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ENHANCED DELAY DEPENDENT STABILITY CRITERIA FOR NEURAL NETWORKS WITH TIME VARYING DELAY

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ABSTRACT. This paper is concerned with stability analysis of neural networks with time varying delay in a given range. The relationship between time varying delay and its lower and upper bounds are taken into consideration while calculating upper bound of the Lyapunov functional derivative. By constructing more general type of Lyapunov functional and employing integral limits containing the lower and upper bound of time delay on activation function, some new less conservative stability criteria are developed in terms of Linear matrix inequality. Finally two numerical examples are used to show the effectiveness and less conservatism of the proposed theorem.

1. INTRODUCTION

Neural networks have found applications in many fields such as as signal processing, image decryption, pattern recognition, associative memories, fixed-point computations, optimization, feedback control, medical diagnosis, and financial applications [1]. Time-delays will be often the source of instability. So, the stability analysis of neural networks with time varying delays has drawn considerable attention. According to the information on delays the stability criteria can be classified as delay dependent or delay independent. Since delay

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independent stability criteria tends to more conservative than the delay dependent many efforts have been paid to derive delay-dependent stability criteria for neural networks with time-delays. For the delay-dependent stability criteria of neural networks with time delays, the main purpose is to obtain a maximum value of the admissible delay such that the concerned systems are asymptotically stable.

One of the important methods to analyze the stability of Neural network is Lyapunov krasovskii method. So for most of the derived results have been based on the Lyapunov stability theory. There are mainly two ways to reduce the conservatism of the derived stability criteria through the Lyapunov approach. The first is based on constructing suitable Lyapunov functionals, and the other is on estimating the derivatives of the Lyapunov functionals as tight as possible [11-14]. For the later one, researchers have mainly focused on developing new techniques such as free-weighting matrices techniques [6], Park's inequality [16], multiple integral approach [17], model transformation [18], convex combination technique [19], reciprocally convex optimization and delay partitioning approach [3, 9, 15]. This paper investigates the stability analysis of neural network with constructing new Lyapunov functional which contains information on the lower bound of delay h_1 and upper bound h_2. Some new delay dependent stability criteria derived in terms of linear matrix inequality. The newly derived criteria gives less conservatism, finally two numerical examples are given to demonstrate the effectiveness of the proposed method.

Notations:
In this paper, R^n denotes the n-dimensional Euclidean space and R^{nxm} is the set of real Matrices. X>0 denotes that the matrix X is a real positive semi definite matrix. * in a matrix represents the elements below the main diagonal of a symmetric matrix. $SymX = X + X^T$. The superscript 'T' denotes the transpose of the matrix. diag$\{\dots\}$ denotes the block diagonal matrix.

2. PROBLEM FORMULATION

Consider the following neural network with an interval time varying delay:

$$\dot{x}\,(t) = -Ax\,(t) + B_1 f\,(x\,(t)) + B_2 f\,(x\,(t - h\,(t)))\,, \tag{2.1}$$

where

$$x(t) = [x_1(t), x_2(t), \dots x_n(t)]^T, f(x(t)) = [f(x_1(t)), f(x_2(t)), \dots, f(x_n(t))]^T$$

represents the neuron state vector and neuron activation function respectively. $A = diag(a_1, a_2, \dots .a_n)$ and $B_1, B_2 \varepsilon R^{n\times n}$ are the known interconnection weight matrices and the time delay h(t) is a continuous differentiable function satisfying $h_1 \le h(t) \le h_2$, $\dot{h}(t) \le \mu$ where h_1, h_2 and μ are known constants. The neuron activation function is assumed to be bounded and satisfy the following assumption.

Assumption 2.1: The activation function f_i (.), i=1, 2,....n is continuous and satisfies the condition

$$l_i^- \le \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \le l_i^+, \forall s_1 \ne s_2, i = 1, 2, \dots, n, \tag{2.2}$$

where l_i^- and l_i^+ are constants.

