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CODING TECHNIQUES THROUGH TWO STAR GRAPHS AND MEAN LABELING

P. HARIPRABAKARAN, A. MANSHATH, V. SWATHY, AND V. BALAJI[1]

ABSTRACT. In this paper, a method of coding technique is developed using graph labeling and GMJ code. There are two illustrations for coding a text messages using two star graphs $K_{1,\lambda} \wedge K_{1,\mu}$ by applying Mean labeling on it for two different cases $\mu = \lambda + 1$ and $\mu = \lambda + 4$. Pictorial message is presented to induce secrecy by transforming text message into picture code through Mean graph.

1. INTRODUCTION

In [4–6], Uma Maheswari et al. have introduced GMJ coding through a two star and three star with super mean labeling and Fibonacci webs with difference cordial labeling. In [3], Somasundaram and Ponraj have introduced the notion of mean labelings of graphs. In [2], they proved that two star $K_{1,\lambda} \wedge K_{1,\mu}$ is a mean graph if and only if $|\lambda - \mu| \leq 4$. By refering the above results we got motivated and found a few techniques of coding by applying mean labeling on two star graph.

Definition 1.1. *A graph G with p nodes and q links is called a mean graph if there is an one to one function ϕ from the nodes of G to $\{0, 1, 2, \cdots q\}$ such that when each link $\alpha\beta$ is labeled with $(\phi(\alpha) + \phi(\beta))/2$ if $\phi(\alpha) + \phi(\beta)$ is even, and*

[1]*corresponding author*

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$(\phi(\alpha)+\phi(\beta)+1)/2$ *if* $\phi(\alpha)+\phi(\beta)$ *is odd, then the resulting link labels* $\{1,2,\cdots q\}$ *are distinct.*

Definition 1.2. *A wedge is a link which is used for connecting two components of a graph. It is denoted as* $\wedge$, $\varpi(G\wedge) < \varpi(G)$. *where* ϖ *denotes the number of components of the graph.*

1.1. **GMJ coding method:** By assigning numbers to the 26 alphabets of English in a different manner, choosing a suitable labeled graph with a given clue mathematical or non-mathematical, finding the number in the graph for each letter of each word of the given message and presenting the letter codes in a unique way in some form, writing it as a horizontal string or in any other way and creating a picture with the codes after mixing the order of the letters to improve the anonymity of the coded message is referred to as GMJ (Graph Message Jumbled) coding method.

1.2. **Description for Graph Labeling:** By reference [1], the two star graph $G = K_{1,\lambda} \wedge K_{1,\mu}$ is a mean graph iff $|\lambda - \mu| \leq 4$.

There are five cases viz. $\mu = \lambda, \mu = \lambda+1, \mu = \lambda+2, \mu = \lambda+3$ and $\mu = \lambda+4$.

Let $V(G) = \{\alpha, \beta\} \cup \{\alpha_i : 1 \leq i \leq \lambda\} \cup \{\beta_j : 1 \leq j \leq \beta\}$ then G has $\lambda+\mu+2$ nodes and $\lambda+\mu+1$ links. Also, let us consider the case: $\mu = \lambda+1$. The node labeling of $\phi : V(G) \to \{0,1,2,\cdots 2\lambda+2\}$ is defined as follows: $\phi(\alpha) = 0;\ \phi(\beta) = 2\lambda+2;\ \phi(\alpha_i) = 2i; \quad for \quad 1 \leq i \leq \lambda$ and $\phi(\beta_j) = 2j-1 \quad for \quad 1 \leq j \leq \lambda+1$.

The link labels are as follows.

The link label of $\alpha\alpha_i$ is i for $1 \leq i \leq \lambda$ and the link label of $\beta\beta_j$ is $\lambda+j+1$ for $1 \leq j \leq \lambda+1$. The wedge $\lambda_\alpha\mu_1$ is $\lambda+1$.

Therefore, the required link labels of

$$G = \{1,2,3,\cdots,\lambda,\lambda+1,\lambda+2,\ldots,2\lambda+2\}$$

and has $2\lambda+2$ distinct links. Hence all the node and link labels of G are distinct.

