$, also J is said to be Jordan triple $*$ derivation, if for all $x, y\in S$ $J(xyx)+J^{'}(x)y^{*}x^{*}+x^{'}J(y)x^{*}+x^{'}yJ(x)=0$. In fact, we prove that an additive mapping $J : S \to S$ on a 2-torsion free semiprime inverse $*$ semiring S, satisfying Jordan triple $*$derivation, then J is a Jordan $*$derivation. For example, Let R be a commutative ring and S is subsemiring of the semiring of all two sided ideals of R, then $S_1 = \{(m, \alpha), m \in R, \alpha \in S\}$, we define $\oplus$ and $\odot$ on S as follows. $(m_1, \alpha) \oplus (m_2, \beta) = (m_1 + m_2, \alpha + \beta)$ and $(m_1, \alpha) \odot (m_2, \beta) = (m_1 m_2, \alpha\beta)$ For all $\alpha, \beta \in S$. Let $u = (m, \{0\}) \in S_1$ be fixed, so $J(x) = [u, x]$ and $x^* = x$ for all $x \in S_1$ is a Jordan $*$ derivation.

We consider the lemmas given below.

2. Results on Inverse Semiring

Lemma 2.1. *[11] If S is an inverse semiring and $u, v\in S$ then $u + v = 0$ implies $u = v^{'}$.*

Lemma 2.2. *In a 2- torsion free semiprime inverse $*$ Semiring S, $mx^*n^*+nxm = 0$ for all $x\in S$ and some $m, n\in S$, then $mn = nm = 0$. Further if S is prime then either $m = 0$ or $n = 0$.*

Proof. We have

$$mx^*n^* + nxm = 0, x\in S. \tag{2.1}$$

Replacing x by ynx in the above we obtain $m(ynx)^*n^*+n(ynx)m = 0$ $(mx^*n^*)y^*n^*+ nynxm = 0$. Applying Lemma 2.1 in (2.1) and using it in the above, we get $(n'xm)y^*n^* + nynxm = 0$.

$$nxnym + nynxm = 0. \tag{2.2}$$

Replacing y by x in (2.2), and S is 2-torsion free we get $nxnxm = 0$, Applying Lemma 2.1 in (2.1) and using it in the above, we get

$$nxm^{'}x^*n^* = 0. \tag{2.3}$$

In (2.2), Replacing y by xmy gives $nxnxmym+nxmynxm = 0$. Applying Lemma 2.1 in (2.1) and using it in above, we get $nxm'x^*n^*ym + nxmynxm = 0$ Using (2.3) in the above, we get $nxmynxm = 0$. By the Semiprimeness of S implies

$$nxm = 0. \tag{2.4}$$

Hence we conclude that $mnxmn = 0$. Thus we get $mn = 0$. By the similar arguments, we obtain $nm = 0$. If S is prime, (2.4) gives $n = 0$ or $m = 0$. Therefore we proved the lemma. □

Lemma 2.3. *Let J be Jordan $*$ derivation of 2- torsion free semiprime inverse $*$ semiring S into itself with the following properties hold:*

(i) $J(mnm) + J^{'}(m)n^*m^* + m^{'}J(n)m^* + m^{'}nJ(m) = 0$,
(ii) $J(mnr+rnm)+J^{'}(m)n^*r^*+m^{'}J(n)r^*+m^{'}nJ(r)+J^{'}(r)n^*m^*+r^{'}J(n)m^*+r^{'}nJ(m) = 0$, *for all* $m, n\in S$.

Proof. We have

$$J(m^2) + J^{'}(m)m^* + m^{'}J(m) = 0.$$

Linearization of the above, we get

$$J(mn + nm) + J^{'}(m)n^* + J^{'}(n)m^* + m^{'}J(n) + n^{'}J(m) = 0. \tag{2.5}$$

From the proof of Lemma 4.1 in [12], we obtain the result (i) by generalizing relation (2.5) in two different ways. We can easily prove (ii) by using the linearization of (i). □

3. Main Result

Theorem 3.1. *Let J be an additive mapping on a suitable torsion free semiprime inverse $*$ semiring S into itself, which satisfies for all $m, n\in S$. $J(mnm)+J^{'}(m)n^*m^* + m^{'}J(n)m^* + m^{'}nJ(m) = 0$. Then J is a Jordon $*$ derivation.*

Proof. We have

$$J(mnm) + J^{'}(m)n^*m^* + m^{'}J(n)m^* + m^{'}nJ(m) = 0 forallm, n\in S. \tag{3.1}$$