Lemma 2.1. *: (Auxillary function based integral inequality [4]) Let x be a differentiable signal in $[a,b] \to R^n$ for a positive definite matrix $R \varepsilon R^{n\times n}$, the following inequality holds:*

$$(b-a)\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \ge \chi_1^T R\chi_1 + \chi_2^T R\chi_2 + \chi_3^T R\chi_3,$$

where χ_1, χ_2 and χ_3 are defined as

$$\chi_1 = x(b) - x(a), \quad \chi_2 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds$$

and

$$\chi_3 = \chi_1 + \frac{6}{b-a}\int_a^b x(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_u^b x(s)dsdu.$$

Theorem 2.1. *For given scalars h_1 , h_2 and μ the system (2.1) is asymptotically stable if there exists positive definite symmetric matrices P ε $R^{n\times n}$, E,F,Gε $R^{2n\times 2n}$, R_1, R_2 ε $R^{n\times n}$ and diagonal matrices $H_i, U_i, \lambda_i \varepsilon R^{n\times n}(i = 1, 2, 3, 4)$ such that the* *following LMI hold:*

(2.3)

$$\Pi=\begin{bmatrix}
\Pi_{1,1} & \Pi_{1,2} & \Pi_{1,3} & 0 & \Pi_{1,5} & \Pi_{1,6} & \Pi_{1,7} & 0 & \Pi_{1,9} & 0 & 0 & \Pi_{1,12} & 0 & 0 \\
* & \Pi_{2,2} & \Pi_{2,3} & 0 & \Pi_{2,5} & \Pi_{2,6} & \Pi_{2,7} & 0 & \Pi_{2,9} & \Pi_{2,10} & 0 & \Pi_{2,12} & 0 & \Pi_{2,14} \\
* & * & \Pi_{3,3} & \Pi_{3,4} & \Pi_{3,5} & \Pi_{3,6} & \Pi_{3,7} & \Pi_{3,8} & 0 & \Pi_{3,10} & \Pi_{3,11} & 0 & \Pi_{3,13} & \Pi_{3,14} \\
0 & 0 & * & \Pi_{4,4} & 0 & 0 & \Pi_{4,7} & \Pi_{4,8} & 0 & 0 & \Pi_{4,11} & 0 & \Pi_{4,13} & 0 \\
* & * & * & 0 & \Pi_{5,5} & \Pi_{5,6} & \Pi_{5,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & * & \Pi_{6,6} & \Pi_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Pi_{7,7} & \Pi_{7,8} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & * & * & 0 & 0 & * & \Pi_{8,8} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{9,9} & 0 & 0 & \Pi_{9,12} & 0 & 0 \\
0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{10,10} & 0 & 0 & 0 & \Pi_{10,14} \\
0 & 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 & \Pi_{11,11} & 0 & \Pi_{11,13} & 0 \\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & \Pi_{12,12} & 0 & 0 \\
0 & 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & \Pi_{13,13} & 0 \\
0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & \Pi_{14,14}
\end{bmatrix}<0$$

