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SEQUENCING OF n JOBS ON THREE MACHINES WITH ARBITRARY LAGS

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ABSTRACT. In this paper the concept of arbitrary lags (start lag and stop lag) in n-jobs, 3-machines flow shop scheduling problem involving the processing time associated with their respective probabilities and transportation time of jobs is being studied. The objective of the study is to propose an algorithm by which we can minimize the make-span in three stage flow shop scheduling problem. A numerical illustration is given to demonstrate the computational efficiency of proposed algorithm as a valuable analytical tool for the researchers.

1. INTRODUCTION

At present period, every manufacturer has to concentrate on innovative ways of production agenda to get success in today's competition of marketing area. For this one should use available resources fruitfully and plan a proper schedule so that the purpose of optimization of production can be achieved. The theory of scheduling is basically focused on Johnson's result of two machines. The analysis of all flow shop scheduling problems can be done by using this result. Proper planning of scheduling in production and marketing is effective in decreasing production price, superior quality of product to satisfy the demands of consumer in the competition of market. By the time lag we meant the minimum

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time delay which is required between the executions of two consecutive operations of the same job. The start lag ($D_i > 0$) is the minimum time which must elapse between starting job i on the first machine and starting it on the second machine.The stop lag ($E_i > 0$) for the job i is the minimum time which elapsed between the completion of it on second machine. [1] Johnson gave procedure for finding the optimal schedule for n-jobs, two machine flow-shop problem with minimization of the make span (i.e. total elapsed time) as the objective. Also Mitten and Johnson [2, 3] separately gave a solution algorithm of obtaining an optimal sequence of n-job, 2-machine flow shop scheduling problem in which each job involves arbitrary time lags (start-lags, stop-lags). Maggu and Das [4] gives solution algorithm of obtaining optimal sequence for an âĂŸn-job, 2- machine' flow shop scheduling problem where in each job involves transportation time.Yoshida and Hitomi [5] were among the first to investigate the flow shop where separated from processing times with their extension of Johnson's rule.There algorithm focused on an extension of Johnson's algorithm originally developed for a two-machine flow shop problem with setup times included. Singh T.P. [6] presents optimal n x 2 flow shop problem solving job block, Transportation times, Arbitrary time and Break-down machine time. Kern, W. and Nawjin, W.M. [7] present optimal production Scheduling with arbitrary time lags on a single machine. Riezebos, J. and Goalman, G.J.C. [8] discussed the flow shop problem with time lags.Singh, T.P. and Gupta, D. [9] worked on Optimal two stage production schedule with group jobs restrictions having set up times separated from processing time associated with probabilities. Gupta, D. and Kumar, R. [10] studied on three stage production schedule the processing time and set up times associated with probabilities including job block. Gupta, D. Bala, S. and Singla, P. [11] discussed on two stage open shop specially structured scheduling to minimize the rental cost, processing time associated with probabilities including transportation time. Gupta, D. Bala, S. Singla P. and Sharma S. [12] discussed on 3- stage specially structured flow shop scheduling to minimize the rental cost including transportation time, job weightage and job block criteria. Gupta, D. and Singh, H. [13] application of idle/ waiting time operator $O_{i,w}$ on two stage flow shop scheduling problem with arbitrary lags. Here we extend the work made by Mitten, L.G. and Johnson, S.M. [3]- [2] by taking three machines in which jobs are processed on these machines. Thus this paper is more wider and practically more applicable and has significant results in the process industry.

2. Problem formulation

2.1. **Assumptions.**

(1) No passing is allowed.
(2) Each operation once started must performed till completion.
(3) Jobs are independent to each other.
(4) No job may be processed by more than one machine at a time.

2.2. **Notations.**

S: Sequence of job 1, 2, 3,...., n.
M_i: Machine j, j=1, 2, 3,...., m.
A_i: Processing time of i^{th} job on machine M_1.
B_i: Processing time of i^{th} job on machine M_2.
C_i: Processing time of i^{th} job on machine M_3.
$A\alpha_i$: Expected processing time of i^{th} job on machine M_1.
$B\alpha_i$: Expected processing time of i^{th} job on machine M_2.
$C\alpha_i$: Expected processing time of i^{th} job on machine M_3.
p_i: Probability associated to the processing time A_i of i^{th} job on machine M_1.
q_i: Probability associated to the processing time B_i of i^{th} job on machine M_2.
r_i: Probability associated to the processing time C_i of i^{th} job on machine M_3.
D_{i1}: Start lag of i^{th} job from machine M_1 to M_2.
E_{i1}: Stop lag of i^{th} job from machine M_1 to M_2.
D_{i2}: Start lag of i^{th} job from machine M_2 to M_3.
E_{i2}: Stop lag of i^{th} job from machine M_2 to M_3.
t_i: Transportation time of i^{th} job from M_1 machine to M_2 machine.
g_i: Transportation time of i^{th} job from M_2 machine to M_3 machine.
t'_i: Effective transportation time of i^{th} job from M_1 machine to M_2 machine.
g'_i: Effective transportation time of i^{th} job from M_2 machine to M_3 machine.

jobs	Machine M_1		Transportation Time	Machine M_2		Transportation Time	Machine M_3		*Start* lag		*Stop* lag	
I	A_i	p_i	t_i	B_i	q_i	g_i	C_i	r_i	D_{i1}	D_{i2}	E_{i1}	E_{i2}
1.	A_1	p_1	t_1	B_1	q_1	g_1	C_1	r_1	D_{11}	D_{12}	E_{11}	E_{12}
2.	A_2	p_2	t_2	B_2	q_2	g_2	C_2	r_2	D_{21}	D_{22}	E_{21}	E_{22}
3.	A_3	p_3	t_3	B_3	q_3	g_3	C_3	r_3	D_{31}	D_{32}	E_{31}	E_{32}
....	....	....	....	....	....	....	....	....	....	....	....	....
n.	A_n	p_n	t_n	B_n	q_n	g_n	C_n	r_n	D_{n1}	D_{n2}	E_{n1}	E_{n2}

Obtained optimal sequence of jobs so as minimize the make-span.