Substituting mnm for n in (3.1) gives

$$J(m^2nm^2) + J^{'}(m)m^*n^*m^{*^2} + m^{'}J(mnm)m^* + m^2nm'J(m) = 0.$$

Applying Lemma 2.1 in (3.1) and using it in the above, we get

$$\begin{aligned} J(m^2nm^2) + J^{'}(m)m^*n^*m^{*^2} + m^{'}J(m)n^*m^{*^2} \\ + m^2J^{'}(n)m^{*^2} + m^2n^{'}J(m)m^* + m^2nm'J(m) = 0 \end{aligned} \tag{3.2}$$

Substituting m^2 for m in (3.1), we get

$$J(m^2nm^2) + J^{'}(m^2)n^*m^{*^2} + m^2J'(n)m^{*^2} + m^2n'J(m^2) = 0.$$

Applying Lemma 2.1 in the above and using it in (3.2), we get

$$\begin{aligned} &(J(m^2) + J^{'}(m)m^* + m^{'}J(m))n^*m^{*^2} + m^2n(J(m^2) + J^{'}(m)m^* \\ &+m^{'}J(m)) + m^2J(n)m^{*^2} + m^2J^{'}(n)m^{*^2} = 0, \end{aligned}$$

which implies

$$G(m)n^*m^{*^2} + m^2nG(m) = 0, \tag{3.3}$$

where $G(m)$ stands for $J(m^2) + J^{'}(m)m^* + m^{'}J(m)$. Relation (3.3) follows to Lemma 2.2 that

$$G(m)m^2 = 0, m \in S \tag{3.4}$$

and

$$m^2G(m) = 0, m \in S. \tag{3.5}$$

Replacing m with $m+n$ in (3.4) and using (3.5), we get

$$\begin{aligned} G(m)n^2 + G(m)(mn+nm) + G(n)m^2 + G(n)(mn+nm) \\ +H(m,n)m^2 + H(m,n)n^2 + H(m,n)(mn+nm) = 0, \end{aligned} \tag{3.6}$$

where $H(m,n)$ stands for $J(mn+nm) + J^{'}(m)n^* + J^{'}(n)m^* + m^{'}J(n) + n'J(m)$ in (3.6), we replace m by m', we get $G(m)n^2 + G(m)(mn+nm)^{'} + G(n)m^2 + G(n)(mn+nm)^{'} + H'(m,n)m^2 + H'(m,n)n^2 + H'(m,n)(mn+nm)^{'} = 0$.

Applying Lemma 2.1 in the above, we get

$$\begin{aligned} G(m)n^2 + G(n)m^2 + H(m,n)(mn+nm) = G(m)(mn+nm) \\ +G(n)(mn+nm) + H(m,n)m^2 + H(m,n)n^2. \end{aligned} \tag{3.7}$$

Substituting (3.7) in (3.6), we get

$$2G(m)(mn+nm) + 2G(n)(mn+nm) + 2H(m,n)m^2 + 2H(m,n)n^2 = 0.$$

Since S is 2-torsion free, we have

$$G(m)(mn+nm) + G(n)(mn+nm) + H(m,n)m^2 + H(m,n)n^2 = 0. \tag{3.8}$$

Replacing m by $2m$ in the above, we get

$$4G(m)(mn+nm)+G(n)(mn+nm)+4H(m,n)m^2+H(m,n)n^2=0.$$

Using (3.8) in the above, we get $3G(m)(mn+nm)+3H(m,n)m^2=0$. Assume S is 3-torsion free, we have

$$G(m)(mn+nm)+H(m,n)m^2=0. \tag{3.9}$$

Post-multiplication of the above relation by $G(m)m$ gives because of (3.5), $G(m)(mn+nm)G(m)m=0$ and

$$G(m)mnG(m)m+G(m)nmG(m)m=0. \tag{3.10}$$

Substituting nm for n in the above $G(m)mnmG(m)m+G(m)nmmG(m)m=0$ and using (3.5), we get $G(m)mnmG(m)m=0$. Pre-multiplying in the above by m, we get $mG(m)mnmG(m)m=0$. It follows that $mG(m)m=0$. Now (3.10) reduces to $G(m)mnG(m)m=0$, which gives

$$G(m)m=0, m\in S. \tag{3.11}$$

Using (3.11) in (3.9), which reduces to $G(m)nm+H(m,n)m^2=0$.

Post- multiplying above by $G(m)$ and using (3.5) , we get $G(m)nmG(m)=0$, and with pre-multiplication of above by m, we get

$$mG(m)=0. \tag{3.12}$$

Putting $m+n$ for m in (3.11) we get $G(m)n+G(n)m=0$ and post-multiplication of the above by $G(m)$ and using (3.12), gives $G(m)nG(m)=0$, since S is Semiprime $G(m)=0$, which gives $J(m^2)+J^{'}(m)m^*+m^{'}J(m)=0$. This means that J is a Jordan $*$ derivation.

This completes the proof of the theorem. □

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