where

$$\begin{aligned}
\prod\nolimits_{1,1} &= sym\left\{-PA + L_m\left(\lambda_1+\lambda_3+\lambda_5\right)A - L_p\left(\lambda_2+\lambda_4+\lambda_6\right)A\right\} + E_{11} \\
&\quad + A^T\left(h_1^2R_1 + h_{12}^2R_2\right)A - L_m\left(H_1+U_1+U_4\right)L_p - 9R_1 \\
\prod\nolimits_{1,2} &= 3R_1 - L_mU_1L_p - \left(L_mU_1L_p\right)^T; \ \prod\nolimits_{1,3} = -L_mU_4L_p - \left(L_mU_4L_p\right)^T; \\
\prod\nolimits_{1,5} &= PB_1 + E_{12} - L_m(\lambda_1+\lambda_3+\lambda_5)B_1 + L_p(\lambda_2+\lambda_4+\lambda_6)B_1 \\
&\quad - A^T(h_1^2R_1 + h_{12}^2R_2)B_1 + L_m(H_1+U_1+U_4) + L_p(H_1+U_1+U_4) \\
&\quad - (\lambda_1+\lambda_3+\lambda_5)A + (\lambda_2+\lambda_4+\lambda_6)A \\
\prod\nolimits_{1,6} &= -L_mU_1 - L_pU_1; \\
\prod\nolimits_{1,7} &= PB_2 - L_m\left(\lambda_1+\lambda_3+\lambda_5\right)B_2 + L_p\left(\lambda_2+\lambda_4+\lambda_6\right)B_2 \\
&\quad - A^T\left(h_1^2R_1 + h_{12}^2R_2\right)B_2 - L_mU_4 - L_pU_4; \\
\prod\nolimits_{1,9} &= -24R_1; \ \prod\nolimits_{1,12} = 60R_1; \\
\prod\nolimits_{2,2} &= F_{11} - E_{11} - 9R_1 - 9R_2 - Sym\left\{L_m(H_2+U_1+U_2)L_p\right\}; \\
\prod\nolimits_{2,3} &= 3R_2 + L_mU_2L_p + (L_mU_2L_p)^T; \ \prod\nolimits_{2,5} = -L_mU_1 - L_pU_1; \\
\prod\nolimits_{2,6} &= F_{12} - E_{12} + L_m(H_2+U_1+U_2) + L_p(H_2+U_1+U_2); \\
\prod\nolimits_{2,7} &= -L_mU_2 - L_pU_2; \ \prod\nolimits_{2,9} = 36R_1; \ \prod\nolimits_{2,10} = -24R_2; \\
\prod\nolimits_{2,12} &= -60R_1; \ \prod\nolimits_{2,14} = 60R_2; \\
\prod\nolimits_{3,3} &= (1-\mu)(G_{11} - F_{11}) - 18R_2 - Sym\left\{L_m(H_3+U_2+U_3+U_4)L_p\right\} \\
\prod\nolimits_{3,4} &= 3R_2 - L_mU_3L_p - (L_mU_3L_p)^T; \ \prod\nolimits_{3,5} = -L_mU_4 - L_pU_4;
\end{aligned}$$

$\prod_{3,6} = -L_mU_2 - L_pU_2;$
$\prod_{3,7} = (1-\mu)(G_{12} - F_{12}) + L_m(H_3 + U_2 + U_3 + U_4) + L_p(H_3 + U_2 + U_3 + U_4);$
$\prod_{3,8} = -L_mU_3 - L_pU_3;\ \prod_{3,10} = 36R_2;\ \prod_{3,11} = -24R_2;\ \prod_{3,13} = 60R_2;$
$\prod_{3,14} = -60R_2;\ \prod_{4,4} = -G_{11} - 9R_2 - Sym\left\{L_m(H_4 + U_3)\right\};$
$\prod_{4,7} = -L_mU_3 - L_pU_3;\ \prod_{4,8} = -G_{12} + L_m(H_4 + U_3) + L_p(H_4 + U_3)$
$\prod_{4,11} = 36R_2;\ \prod_{4,13} = -60R_2;$
$\prod_{5,5} = Sym\left\{B_1(\lambda_1 + \lambda_3 + \lambda_5) - B_1(\lambda_2 + \lambda_4 + \lambda_6) - (H_1 + U_1 + U_4)\right\}$
$\quad +B_1^T(h_1^2R_1 + h_{12}^2R_2)B_1 + E_{22};$
$\prod_{5,6} = 2U_1;$
$\prod_{5,7} = B_2(\lambda_1 + \lambda_3 + \lambda_5) - B_2(\lambda_2 + \lambda_4 + \lambda_6) + B_1^T(h_1^2R_1 + h_{12}^2R_2)B_2 + 2U_4;$
$\prod_{6,6} = F_{22} - E_{22} - H_2 - H_2^T - U_1 - U_1^T - U_2 - U_2^T;\ \prod_{6,7} = 2U_2;$
$\prod_{7,7} = B_2^T(h_1^2R_1 + h_{12}^2R_2)B_2 - Sym(H_3 + U_2 + U_3 + U_4) + (1-\mu)(G_{22} - F_{22});$
$\prod_{7,8} = 2U_3;\ \prod_{8,8} = -Sym(H_4 + U_3) - G_{22};\ \prod_{9,9} = -192R_1;\ \prod_{9,12} = 360R_1;$
$\prod_{10,10} = -192R_2;\ \prod_{10,14} = 360R_2;\ \prod_{11,11} = -192R_2;\ \prod_{11,13} = 360R_2;$
$\prod_{12,12} = -720R_1;\ \prod_{13,13} = -720R_2;\ \prod_{14,14} = -720R_2;$