Let us consider the Case : $\mu = \lambda+4$. The node labeling of $\phi : V(G) \to \{0,1,2,\cdots 2\lambda+5\}$ is defined as follows: $\phi(\alpha) = 1;\ \phi(\beta) = 2\lambda+4;\ \phi(\alpha_i) = 2i+1; \quad for \quad 1 \leq i \leq \lambda;$
$\phi(\beta_j) = 2j-2 \quad for \quad 1 \leq j \leq \lambda+2$; $\phi(\beta_{\lambda+3}) = 2\lambda+3;\ \phi(\beta_{\lambda+4}) = 2\lambda+5$.

The link labels are as follows.

The link label of $\alpha\alpha_i$ is i + 1 for $1 \leq i \leq \lambda$; The link label of $\beta\beta_j$ is $\lambda + j + 1$ for $1 \leq j \leq \lambda + 2$; The link label of $\beta\beta_{\lambda+3}$ is $2\lambda + 4$ and the link label of $\beta\beta_{\lambda+4}$ is $2\lambda + 5$. The wedge $\lambda\mu_1$ is 1. Therefore, the required link labels of $G = \{1, 2, 3, \cdots, \lambda, \lambda + 1, \lambda + 2, \ldots, 2\lambda + 5\}$ and has $2\lambda + 5$ distinct links.

Hence, all the node and link labels of G are distinct.

1.3. **Procedure for encoding:** Coding Method through graph labeling and GMJ (Graph Message Jumbled) code is referred by [5]. Sharing secrecy becomes very much limited between the sender and the receiver and not to be understood by others. The original intelligible message is known as Plain text. The transformed message is known as Cipher text. An algorithm for transforming an intelligible message into one that is unintelligible by transposition and substitution methods is known as Cipher.

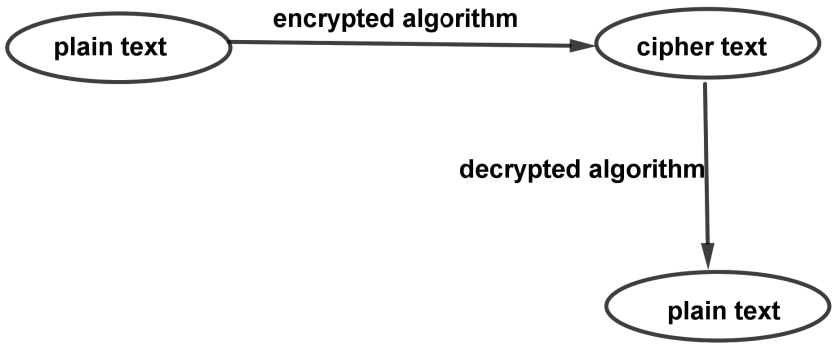

FIGURE 1. Transformation

2. RESULTS AND DISCUSSIONS

2.1. **Illustration:**

(1) **Message:** Twinkling Stars Are Sprinkling in the Whirlpool Galaxy.
(2) **Clue:** The Double square number is pairing up with next one. [The double square number is 16 because it is the only number satisfying $m^n = n^m$ for $n \neq m$. The next number of 16 is 17].
(3) **Graph:** The two star graph $K_{1,16} \wedge K_{1,17}$.

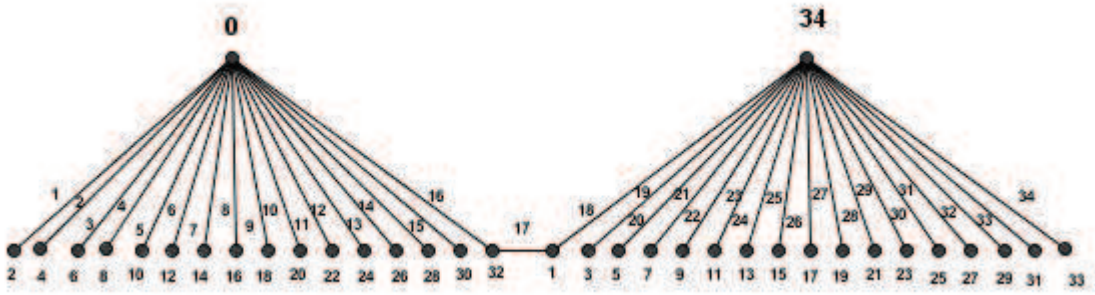

FIGURE 2. $K_{1,\,16} \wedge K_{1,\,17}$

(4) **Labeling:** The Mean Labeling done for $K_{1,16} \wedge K_{1,17}$.