3. Algorithm

Step 1: Define expected processing time $A\alpha_i$, $B\alpha_i$ and $C\alpha_i$ on machine M_1, M_2 and M_3 respectively as follows:

1. $A\alpha_i = A_i \times p_i$
2. $B\alpha_i = B_i \times q_i$
3. $C\alpha_i = C_i \times r_i$

Step 2: Define effective transportation time t'_i and g'_i of i^{th} job from machine M_1 to M_2 and machine M_2 to M_3 respectively as follows:

1. $t'_i = \max(D_{i1} - A\alpha_i, E_{i1} - B\alpha_i, t_i)$
2. $g'_i = \max(D_{i2} - B\alpha_i, E_{i2} - C\alpha_i, g_i)$

Step 3: Compute processing time by creating two factitious machines G and H with their processing time G_i and H_i respectively as follows:

$G_i = |A\alpha_i + B\alpha_i + t'_i + g'_i|$ and $H_i = |B\alpha_i + C\alpha_i + t'_i + g'_i|$

if, either min$(A\alpha_i + t'_i) \geq \max(B\alpha_i + t'_i)$ or min$(C\alpha_i + g'_i) \geq \max(B\alpha_i + g'_i)$. **Step 4:** Apply Johnson's (1954) technique to obtain the optimal string S_i for the new reduced problem obtained in step 3.

Step 5: Compute the In-Out table for the sequence obtained in step 4.

4. Numerical Illustration

Consider the 5 - jobs and 3 - machines problem with processing time associated with their respective probabilities, transportation time and arbitrary lags (start lag and stop lag) of jobs given as:

jobs	Machine M_1		Transportation Time	Machine M_2		Transportation Time	Machine M_3		*Start* lag		*Stop* lag	
I	A_i	p_i	t_i	B_i	q_i	g_i	C_i	r_i	D_{i1}	D_{i2}	E_{i1}	E_{i2}
1.	40	.3	5	30	.2	4	40	.1	18	12	13	11
2.	55	.2	4	40	.2	3	60	.1	14	9	11	10
3.	40	.2	7	30	.1	2	35	.2	15	11	9	12
4.	35	.2	6	25	.2	5	40	.2	8	6	7	9
5.	80	.1	5	20	.3	4	15	.4	12	8	7	8

Obtained optimal sequence of jobs so as minimize the make-span.

Solution:

Step1:First of all we find the expected processing time by multiplying the processing time with their respective probabilities.

Jobs	Machine M_1	Transportation Time	Machine M_2	Transportation Time	Machine M_3	*Start* lag		*Stop* lag	
I	$A\alpha_i$	t_i	$B\alpha_i$	g_i	$C\alpha_i$	D_{i1}	D_{i2}	E_{i1}	E_{i2}
1.	12	5	6	4	4	18	12	13	11
2.	11	4	8	3	6	14	9	11	10
3.	8	7	3	2	7	15	11	9	12
4.	7	6	5	5	8	8	6	7	9
5.	8	5	6	4	6	12	8	7	8

Step 2: Define effective transportation time t'_i and g'_i of i^{th} job from machine M_1 to M_2 and machine M_2 to M_3 respectively as follows:

1. $t'_i = \max(D_{i1} - A\alpha_i, E_{i1} - B\alpha_i, t_i)$ 2. $g'_i = \max(D_{i2} - B\alpha_i, E_{i2} - C\alpha_i, g_i)$

Jobs	Machine M_1	Effective Transportation Time	Machine M_2	Effective Transportation Time	Machine M_3
I	$A\alpha_i$	t'_i	$B\alpha_i$	g'_i	$C\alpha_i$
1.	12	7	6	7	4
2.	11	4	8	4	6
3.	8	7	3	8	7
4.	7	6	5	5	8
5.	8	5	6	4	6

Step 3: Now we check the condition discussed in the algorithm, and here $\min(A\alpha_i + t'_i) \geq \max(B\alpha_i + t'_i)$ satisfied. So we create two factious machines G and H with their processing time G_i and H_i as follows:

$G_i = |A\alpha_i + B\alpha_i + t'_i + g'_i|$ and $H_i = |B\alpha_i + C\alpha_i + t'_i + g'_i|$

Jobs	Factious Machine G	Factious Machine H
I	G_i	H_i
1.	32	24
2.	27	22
3.	26	25
4.	23	24
5.	23	21

Step 4: Now by using Johnson's technique, optimal sequence is S=4,3,1,2,5.
Step 5: In-Out table for sequence S is as follows:

Jobs	Machine M_1	Effective Transportation Time	Machine M_2	Effective Transportation Time	Machine M_3
I	In-Out	t'_i	In-Out	g'_i	In-Out
4.	0-7	6	13-18	5	23-31
3.	7-15	7	22-25	8	33-40
1.	15-27	7	34-40	7	47-51
2.	27-38	4	42-50	4	54-60
5.	38-46	5	51-57	4	61-67

Minimum Make-Span for the given problem is 67 units.

5. Conclusion

The present study deal with the flow shop scheduling problem with the main idea to minimize the total elapsed time of jobs. The work can be extended by considering various parameters such as equivalent job block, setup times separated from processing time, break-down interval, mean weightage time etc.

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