where $h_{12} = h_2 - h_1;\ \tilde{h}_2 = h_2 - h(t);\ \tilde{h}_1 = h(t) - h_1.$

Proof. Consider the following Lyapunov Krasovskii Functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),$$

where

$$V_1(t) = x^T(t)Px(t)$$
$$\begin{aligned}V_2(t) = {} & 2\sum_{i=1}^n \left\{\lambda_{1i}\int_0^{x_i(t)}(f_i(s) - l_i^-)ds + \lambda_{2i}\int_0^{x_i(t)}(f_i(s) - l_i^+)ds\right\}\\ & +2\sum_{i=1}^n \left\{\lambda_{3i}\int_0^{x_i(t-h_1)}(f_i(s) - l_i^-)ds + \lambda_{4i}\int_0^{x_i(t-h_1)}(f_i(s) - l_i^+)ds\right\}\\ & +2\sum_{i=1}^n \left\{\lambda_{5i}\int_0^{x_i(t-h_2)}(f_i(s) - l_i^-)ds + \lambda_{6i}\int_0^{x_i(t-h_2)}(f_i(s) - l_i^+)ds\right\}\end{aligned}$$
$$V_3(t) = \int_{t-h_1}^{t}\eta^T(s)E\eta(s)ds + \int_{t-h(t)}^{t-h_1}\eta^T(s)F\eta(s)ds + \int_{t-h_2}^{t-h(t)}\eta^T(s)G\eta(s)ds$$

where

$$\eta(t) = col\left[x(t), f(x(t))\right]$$
$$V_4(t) = h_1\int_{t-h_1}^{t}\int_u^t \dot{x}^T(s)R_1\dot{x}(s)dsdu + h_{12}\int_{t-h_2}^{t-h_1}\int_u^t \dot{x}^T(s)R_2\dot{x}(s)dsdu.$$

Calculating the time derivative of V(t) along the given system yields

$$\dot{V}_1(t) = 2x^T(t)P\left[-Ax(t) + B_1f(x(t)) + B_2f(x(t-h(t)))\right] \tag{2.4}$$

$$\begin{aligned}\dot{V}_2(t) = {} & 2\left[f^T(x(t))\left[(\lambda_1 + \lambda_3 + \lambda_5) - (\lambda_2 + \lambda_4 + \lambda_6)\right]\dot{x}(t)\right]\\ & +2\left[x^T(t)[L_p(\lambda_2 + \lambda_4 + \lambda_6) - L_m(\lambda_1 + \lambda_3 + \lambda_5)]\dot{x}(t)\right]\end{aligned} \tag{2.5}$$

$$\dot{V}_3(t) = \eta^T(t)E\eta(t) + \eta^T(t-h_1)(F-E)\eta(t-h_1) - \eta^T(t-h_2) \\ G\eta(t-h_2) + (1-h^{\cdot}(t))\eta^T(t-h(t))(G-F)\eta(t-h(t)) \tag{2.6}$$

$$\dot{V}_4(t) = h_1^2\dot{x}^T(t)R_1\dot{x}(t) - h_1\int_{t-h_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds + \\ h_{12}^2\dot{x}^T(t)R_2\dot{x}(t) - h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)ds \tag{2.7}$$