(5) **Numbering of Alphabets:** First Prime Next Others (FPNO)

9	0	1	10	2	11	3	12	13	14	4	15	5
A	B	C	D	E	F	G	H	I	J	K	L	M
16	17	18	6	19	7	20	21	22	8	23	24	25
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

The numbers 0 to 8 are allotted to the letters in the prime position(2, 3, 5, 7, 11, 13, 17, 19, 23) and rest of the alphabets starting with A are given the numbers 9 to 25.
The function for encoding is given below:

$$h(d_{f_i}) = i - 1,\ \ i = 1, 2, \cdots 9$$

Here f_i denotes the i^{th} prime number from 1 to 26 and d_{f_i} denotes f_i^{th} alphabetical letter.

$$h(d_{g_j}) = 8 + j,\ \ j = 1, 2, \cdots 17$$

Here g_j denotes the j^{th} non prime number from 1 to 26 and d_{g_j} denotes g_j^{th} alphabetical letter. $d_{f_i} \neq d_{g_j}$.
Here d_{f_i} and d_{g_j} denotes the 26 alphabets.

(6) **Coding a Letter:** Here ◯ and □ are used to refer the first and second star

respectively. T, P_i, and E_i denote the top vertex, the i^{th} pendent vertex and the i^{th} edge value in order. (W) denotes the wedge value.

For example (T) denotes the number assigned to the top vertex of first star and $\boxed{P_i}$ denotes the number assigned to the i^{th} pendent vertex of second star respectively.

(7) **Coding(wordwise):**

TWINKLING - (P_{10}) (P_4) $\boxed{P_7}$ (E_{16}) (E_4) $\boxed{P_8}$ $\boxed{P_7}$ (P_8) $\boxed{P_2}$

STARS - $\boxed{P_4}$ $\boxed{E_3}$ (E_9) $\boxed{P_{10}}$ $\boxed{P_4}$

ARE - $\boxed{p_5}$ $\boxed{E_2}$ (P_1)

SPRINKLING - $\boxed{P_4}$ (P_9) $\boxed{P_{10}}$ (E_{13}) (E_{16}) (E_4) $\boxed{P_8}$ (E_{13}) (P_8) $\boxed{P_2}$

IN - (E_{13}) (P_8)

THE - $\boxed{E_3}$ (P_6) (P_1)

WHIRLPOOL - (E_8) (P_6) $\boxed{P_7}$ $\boxed{P_{10}}$ (E_{15}) $\boxed{E_1}$ $(\boxed{W})$ $(\boxed{W})$ (E_{15})

GALAXY - (E_3) $\boxed{P_5}$ $\boxed{P_8}$ (E_9) $\boxed{E_6}$ (P_{12})

(8) **Horizontal string:**

(P_{10}) (P_4) $\boxed{P_7}$ (E_{16}) (E_4) $\boxed{P_8}$ $\boxed{P_7}$ (P_8) $\boxed{P_2}$ # $\boxed{P_4}$ $\boxed{E_3}$ (E_9) $\boxed{P_{10}}$ $\boxed{P_4}$ # $\boxed{p_5}$ $\boxed{E_2}$ (P_1) # $\boxed{P_4}$ (P_9) $\boxed{P_{10}}$ (E_{13}) (E_{16}) (E_4) $\boxed{P_8}$ (E_{13}) (P_8) $\boxed{P_2}$ # (E_{13}) (P_8) # $\boxed{E_3}$ (P_6) (P_1) # (E_8) (P_6) $\boxed{P_7}$ $\boxed{P_{10}}$ (E_{15}) $\boxed{E_1}$ $(\boxed{W})$ $(\boxed{W})$ (E_{15}) # (E_3) $\boxed{P_5}$ $\boxed{P_8}$ (E_9) $\boxed{E_6}$ (P_{12})

(9) **Picture Coding:**

2.2. **Illustration:**

(1) **Message:** Twinkling Stars Are Sprinkling in the Whirlpool Galaxy.

(2) **Clue:** The Jupiter is compositing with Uranus. [Jupiter is the fifth planet and Uranus is the seventh planet in our solar system. So the fifth composite number is 10 and seventh composite number is 14].

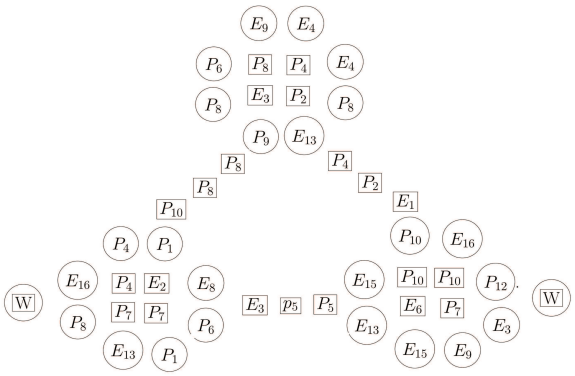

FIGURE 3. Picture Coding FPNO

(3) **Graph:** The two star graph $K_{1,10} \wedge K_{1,14}$.

(4) **Labeling:** The Mean Labeling done for $K_{1,10} \wedge K_{1,14}$.

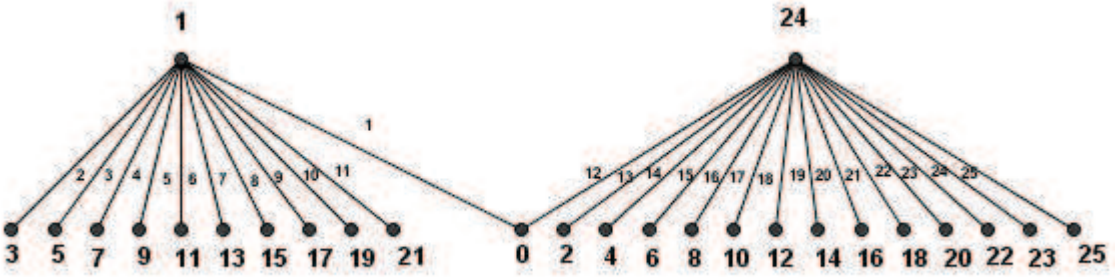

FIGURE 4. $K_{1,\,10} \wedge K_{1,\,14}$

(5) **Numbering of Alphabets:** Multiple of Three And Others (MTAO)

8	9	0	10	11	1	12	13	2	14	15	3	16
A	B	C	D	E	F	G	H	I	J	K	L	M
17	4	18	19	5	20	21	6	22	23	7	24	25
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

The alphabets in the position of multiple of three(C,F,I,L,O,R,U,X) are given the numbers 0 to 7 and rest of the alphabets starting with A are given the numbers 8 to 25.

The function for encoding is given below:

$h(a_{3i}) = i - 1, \quad i = 1, 2, \cdots 8 \qquad h(a_i) = 7 + i, \quad i = 1, 2$
$h(a_i) = 6 + i, \quad i = 4, 5 \qquad h(a_i) = 5 + i, \quad i = 7, 8$
$h(a_i) = 4 + i, \quad i = 10, 11 \qquad h(a_i) = 3 + i, \quad i = 13, 14$
$h(a_i) = 2 + i, \quad i = 16, 17 \qquad h(a_i) = 1 + i, \quad i = 19, 20$
$h(a_i) = i, \quad i = 22, 23 \qquad h(a_i) = i - 1, \quad i = 25, 26.$

(6) **Coding a Letter:** Here F and S are used to refer the first and second star respectively. T, P_i, and E_i denote the top vertex, the i^{th} pendent vertex and the i^{th} edge value in order. W denotes the wedge value.