By utilizing the lemma 2.1 to the above integrals we have

$$-h_1\int_{t-h_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \leq -\begin{bmatrix} x(t) \\ x(t-h_1) \\ \frac{2}{h_1}\int_{t-h_1}^{t}x(s)ds \\ \frac{12}{h_1^2}\int_{t-h_1}^{t}\int_{u}^{t}x(s)dsdu \end{bmatrix}^T$$

$$\begin{bmatrix} 9R_1 & -3R_1 & 12R_1 & -5R_1 \\ * & 9R_1 & -18R_1 & 5R_1 \\ * & * & 48R_1 & -15R_1 \\ * & * & * & 5R_1 \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h_1) \\ \frac{2}{h_1}\int_{t-h_1}^{t}x(s)ds \\ \frac{12}{h_1^2}\int_{t-h_1}^{t}\int_{u}^{t}x(s)dsdu \end{bmatrix} \tag{2.8}$$

Observe that

$$-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)ds \leq -h_{12}\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)ds \\ -h_{12}\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)ds.$$

Since $h_{12} \geq h_2 - h(t)$ and by Lemma 2.1,

$$-h_{12}\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)ds \leq -\begin{bmatrix} x(t-h(t)) \\ x(t-h_2) \\ \frac{2}{\tilde{h}_2}\int_{t-h_2}^{t-h(t)}x(s)ds \\ \frac{12}{\tilde{h}_2^2}\int_{t-h_2}^{t-h(t)}\int_{u}^{t-h(t)}x(s)dsdu \end{bmatrix}^T$$

$$\begin{bmatrix} 9R_2 & -3R_2 & 12R_2 & -5R_2 \\ * & 9R_2 & -18R_2 & 5R_2 \\ * & * & 48R_2 & -15R_2 \\ * & * & * & 5R_2 \end{bmatrix}\begin{bmatrix} x(t-h(t)) \\ x(t-h_2) \\ \frac{2}{\tilde{h}_2}\int_{t-h_2}^{t-h(t)}x(s)ds \\ \frac{12}{\tilde{h}_2^2}\int_{t-h_2}^{t-h(t)}\int_{u}^{t-h(t)}x(s)dsdu \end{bmatrix} \tag{2.9}$$

In the same manners one infers that

$$-h_{12}\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)ds \le -\begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ \frac{2}{\tilde{h}_1}\int_{t-h(t)}^{t-h_1}x(s)ds \\ \frac{12}{\tilde{h}_1^2}\int_{t-h(t)}^{t-h_1)}\int_{u}^{t-h_1}x(s)dsdu \end{bmatrix}^T$$

$$\begin{bmatrix} 9R_2 & -3R_2 & 12R_2 & -5R_2 \\ * & 9R_2 & -18R_2 & 5R_2 \\ * & * & 48R_2 & -15R_2 \\ * & * & * & 5R_2 \end{bmatrix}\begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ \frac{2}{\tilde{h}_1}\int_{t-h(t)}^{t-h_1}x(s)ds \\ \frac{12}{\tilde{h}_1^2}\int_{t-h(t)}^{t-h_1)}\int_{u}^{t-h_1}x(s)dsdu \end{bmatrix} \tag{2.10}$$

By the assumption of activation function (2.2) we have

$\alpha_i(S) : 2[L_m x(s) - f(x(s))]^T H_i[f(x(s)) - L_p x(s)] \ge 0$

$u_i(S_1, S_2) : 2[L_m(x(s_1) - x(s_2)) - (f(x(s_1)) - f(x(s_2)))]^T$

$U_i[(f(x(s_1)) - f(x(s_2))) - L_p(x(s_1) - x(s_2))] \ge 0,$

where $H_i = diag[\alpha_{1i}, \alpha_{2i}, ..., \alpha_{ni}] \ge 0, U_i = diag[u_{1i}, u_{2i}, \dots, u_{ni}] \ge 0, i = 1, 2, 3, 4.$

Then the following inequalities hold

$$\alpha_1(t) + \alpha_2(t-h_1) + \alpha_3(t-h(t)) + \alpha_4(t-h_2) \ge 0 \tag{2.11}$$

$$u_1(t, t-h_1) + u_2(t-h_1, t-h(t)) + u_3(t-h(t), t-h_2) + u_4(t, t-h(t)) \ge 0 \tag{2.12}$$

combining the equations (2.4)-(2.12) we get $\dot{V}(t) \le \xi^T(t)\prod\xi(t),$ where $\prod$ is defined in (2.3) and

$$\begin{aligned}\xi^T(t) = [&x^T(t), x^T(t-h_1), x^T(t-h(t)), x^T(t-h_2), f^T(x(t)), f^T(x(t-h_1)), \\ &f^T(x(t-h(t))), f^T(x(t-h_2)), \frac{1}{h_1}\int_{t-h_1}^{t}x^T(s)ds, \frac{1}{\tilde{h}_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds, \\ &\frac{1}{\tilde{h}_2}\int_{t-h_2}^{t-h(t)}x^T(s)ds, \frac{1}{h_1^2}\int_{t-h_1}^{t}\int_{u}^{t}x^T(s)dsdu, \\ &\frac{1}{\tilde{h}_2^2}\int_{t-h_2}^{t-h(t)}\int_{u}^{t-h(t)}x^T(s)dsdu, \frac{1}{\tilde{h}_1^2}\int_{t-h(t)}^{t-h_1}\int_{u}^{t-h_1}x^T(s)dsdu].\end{aligned}$$

Thus, if $\prod < 0$, the system (2.1) is asymptotically stable. This completes the proof. □

3. Numerical Examples

This section provides two numerical examples to show that the proposed results are less conservative than some existing ones.

Example 1. *Consider the system* $\dot{x}(t) = -Ax(t) + B_1 f(x(t)) + B_2 f(x(t - h(t)))$, *where*

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B_1 = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix} B_2 = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix}.$$

Neuron activation function are assumed to satisfy with $L_m = diag(0,0)$ $L_p = diag(0.4, 0.8)$. *The maximum delay bounds for guaranteeing the asymptotic stability of the given system with various* h_1 *and* μ *are listed in Table1 including the results of [7], [8] and our method.*

Example 2. *Consider the system* $\dot{x}(t) = -Ax(t) + B_1 f(x(t)) + B_2 f(x(t - h(t)))$, *where*

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} B_1 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} B_2 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}.$$

Neuron activation function are assumed to satisfy with $L_m = diag(0,0)$ $L_p = diag(0.4, 0.8)$. *It is seen from Table 2 that the results obtained by our method are less conservative than those of [2] ,[5] and [8].*

TABLE 1. Upper bounds h_2 for various h_1 and μ

h_1	Methods	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
0.5	[7]	0.8262	0.8215	0.8183
	[8]	1.1217	0.9984	0.9037
	Theorem 2.1	1.1368	1.1304	1.1281
0.75	[7]	0.9669	0.9625	0.9592
	[8]	1.2213	1.1021	1.0102
	Theorem 2.1	1.3319	1.3202	1.1532
1	[7]	1.1152	1.1108	1.1075
	[8]	1.3432	1.2238	1.1318
	Theorem 2.1	1.5816	1.5301	1.5181

TABLE 2. Upper bounds h_2 for various h_1 and μ

h_1	Methods	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
0	[2]	1.2281	0.8639	0.8298
	[5]	1.6831	1.1493	------
	[8]	1.6831	1.1494	1.0880
	Theorem 2.1	2.0509	1.1615	1.1605
1	[8]	2.5967	2.0443	1.9621
	Theorem 2.1	2.7987	2.1321	2.1223
100	[8]	101.5946	101.0443	100.9621
	Theorem 2.1	101.7987	101.1321	101.1223

4. CONCLUSION

This paper investigates stability problem of neural networks with time varying delay in a given range. By constructing more general type of Lyapunov functional and employing integral limits containing the lower and upper bound of delay on activation function some new less conservative stability criteria are developed in terms of Linear matrix inequality. Finally two numerical examples are given to show the effectiveness of the proposed theorem.

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