For example FT denotes the number assigned to the top vertex of first star and SP_i denotes the number assigned to the i^{th} pendent vertex of second star respectively.

(7) **Coding(wordwise):**

TWINKLING	-	$FP_{10}SP_{13}SP_2SE_6FP_7FP_1FE_1FP_8SE_1$
STARS	-	$SP_{11}FP_{10}FE_7FE_4SE_9$
ARE	-	$SP_5FE_4FE_{10}$
SPRINKLING	-	$SP_{11}SE_7FE_4FE_1SE_6FP_7FP_1SP_2FP_8SE_1$
IN	-	SP_2FP_8
THE	-	$FP_{10}SE_2FE_{10}$
WHIRLPOOL	-	$SP_{13}SE_2FE_1FE_4FP_1SP_{10}FE_3FE_3FP_1$
GALAXY	-	$SE_1SP_5FE_2SP_5FE_6ST$

(8) **Horizontal string:**

$FP_{10}SP_{13}SP_2SE_6FP_7FP_1FE_1FP_8SE_1 * SP_{11}FP_{10}FE_7FE_4SE_9$
$* SP_5FE_4FE_{10} * SP_{11}SE_7FE_4FE_1SE_6FP_7FP_1SP_2FP_8SE_1 * SP_2FP_8$
$* FP_{10}SE_2FE_{10} * SP_{13}SE_2FE_1FE_4FP_1SP_{10}FE_3FE_3FP_1$
$* SE_1SP_5FE_2SP_5FE_6ST$

(9) **Picture Coding:**

3. ALGORITHM

An appropriate two star graph with wedge must be taken. A Hint, mathematical or non-mathematical to find the two star graph $K_{1,\lambda} \wedge K_{1,\mu}$ is specified which is to be used.

FIGURE 5. Picture Coding MTAO

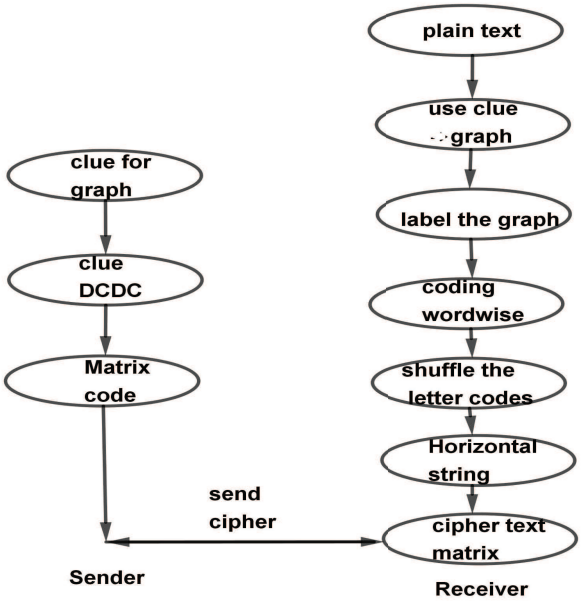

FIGURE 6. Flow Chart

4. CONCLUSION

We investigate Mean labeling on the two star graph for communicating some messages using a two methods of numbering of alphabets (FPNO) and (MTAO) and two methods for letter codings and a suitable picture coding are given in this paper. In Future we planned to develop GMJ code on two star graph with Mean labeling for other two cases.

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DEPARTMENT OF MATHEMATICS
THIRUVALLUVAR UNIVERSITY CONSTITUENT COLLEGE OF ARTS AND SCIENCE
TIRUPATTUR - 635 901

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE
B. S. ABDUR RAHMAN CRESCENT INSTITUTE OF SCIENCE AND TECHNOLOGY
CHENNAI - 600 048

DEPARTMENT OF MATHEMATICS
SACRED HEART COLLEGE
TIRUPATTUR - 635 601

DEPARTMENT OF MATHEMATICS
SACRED HEART COLLEGE
TIRUPATTUR - 635 601
E-mail address: pulibala70@gmail.com